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Design diagrams for wavelength discrepancy in tank testing with inconsistently scaled intermediate water depth

Donald R. Noble [a,b,*], Samuel Draycott [a], Thomas A.D. Davey [a], Tom Bruce [b]

[a] *FloWave Ocean Energy Research Facility, The University of Edinburgh, Edinburgh EH9 3BF, United Kingdom*
[b] *School of Engineering, The University of Edinburgh, Edinburgh EH9 3FB, United Kingdom*

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A B S T R A C T

The well-known dispersion relation links the length and period of a water wave with the depth in which it propagates. When model testing in tanks, the water depth should be consistently scaled to correctly replicate the waves. While this is done routinely by scaling foreshore bathymetry in coastal engineering physical model studies, and is not significant for deep water scenarios, this is not always considered when testing marine renewable energy devices, which are often in intermediate depth. Where water depth is not scaled consistently there will be resulting errors in wave parameters including wavelength, steepness, celerity, group velocity, and power. Design diagrams are presented to quantify and visualise these discrepancies over a typical range for testing offshore renewable energy devices. This design tool will facilitate experimental planning, quantification of uncertainties, and correlation of model test results with field data.

1. Introduction

When re-creating waves in a tank using Froude scaling laws, it is important to consider the depth ratio between the deployment site of interest and the tank, and to scale this correctly where possible. There will of course be instances where this is not possible, for example the deployment site water depth is not known, the tank has a fixed depth, or there are other constraints on the model size.

Whilst it is common knowledge that the wavelength of a water wave is a function of water depth, there has been little published regarding the incorrect scaled reproduction of wavelength resulting from inconsistent depth scaling. This issue was mentioned in [1], and expanded upon in [2], but the authors are not aware of other published discussions.

A number of authors, including [3,4], highlight the issue of scaling water depth when tank testing in the context of modelling mooring systems. These do not, however, highlight the consequences for wavelength error. Water depth scaling in relation to distorted hydraulic models is discussed in [5], noting that these models cannot be used for the study of water waves, as wavelength depends on water depth. For similitude in waves, the horizontal and vertical scales must be the same.

This technical note highlights the potential discrepancy in wavelength, group velocity, and power by restating the relevant aspects of small-amplitude wave theory. A method for calculating and visualising the errors resulting from incorrectly scaling the water depth when testing is then presented. This is followed by a brief discussion of implications for testing, focusing on marine renewable energy converters, which may be particularly sensitive to this issue.

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[*] Corresponding author at: FloWave Ocean Energy Research Facility, The University of Edinburgh, Edinburgh EH9 3BF, United Kingdom.
E-mail address: D.Noble@ed.ac.uk (D.R. Noble).

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2. Background theory

When re-creating waves in a test tank, the Froude scaling law is used to match the ratio of inertial to gravitational forces that dominate this problem. The ratio of depth at the site of interest to the tank depth is important because gravity waves in water of finite depth can only be correctly re-created when the water depth is also scaled. The wavelength, celerity, and group velocity are all influenced by water depth, which in turn affect wave steepness and power. If the depth is not correctly scaled, this will lead to frequency dependent errors in these parameters, as discussed below. This situation may arise from constraints in the test facility, or from the deployment site depth not being known or considered when the model testing was being conducted.

It is well known that the properties of water waves are related by the dispersion relation, Eq. (1)

$$\omega^2 = gk\tanh(kh) \tag{1}$$

where ω is rotational frequency, k the wavenumber, g acceleration due to gravity, and h water depth. The dispersion relation can also be expressed in terms of period T, and wavelength L, using Eq. (2) to obtain Eq. (3).

$$\omega = 2\pi/T, \quad k = 2\pi/L \tag{2}$$

$$\Rightarrow L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi h}{L}\right) \tag{3}$$

Eq. (3) gives a unique relationship between the three quantities of wavelength, period, and depth. Therefore, if the depth is not scaled consistently, then wavelength will also be incorrect for a given Froude scaled period.

The wavelength at a site L_{site} can be calculated from wave period and depth using the dispersion relation Eq. (3), expressed here in terms of wavelength and period at the site.

$$L_{\text{site}} = \frac{gT_{\text{site}}^2}{2\pi}\tanh\left(\frac{2\pi h_{\text{site}}}{L_{\text{site}}}\right) \tag{4}$$

Using Froude scaling, where λ is the scale factor, these properties at tank scale should thus be Eq. (5) giving Eq. (6).

$$T_{\text{tank}} = T_{\text{site}}\sqrt{\lambda}, \quad h_{\text{tank}} = h_{\text{site}}\lambda, \quad L_{\text{tank}} = L_{\text{site}}\lambda \tag{5}$$

$$L_{\text{tank}} = \frac{gT_{\text{site}}^2\lambda}{2\pi}\tanh\left(\frac{2\pi h_{\text{site}}\lambda}{L_{\text{tank}}}\right) \tag{6}$$

However, if the tank depth is not correctly scaled, $h_{\text{tank}} \neq h_{\text{site}}\lambda$, the (incorrect) wavelength in the tank $L_{\text{tank}*}$will instead be given by Eq. (7), assuming the period is correctly Froude scaled.

$$L_{\text{tank}*} = \frac{gT_{\text{site}}^2\lambda}{2\pi}\tanh\left(\frac{2\pi h_{\text{tank}}}{L_{\text{tank}*}}\right) \tag{7}$$

The error in wavelength ε_L is taken as the ratio of wavelength actually generated in the tank $L_{\text{tank}*}$ to the correctly scaled wavelength $L_{\text{tank}} = L_{\text{site}}\lambda$.

$$\varepsilon_L \equiv \frac{L_{\text{tank}*}}{L_{\text{tank}}} \tag{8}$$

The group velocity of a wave C_g is a more complex function of wavelength and water depth, given by Eq. (9).

$$C_g = \frac{1}{2}\sqrt{\frac{gL}{2\pi}\tanh\left(\frac{2\pi h}{L}\right)}\left[1 + \frac{4\pi h}{L\sinh\left(\frac{4\pi h}{L}\right)}\right] \tag{9}$$

The error in group velocity can be computed in the same manner, by calculating $C_{g,\text{tank}*}$ based on the wavelength actually generated in the tank, and $C_{g,\text{tank}}$ from the correctly scaled wavelength. The error is simply the ratio between these, Eq. (10)

$$\varepsilon_{Cg} \equiv \frac{C_{g,\text{tank}*}}{C_{g,\text{tank}}} \tag{10}$$

The speed, or celerity, of an individual wave is given by $C = L/T$, and steepness by $S = H/L$. Provided the period and height are correctly Froude scaled, the corresponding relative errors in celerity and steepness will thus equal the error in wavelength. As wave power $P = E_A C_g$, where E_A is wave energy per unit horizontal area [5], the relative error in power will equal that of group velocity. Any discrepancy in wave power is particularly important in tank testing wave energy converters, as wave power (kW/m) is scaled by $\lambda^{2.5}$, magnifying the projected full scale power discrepancy.

3. Graphical visualisation of errors

Whilst the theory covered in Section 2 is not new knowledge, a method to calculate and visualise these discrepancies has been developed in order to assist with test design and aid understanding of this potential source of error. This method utilises a new term 'scale depth discrepancy' (*SDD*), defined as Eq. (11), and shown graphically in Fig. 1.

$$SDD \equiv \lambda \frac{h_{\text{site}}}{h_{\text{tank}}} \tag{11}$$

This term aggregates the scale factor plus relative water depths between deployment site and tank into one variable. A value of *SDD* less than unity corresponds to the tank being too deep for the scaled site depth, resulting from a relatively shallower deployment site and/or a smaller model scale.

The frequency dependent errors resulting from this discrepancy can then be plotted over the range of *SDD* and non-dimensional tank-scale period, Eq. (12).

$$\tau_{\text{tank}} = T_{\text{tank}}\sqrt{g/h_{\text{tank}}} = T_{\text{site}}\lambda\sqrt{g/h_{\text{tank}}} \tag{12}$$

The non-dimensional period is only the same in the tank as at the site when the depth is scaled consistently, i.e. $SDD = 1$. Discrepancy in wavelength, steepness, or celerity is shown in Fig. 2, noting that these are of the same magnitude, whilst the discrepancy in group velocity or power is shown in Fig. 3.

A deep water simplification is often used in offshore engineering, based on the fact that $\tanh(kh) \rightarrow 1$ for large kh. This simplification is usually applied for $kh > \pi$ where the discrepancy is $< 0.4\%$ [6]. Expressed in terms of depth and wavelength, this limit equates to $h/L > 1/2$. For situations where both the deployment site and test tank can be considered deep water, i.e. below both dashed lines in Fig. 2, the error in wavelength is negligible and correct depth scaling is not required. It is interesting to note that the errors in wavelength are compounded when calculating the group velocity, resulting in a discrepancy for $C_g \approx 2\%$ at the deep water limit, although this is still likely to be acceptable when tank testing.

Wave energy converters typically operate in wave periods of 3–15 s, and depths around 20–80 m, which equates to full-scale non-dimensional periods of about 1–10. At tank scale, the non-dimensional period should be similar, providing *SDD* is close to unity. Typical model scales for testing model renewable energy devices are between 1:100 and 1:10, tested in tanks 0.5–5 m deep, although large models are unlikely to be tested in small tanks and vice versa. This may result in scaled depth discrepancies in the order of 0.3–3, obviously depending on the specifics of model, device, and tank. Therefore, errors in wavelength/steepness/celerity of up to $\pm 30\%$ may be experienced in testing. This results in a corresponding $\pm 20\%$ discrepancy in wave power and group velocity.

To facilitate understanding of Figs. 2 and 3, examples are shown on the secondary axes with a 1:25 scale model, 50 m deep site, and 2 m deep test facility. A 12 s full-scale wave of interest to a wave-energy device would have non-dimensional period $\tau_{\text{tank}} = 5.3$. If the site depth was 40 m or 100 m instead of 50 m, as shown on the lower secondary x-axis, the wavelength/steepness/celerity (Fig. 2) would be wrong by a factor ε_L of 0.95 or 1.09 respectively, and group velocity/wave power (Fig. 3) would be wrong by a factor ε_{Cg} of 1.02 or 0.88, as shown in Table 1 along with additional examples.

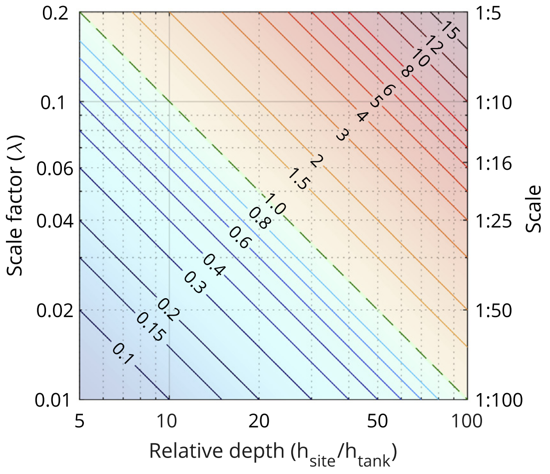

Fig. 1. Contours of scale depth discrepancy (SDD) shown for a range of relative depths (site to tank) and scale factors. A value of SDD less than unity corresponds to the tank being too deep for the scaled site depth, resulting from a relatively shallower deployment site and/or a smaller model scale.

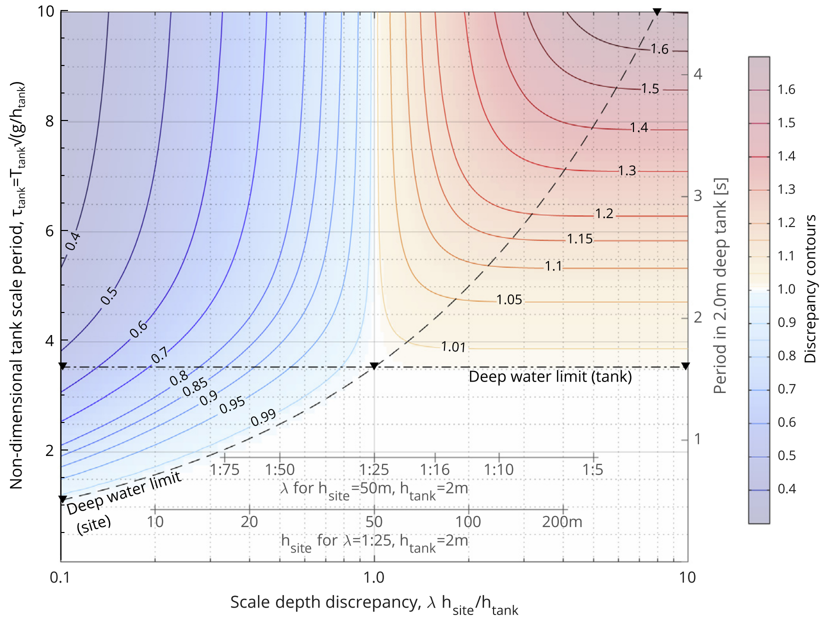

Fig. 2. Contours of relative error in wavelength or steepness or celerity for a range of scaled depth discrepancies and non-dimensional tank scale periods. Secondary axes show example scale factor and site depth for other parameters fixed. Dashed and dash-dot lines shows deep water limits for site and tank respectively.

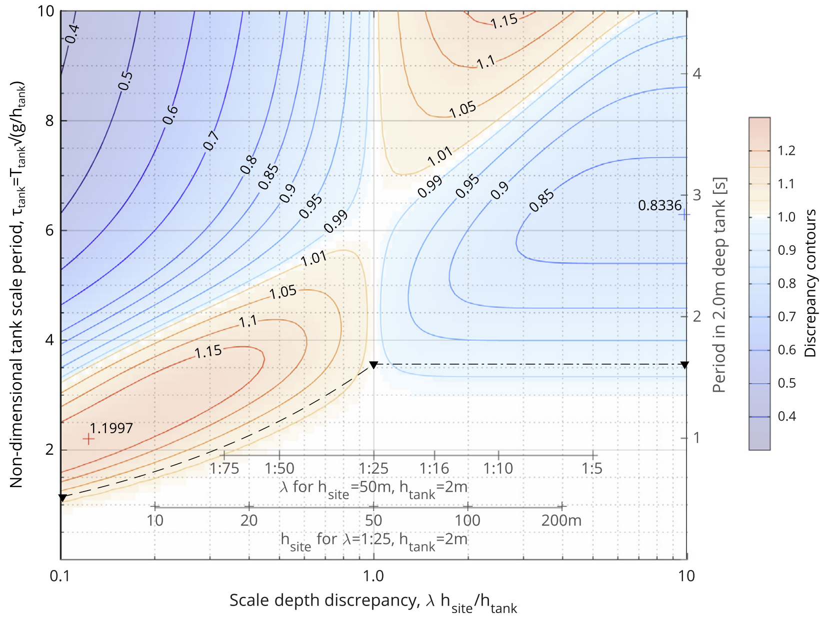

Fig. 3. Contours of relative error in group velocity or wave power for a range of scaled depth discrepancies and non-dimensional tank scale periods. Secondary axes show example scale factor and site depth for other parameters fixed. Dashed and dash-dot lines shows deep water limits for site and tank respectively.

4. Discussion

For marine renewable energy devices, such as wave energy converters or floating offshore wind turbines, wave steepness can be particularly important as this affects floating device response. It is also beneficial to understand how the power of waves is scaled and what errors may be present when modelling a device to extract this power. The method presented here is designed to be an aid to tank test planning, allowing the range of discrepancies to be quantified when selecting a facility

Table 1
Example discrepancies in a 12 s full-scale wave, for wavelength/steepness/celerity ε_L Fig. 2 and group velocity/wave power ε_{Cg} Fig. 3

Scale	h_{site}	h_{tank}	SDD	τ_{tank}	ε_L	ε_{Cg}
1:25	50 m	2 m	1	5.31	1	1
1:25	40 m	2 m	0.8	5.31	0.95	1.02
1:25	75 m	2 m	1.5	5.31	1.07	0.93
1:25	100 m	2 m	2.0	5.31	1.09	0.88
1:50	50 m	2 m	0.5	3.76	0.92	1.13
1:16	50 m	2 m	1.56	6.64	1.13	0.98
1:25	50 m	3 m	0.67	4.34	0.94	1.08

and model scale. It will also be of benefit when correlating tank test measurements to real site deployment where the scaled depth ratio is not unity, which is a likely scenario.

This issue of incorrect depth scaling may not have received much attention previously, as it is less critical for other applications of tank testing. For coastal models where water depth is paramount, the bathymetry is usually re-created in the test facility, removing any depth error and corresponding wavelength issues. When testing ships, there is not a unique depth in which they operate, and this can usually be classified as 'deep water' reducing the importance of understanding this depth scaling discrepancy. Oil platforms also typically operate in deep water relative to the waves experienced.

While the errors presented here have been calculated using linear wave theory, they also hold for second order Stokes waves, where although the wave shape is different the dispersion relation remains the same [6]. A correction to the dispersion relation is required for third and higher orders.

Discrepancies in wave spectra are frequency dependent, and can be visualised as vertical sections through the contoured surfaces shown in Figs. 2 and 3. For wavelength, steepness, or celerity the error has the same direction as the scale depth discrepancy, i.e. $\varepsilon_L < 1$ for $SDD < 1$. For group velocity or wave power it is possible to have both errors smaller and larger than unity for a given SDD.

Scale dependencies are routinely accounted for in the analysis of test results. For example, when using Froude scaling in tank testing, time is scaled by $\sqrt{\lambda}$. It is also common to test in fresh water which is approximately 2.5% less dense than typical sea water. In the same manner, the design methods presented here could also be applied to any discrepancy in the scaled water depth.

5. Conclusion

A method has been developed to quantify and visualise the errors that may arise while tank testing if the scaled water depth is not correct. This issue may be of particular relevance to marine renewable energy, where devices sensitive to wavelength and power are moored in finite depth water conditions. For typical model tests, this may result in wavelength/steepness errors of up to $\pm 30\%$, and up to $\pm 20\%$ in wave power.

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