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Modelling Velocity Profiles using the Law of the Wake for Tidal Currents and Winds

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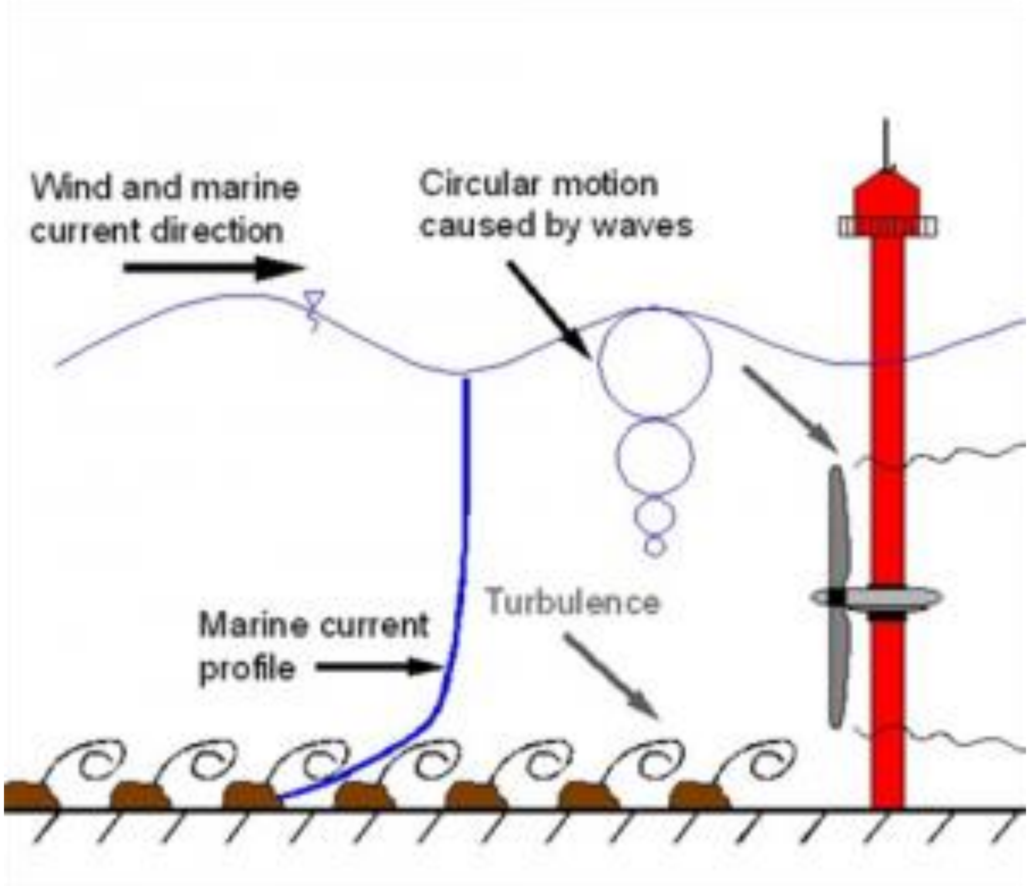
Why examine the vertical profile?

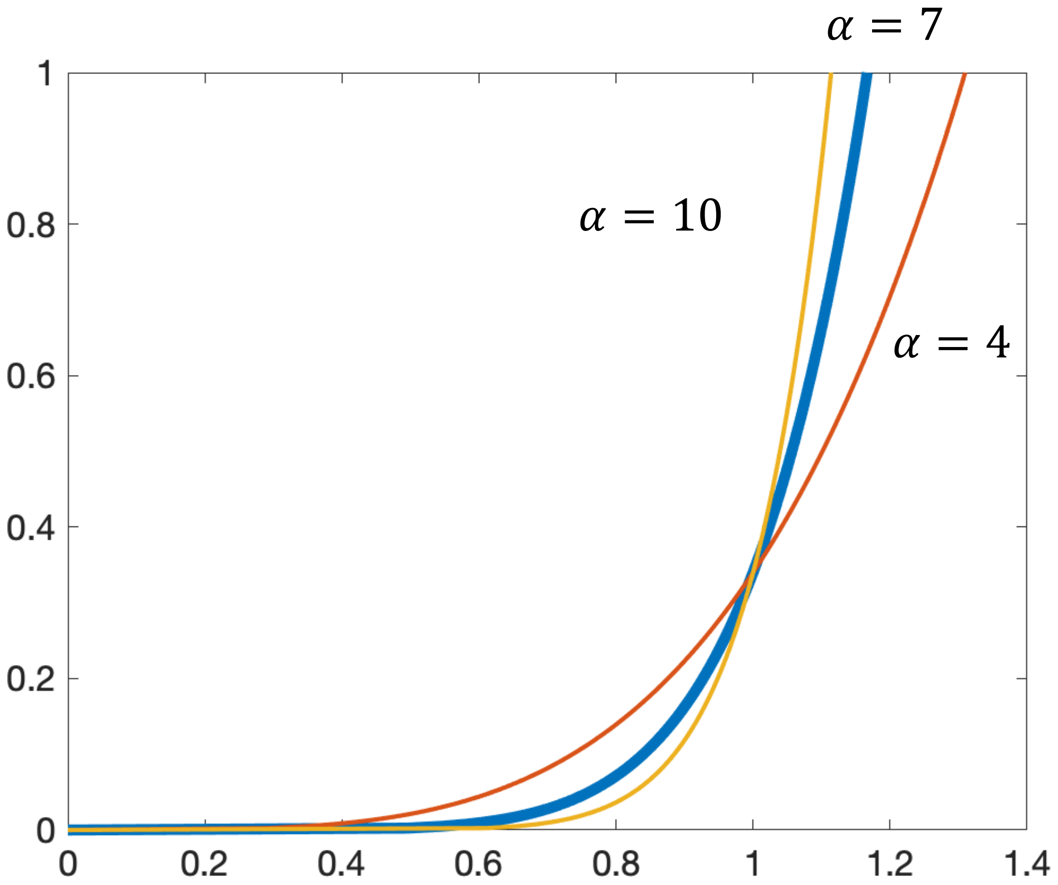
- Many practical reasons:
  - Interpolate/Extrapolate missing measurements
  - Calculate shear across face of turbines
  - Identify anomalous flow
  - Understand physics
  - Improve numerical modelling
- Settle an argument



Mathematical models of vertical profiles

Power Law: $u(\eta) = \left(\frac{\eta}{\beta}\right)^{\frac{1}{\alpha}}$ $, \eta = z/h$

- Used by engineers for many flows
- Called: 1/7th power law turbulent velocity profile
- Simple, two parameters with shape determined by α
- Does not allow for reverse shear

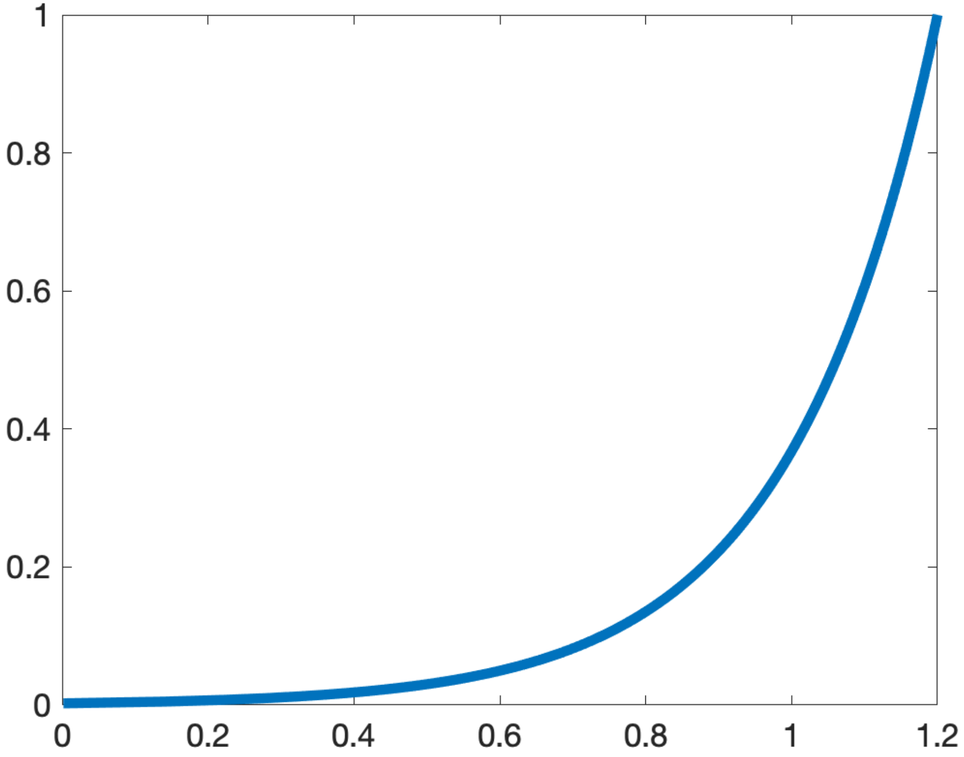


Mathematical models of vertical profiles

Log Law/ law of the wall: $u(\eta) = \frac{u_*}{\kappa}[\ln(\eta/z_0)]$

$$= \frac{u_*}{\kappa}[\ln(\eta) + B]$$

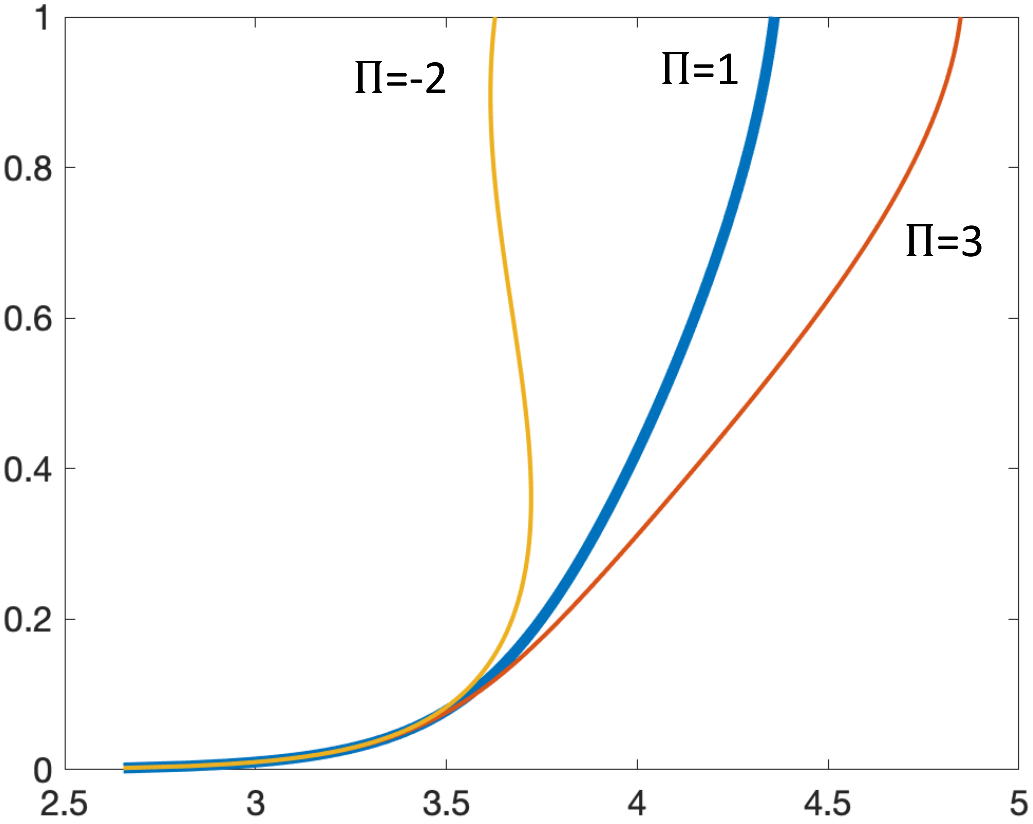
- Used by oceanographers for near bottom flow
- Allows calculation of the bottom drag, $C_D = \left(\frac{u_*}{U}\right)^2$
- Does not allow for reverse shear



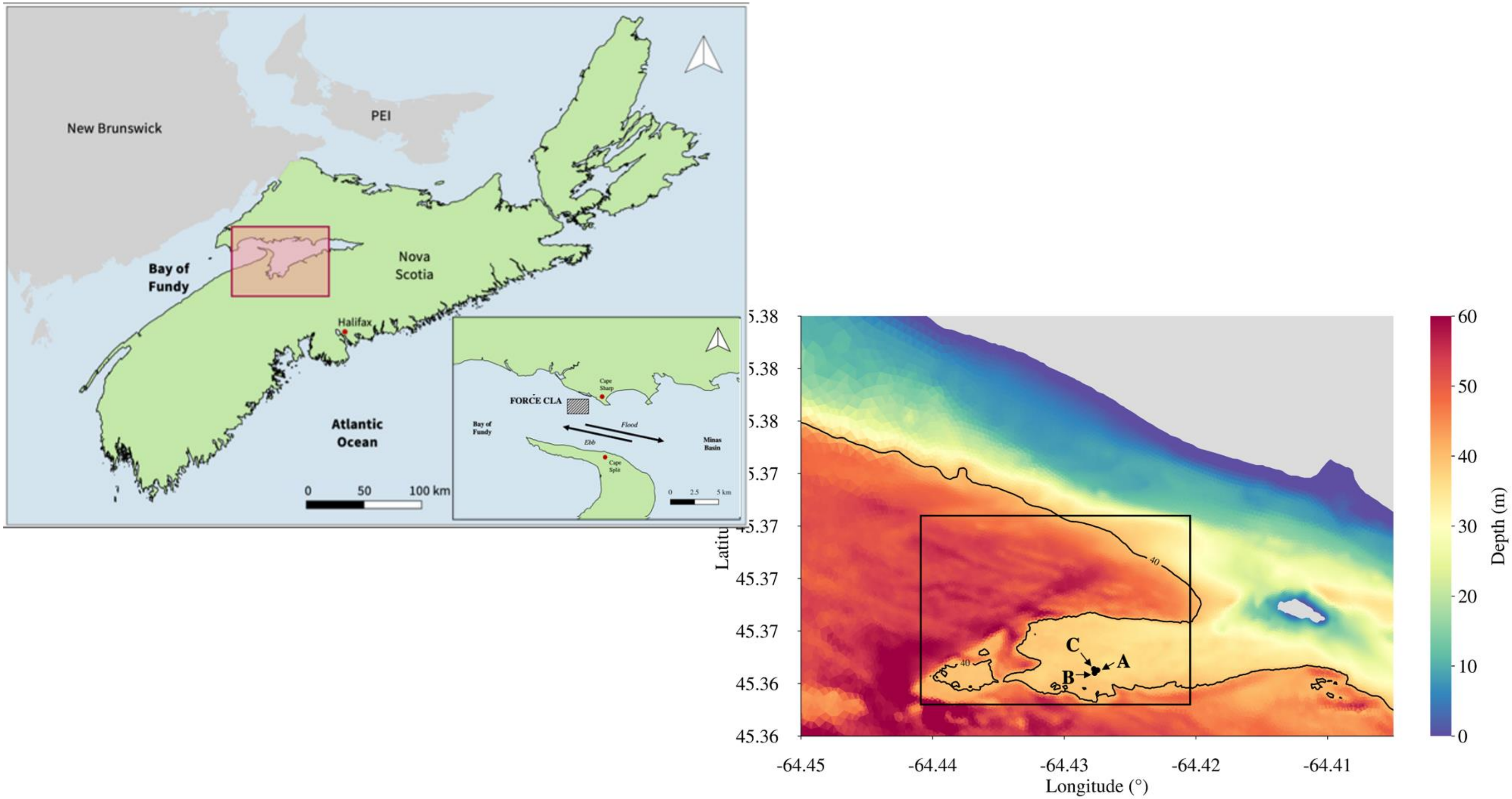
Mathematical models of vertical profiles

Wake Law: $u(\eta) = \frac{u_*}{\kappa}[\,\ln(\eta) + B + \Pi\, w(\eta)]$

- Introduced in 1950s by Coles
- Adds a ‘wake’ term to law of wall
- Allows for reverse shear, if Π is negative



ADCP data from Minas Passage



Vertical Structure of Tidal Flow

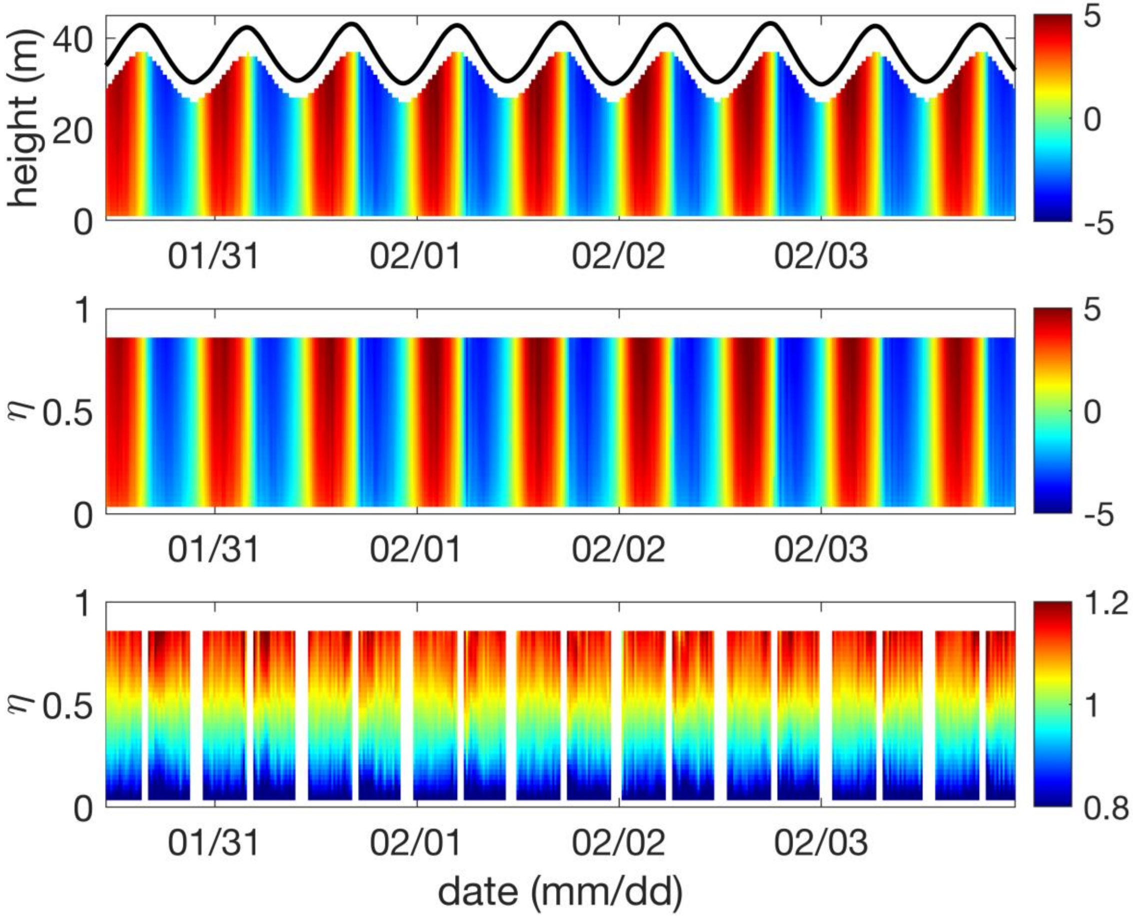
No obvious vertical structure in raw data

Remove change in depth by changing to normalized coordinates:

$$\eta = z/h$$

Normalize velocity by vertical mean speed:

$$\mathrm{u} = \frac{u}{\frac{1}{h}\int_0^h u\,dz}$$



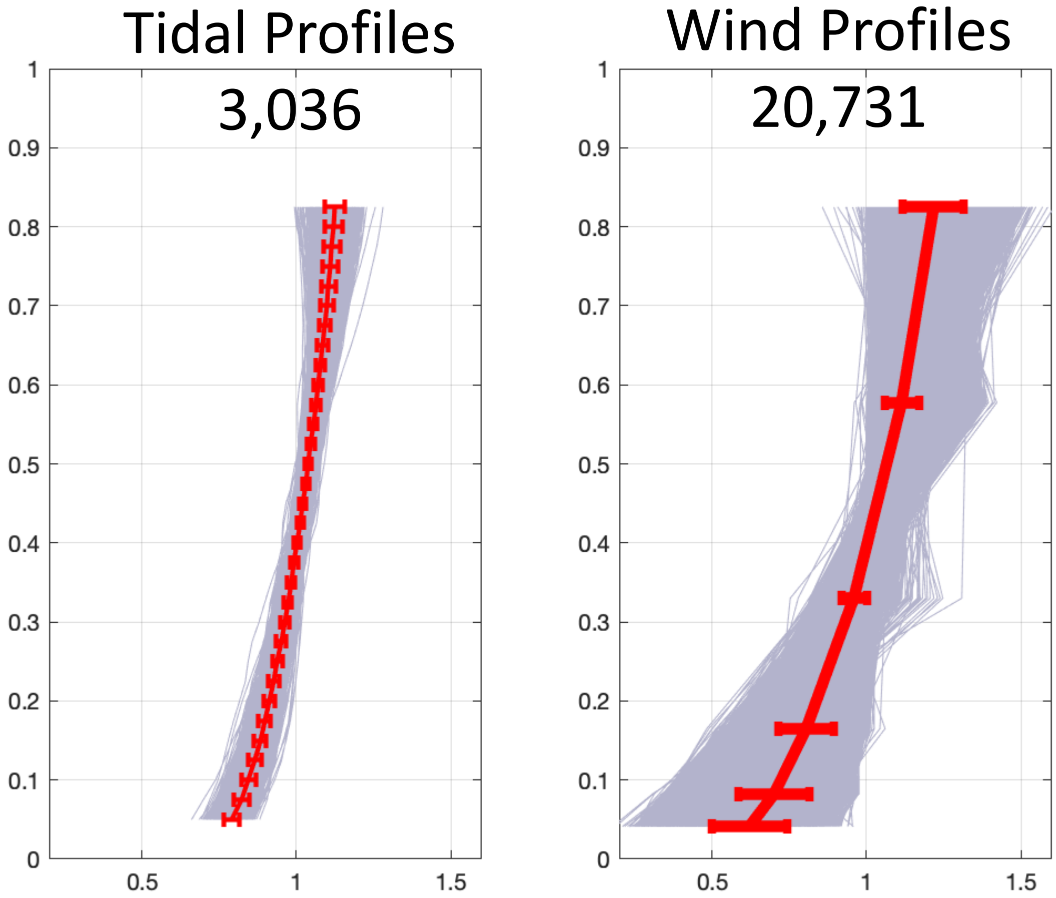
Tidal and Wind profiles

Tidal: FORCE

  - Flood tides over 65 days
  - speeds > 1.5 m/s (75% of data)

- Wind: Cabauw
  - Full year (2020)
  - speeds > 8 m/s (35% od data)

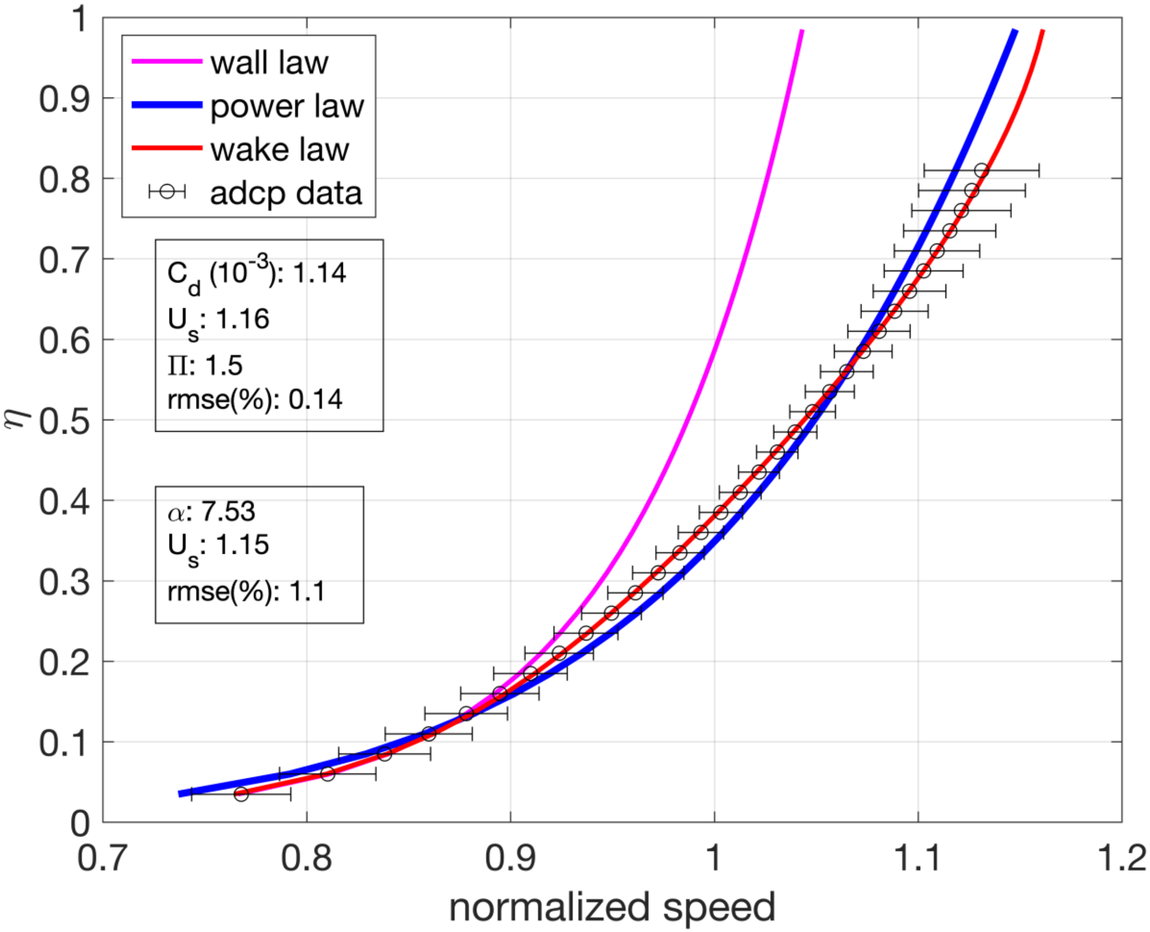
- 10 minute-averages



Red = mean of all measurements at given depth +1 std

Fitting Models

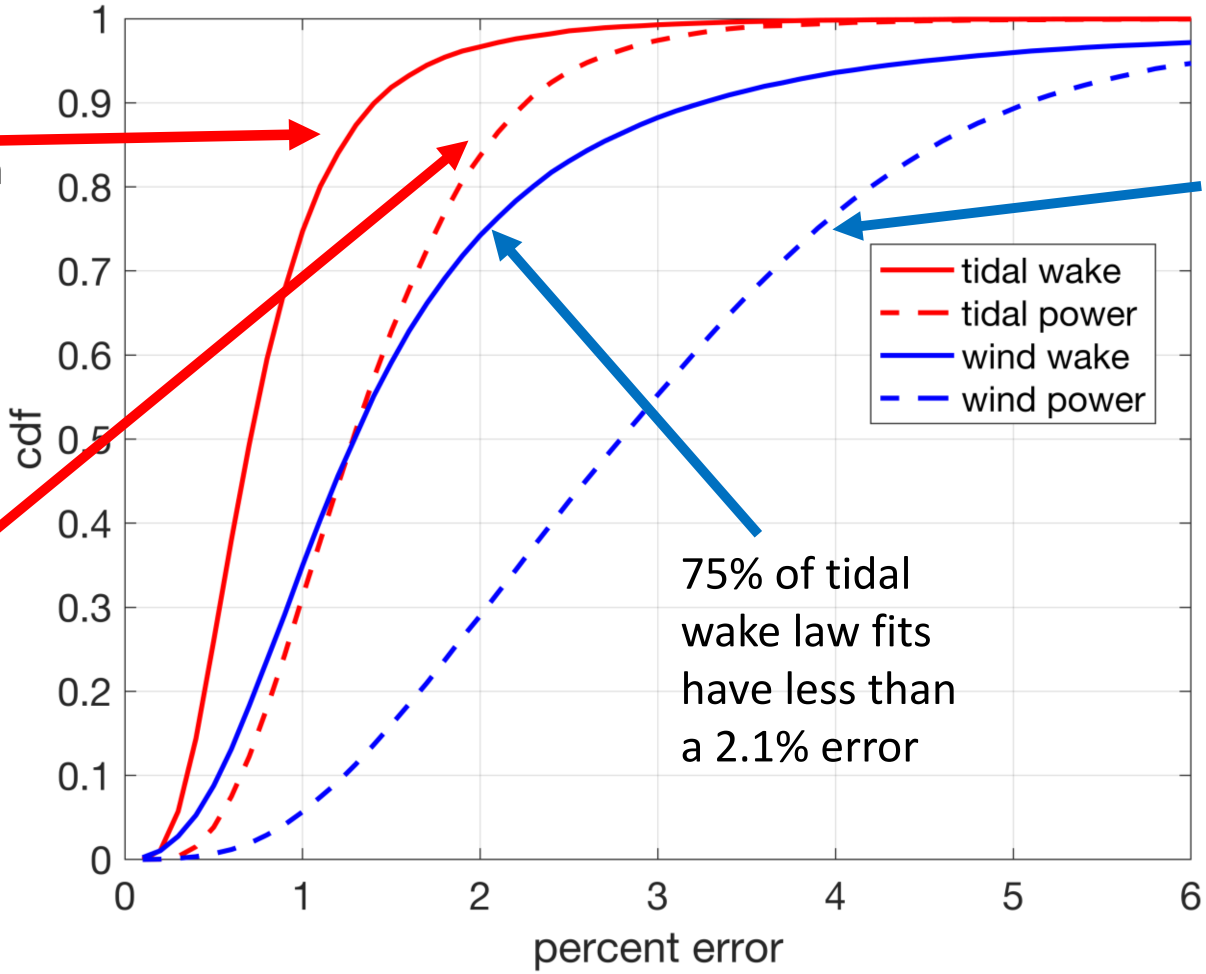
- Fit three models to each 10-minute ensemble
- Choose parameters to minimize RMSE
- Wall law only fit to near bottom flow.
- RMSE:
  - Power law 1.1%
  - Wake law 0.14%



Cumulative Distribution Function

87% of tidal wake law fits have less than a 1.35% error

87% of tidal power law fits have less than a 2.15% error

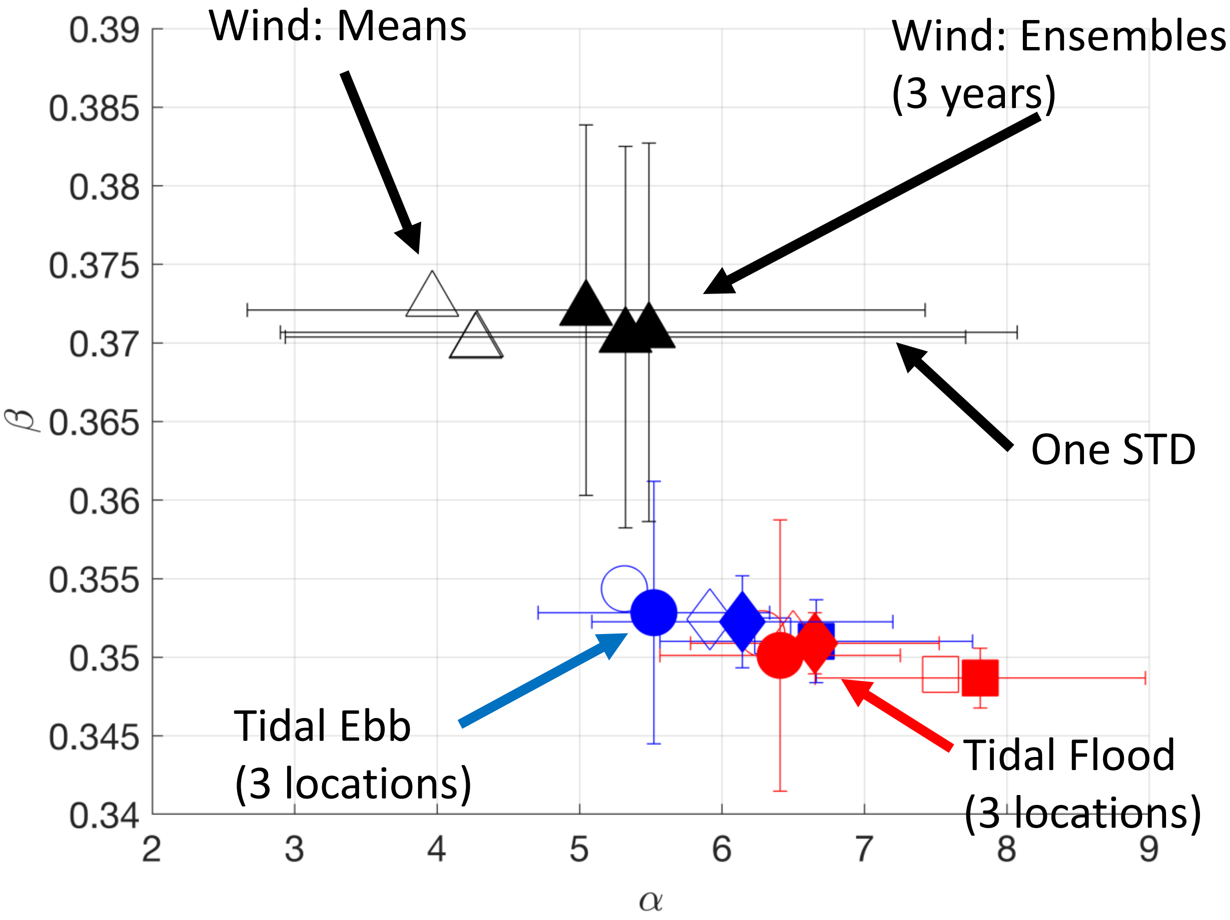


75% of tidal wake law fits have less than a 2.1% error

75% of tidal power law fits have less than a 4% error

Power Law Parameters

- Tidal:
  - α varies over ebb/flood and location
  - Ensemble and mean similar
- Wind
  - α mean different than ensemble
  - more variation
- β varies very little



Wind: Means

Wind: Ensembles (3 years)

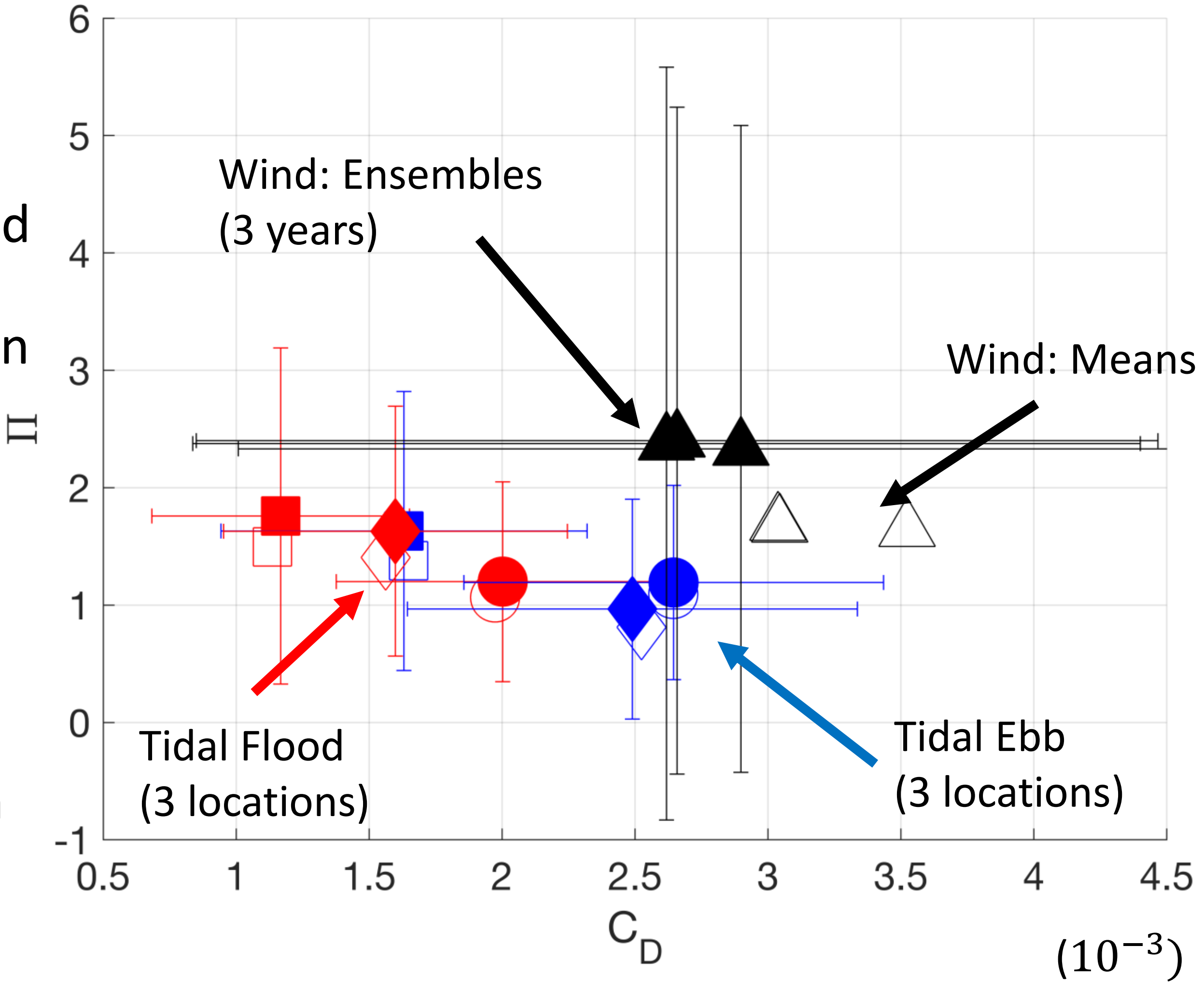
One STD

Tidal Ebb (3 locations)

Tidal Flood (3 locations)

Wake Law Parameters

- Tidal:
  - Drag, C_D, varies over ebb/flood and location
  - Smaller variation in Wake parameter
  - Ensembles and mean similar
- Wind
  - Large error bars
  - Small amount of variation between different years



Conclusions

- Vertical profile models remarkably robust for Minas Passage flow
- Wake laws provides a better fit and more information than power law, at the cost of some complexity
- Similar results for Wind data, but there is more variability in the parameters across ensembles.

Research Papers

- D. S. Coles, “The law of the wake in the turbulent boundary layer,” J. Fluid Mech., vol. 1, no. 2, pp. 191–226, doi: 10.1017/S0022112056000135.
- L. Enders, “Flow Characterization at a Turbulent Tidal Energy Site in Minas Passage, Bay of Fundy,” MSc Thesis, Acadia University, 2022.
- Lilli Enders, and Richard Karsten, “Improved Modelling for Vertical Profiles of Flow Speed in a Turbulent Tidal Channel,” in Proceedings of the 15th European Wave and Tidal Energy Conference, Bilbao, Spain, 2023.