On The Modelling of Arrays of Wave Energy Converters

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***Abstract*—This paper presents the concept of developing a surrogate model for the hydrodynamic interactions between heaving bodies including radiation wave effects. The surrogate model for an array of n heaving bodies is developed by creating an equivalent mechanical system that consists of n bodies in addition to other smaller intermediate bodies, springs, and dampers connected to each pair of the main n bodies. This equivalent mechanical system is designed such that the motions of its main bodies are the same as the motions of the actual array heaving bodies. The purpose is to develop dynamic models for wave energy converters (WECs) arrays that are more convenient to use for control design and optimization. The small moving mass that is connected between each two bodies in the surrogate mechanical system enables the modelling of the hydrodynamic interaction forces between the actual two buoys. A simulation tool is developed to simulate the motions of all bodies in the surrogate model. Also the motions of the actual WECs in the array are simulated using AQWA software. An optimization problem is then solved to minimize the error between the simulated motions of the WECs array in ocean, and the surrogate mechanical system. In this case study, the system design parameters to be optimized are: springs coefficients, dampers coefficients, and the additional body masses. Slack mooring dynamics are included in the derived equations of the motion of floating bodies. The proposed surrogate model is a time domain model that can be used for time-domain control design. The main advantage in the proposed technique is the ability to use a simple dynamic model for control design of arrays of wave energy converters. Simulations are presented that highlight the utility of the proposed model in control design.**

***Index Terms*—Ocean wave energy conversion, WEC array modelling, Surrogate model of WEC arrays, interactive WECs, Control of WEC arrays**

I. Introduction

Waves can be a reliable source of renewable energy if wave energy converters can be operated in an economic way in various sea conditions. WECs are likely to operate in oceans in large arrays. Due to the oscillating behavior of each WEC, and the interaction with water waves, each WEC is affecting the other surrounding WECs with its radiated waves, in addition to the scattered waves that are also affecting the wave field. This interaction may affect the annual power absorption constructively or destructively. Such a complex dynamic system need to have an accurate model in order to study the effect of operating WECs on each others. There has been extensive studies in the literature on the hydrodynamic modelling of WECs arrays. While significant developments have been done on the ocean farms layout optimization [1]–[5], there is a clear

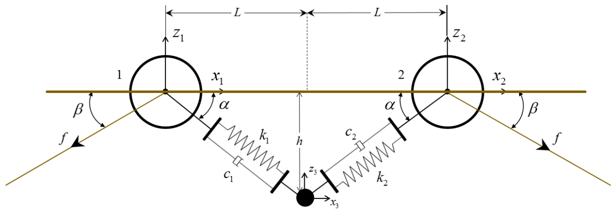

Fig. 1. Surrogate system for an array of two hydrodynamically coupled buoys. The two main bodies, in this surrogate system, are connected to an intermediate mass through a spring-damper system. Each body is acted upon by a force that is equivalent to the mooring force in the original system

gap on the collective control of such a system; this gap can be attributed to the absence of a suitable form of the dynamic model as highlighted in [6], [7]. Hydrodynamic modelling is not the subject of this paper. This paper presents a surrogate model that replaces the hydrodynamic model of a WECs array. This surrogate model is the model of a purely mechanical surrogate system. Consider, for example, an array if two buoys in the ocean. The motion of each these two buoys can be computed from another surrogate system. This surrogate system in this case is shown in Figure 1. The surrogate system consists of two main bodies (bodies 1 and 2), one intermediate body (body 3), two springs and two dampers, all connected as shown in Figure 1. The parameters of this surrogate model include the masses of all bodies, the coefficients of springs and dampers, and the geometry dimensions; these parameters are computed as functions of the actual masses and dimensions of the actual buoys array in the ocean, and such that the difference between the motion of body-1/body-2 in the surrogate system and the motion of buoy-1/buoy-2 in the actual buoys array in the ocean is minimal. This is achieved through an optimization process as described in section II.

Once the surrogate model is obtained, it can be used for control systems analysis and design. The advantage of using the surrogate model for control design is that no hydrodynamic calculations are needed. The surrogate system is purely mechanical, and a large literature of control of mechanical systems can be leveraged. Section III presents a collective control design test case using the proposed surrogate model.

II. The Surrogate System and Its Model

Consider a WEC array composed of two identical, hydrodynamically coupled, heaving buoys. The proposed surrogate system in this paper is composed of two bodies connected with each other through a mechanical system of two distinct springs and dampers in addition to an intermediate mass, as shown in Fig.1. Each body in the surrogate system corresponds to a buoy. A body mass in the surrogate system is slightly bigger than the mass of the corresponding buoy, as it includes the buoy's added mass at infinity as well. Another force f affects each of these two bodies that is equal to the mooring force of the original buoys. Another force is applied on each body that is equal to the hydrostatic force on each of the two buoys. The unknown parameters in the surrogate system are: the mass and volume of the intermediate bodies (body 3 in this case study), the springs coefficients, and the dampers coefficients. These unknown parameters will be designed such that the motions of the main bodies in the surrogate model (body 1 and body 2) are the same as the motions of the two buoys in the WEC array. The equation of motion for each body in this surrogate model is detailed below.

A. Body 1

1) Equation of motion in x-direction: Newton's law applied on body 1 in the surrogate model results in the following equation of motion for body 1:

$$(M+\bar{a})\ddot{x}_1 = F_{ex,x1} - f_{x1} + A_{x1}p_1^\star\bar{x}_1 + u_{x1} \tag{1}$$

where:

$$A_{x1} = [Al_{x1} \quad Ar_{x1}] \tag{2}$$

$$Al_{x1} = \begin{bmatrix}-k_1p_1^\star & k_1w_1^\star & k_1p_1^\star & -k_1w_1^\star\end{bmatrix} \tag{3}$$

$$Ar_{x1} = \begin{bmatrix}-c_1p_1^\star & c_1w_1^\star & c_1p_1^\star & -c_1w_1^\star\end{bmatrix} \tag{4}$$

$$\begin{cases} w\bar{\delta_2} = -x_2cos(\beta) + z_2sin(\beta)\\ f_{x2} = -kx_2\cos^2(\beta) + \frac{1}{2}kz_2\sin(2\beta)\\ p_1^\star = \cos(\alpha_o) - \epsilon_1\sin(\alpha_o)\\ w_1^\star = \epsilon_1\cos(\alpha_o) + \sin(\alpha_o)\end{cases} \tag{5}$$

$$\begin{cases} \delta_1 = \cos\alpha_o(z_1\epsilon_1 - x_1) + \sin\alpha_o(z_1 + x_1\epsilon_1),\\ \dot{\delta}_1 = \cos\alpha_o(v_1\epsilon_1 - u_1) + \sin\alpha_o(v_1 + u_1\epsilon_1),\\ \bar{\delta}_3 = \cos\alpha_o(x_3 - z_3\epsilon_1) - \sin\alpha_o(z_3 + x_3\epsilon_1),\\ \dot{\bar{\delta}}_3 = \cos\alpha_o(u_3 - v_3\epsilon_1) - \sin\alpha_o(v_3 + u_3\epsilon_1),\\ f_{x1} = kx_1\cos^2(\beta) + \frac{1}{2}kz_1\sin(2\beta),\end{cases} \tag{6}$$

$$\bar{x}_1^T = \begin{bmatrix}x_1 & z_1 & x_3 & z_3 & u_1 & v_1 & u_3 & v_3\end{bmatrix} \tag{7}$$

$$\epsilon_{1,2} = \frac{z_{1,2} - z_3}{lcos(\alpha_0)} \tag{8}$$

where $l = \sqrt{h^2 + L^2}$ is the initial length of the spring-damper connector between both bodies representing the original floating spheres and origin of the local axis 3. Initially (at $t = 0$), $\alpha_0 = \arccos(L/l)$, while in general $\alpha(t) = \arccos(L/l + \Delta l_i(t))$, where $\Delta l_i(t)$ is the change in spring-damper length at each time instant which is relevant to the perturbation of α.

2) Equation of motion in z-direction: The heave equation of motion for body 1 can be written as follows:

$$(M+\bar{a})\ddot{z}_1 + K_hz_1 = F_{ex,z1} - f_{z1} + A_{z1}p_1^\star\bar{z}_1 + u_{z1} \tag{9}$$

where:

$$\bar{z}_1^T = \begin{bmatrix}x_1 & z_1 & x_3 & z_3 & u_1 & v_1 & u_3 & v_3\end{bmatrix} \tag{10}$$

$$A_{z1} = \begin{bmatrix}Al_{z1} & Ar_{z1}\end{bmatrix} \tag{11}$$

$$Al_{z1} = \begin{bmatrix}-k_1p_1^\star & k_1w_1^\star & k1p_1^\star & -k_1w_1^\star\end{bmatrix} \tag{12}$$

$$Ar_{z1} = \begin{bmatrix}-c_1p_1^\star & c_1w_1^\star & c_1p_1^\star & -c_1w_1^\star\end{bmatrix} \tag{13}$$

$$f_{z1} = kx_1sin(2\beta) + kz_1sin^2(\beta) \tag{14}$$

B. Body 2

1) Equation of motion in x-direction: The equation of motion for body 2 in the surrogate system can be written as:

$$(M+\bar{a})\ddot{x}_2 = Fex, x_2 + f_{x2} + A_{x2}p^\star\bar{x}_2 + u_{x2} \tag{15}$$

where:

$$\bar{x}_2^T = \begin{bmatrix}x_2 & z_2 & x_3 & z_3 & u_2 & v_2 & u_3 & v_3\end{bmatrix} \tag{16}$$

$$A_{x2} = \begin{bmatrix}Al_{x2} & Ar_{x2}\end{bmatrix} \tag{17}$$

$$Al_{x2} = \begin{bmatrix}-k_2p^\star & -k_2w^\star & k_2p^\star & k_2w^\star\end{bmatrix} \tag{18}$$

$$Ar_{x2} = \begin{bmatrix}-c_2p^\star & -c_2w^\star & c_2p^\star & c_2w^\star\end{bmatrix} \tag{19}$$

$$\begin{cases} f_{x2} = -kx_2\cos^2(\beta) + \frac{1}{2}kz_2\sin(2\beta)\\ p^\star = \cos(\alpha_o) - \epsilon_2\sin(\alpha_o)\\ w^\star = \epsilon_2\cos(\alpha_o) + \sin(\alpha_o)\end{cases} \tag{20}$$

2) Equation of motion in z-direction: The equation of motion for body 2 in the z-direction can be written as:

$$(M+\bar{a})\ddot{z}_2 + K_hz_2 = Fex, x_2 + f_{z2} + A_{z2}w^\star\bar{x}_2 + u_{z2} \tag{21}$$

$$A_{z2} = \begin{bmatrix}Al_{z2} & Ar_{z2}\end{bmatrix} \tag{22}$$

$$Al_{z2} = \begin{bmatrix}-k_2p^\star & -k_2w^\star & k_2p^\star & k_2w^\star\end{bmatrix} \tag{23}$$

$$Ar_{z2} = \begin{bmatrix}-c_2p^\star & -c_2w^\star & c_2p^\star & c_2w^\star\end{bmatrix} \tag{24}$$

$$f_{z2} = -\frac{1}{2}kx_2sin(2\beta) + kz_2sin^2(\beta) \tag{25}$$

C. Body 3

1) Equation of motion in x-direction: The equation of motion of the artificial mass (body 3) in the x-direction can be written as:

$$M_3\ddot{x}_3 = Fex, x_3 + A1_{x3}\bar{x}_2 p^\star + A2_{x3}\bar{x}_1 p_1^\star \tag{26}$$

where:

$$\begin{aligned} A1_{x3} &= \begin{bmatrix} A1l_{x3} & A1r_{x3} \end{bmatrix} \\ A1l_{x3} &= \begin{bmatrix} k_2 p^\star & k_2 w^\star & -k_2 p^\star & -k_2 w^\star \end{bmatrix} \\ A1r_{x3} &= \begin{bmatrix} c_2 p^\star & c_2 w^\star & -c_2 p^\star & -c_2 w^\star \end{bmatrix} \\ A2_{x3} &= \begin{bmatrix} A2l_{x3} & A2r_{x3} \end{bmatrix} \\ A2l_{x3} &= \begin{bmatrix} k_1 d_1^\star & k_1 w_1^\star & -k_1 p^\star & -k_1 w^\star \end{bmatrix} \\ A2r_{x3} &= \begin{bmatrix} -c_1 f^\star & -c_1 w_1^\star & c_1 p^\star & c_1 w^\star \end{bmatrix} \end{aligned} \tag{27}$$

2) Equation of motion in z-direction: The equation of motion of the artificial mass (body 3) in the z-direction can be written as:

$$M_3\ddot{z}_3 = Fex, z_3 + A1_{z3}\bar{x}_2 w^\star + A2_{z3}\bar{x}_1 w_1^\star \tag{28}$$

where:

$$\begin{aligned} A1_{z3} &= \begin{bmatrix} A1l_{z3} & A1r_{z3} \end{bmatrix} \\ A1l_{z3} &= \begin{bmatrix} k_2 p^\star & k_2 w^\star & -k_2 p^\star & -k_2 w^\star \end{bmatrix} \\ A1r_{z3} &= \begin{bmatrix} c_2 p^\star & c_2 w^\star & -c_2 p^\star & -c_2 w^\star \end{bmatrix} \\ A2_{z3} &= \begin{bmatrix} A2l_{z3} & A2r_{z3} \end{bmatrix} \\ A2l_{z3} &= \begin{bmatrix} -k_1 p_1^\star & k_1 w_1^\star & k_1 p_1^\star & -k_1 w_1^\star \end{bmatrix} \\ A2r_{z3} &= \begin{bmatrix} -c_1 p_1^\star & c_1 w_1^\star & c_1 p_1^\star & -c_1 w_1^\star \end{bmatrix} \end{aligned} \tag{29}$$

Here $\bar{a}$ is the infinite frequency added mass of each of the two identical buoys, $F_{ex,xi}$ and $F_{ex,zi}$ are the wave excitation force components on each floating body in surge and heave directions respectively. These two force components are used to excite the two alternative bodies. The u_i and v_i are the absolute surge and heave velocities of body i, respectively, $(i = 1 \cdots 3)$. Finally f_{xi} and f_{zi} are the mooring tension forces on floater i in x and z directions. Both bodies, 1 and 2, in the surrogate model are forced by the same hydrostatic forces of the original floaters as shown in Eqs. (9) and (21).

Previous studies showed the linearization process of the mooring forces and highlighted its dependency on the body motion amplitude and the cables initial length [8]. Also different mooring configurations and design approaches were introduced and discussed in [9], [10]. In this paper, cables connecting the buoys to the seabed are modeled as weightless nonlinear mooring lines, initially unstretched in the calm water condition. A polynomial of order three is defining the tension force as

$$F_t = \begin{cases} \zeta_1 \Delta L + \zeta_2 (\Delta L)^2 + \zeta_3 (\Delta L)^3, & if \Delta L > 0 \\ 0, & else \end{cases} \tag{30}$$

where ζ_i $(i = 1 \cdots 3)$ are the polynomial function coefficients, which are representing the spring effect, linear damping and cable inertia, respectively, and $\Delta L = L - L_0$
A simple control force that consists of only linear damping acting on each body by applying a vertical force proportional to the heaving velocity, $-C\dot{z}_i$, $(i = 1, 2)$.

The extracted energy over a certain time period, T, is calculated as:

$$E = \int_0^T u(t)\dot{z}(t)dt \tag{31}$$

The surrogate model described above can be extended to arrays of any size. In a WEC array of N buoys, we will have a surrogate model of n main bodies. Each main body corresponds to a buoy in the WEC array. Each two bodies in the surrogate model will be coupled in a similar way to the case described above. Once the surrogate system is designed, the design is fixed and can be used to obtain approximate response for the WEC array.

D. The Model Identification

To approximate the hydrodynamic model of the WEC array accurately, the parameters of the surrogate model need to be identified. The unknown parameters of the surrogate model of a WEC array are the springs, dampers coefficients, and the masses of the intermediate bodies. The objective function of the system identification is set up as:

$$Min: \quad J = \sum_{i=1}^{3} e_i^T e_i \tag{32}$$

$$i = 1,\ 2,\ N \tag{33}$$

where e_i represents the approximation error of the displacement of the ith body:

$$e_i = \tilde{z}_i - z_i \tag{34}$$

where $\tilde{z}_i$ represents the displacement propagated based on the surrogate model and z_i is the displacement of the ith body which is obtained from a simulation conducted using the AQWA software.

III. The Collective Control

Consider an array of 3 WECs which surrogate model is shown in Figure 2.

Since we have the surrogate model to approximate the hydrodynamic behavior, we need to design the controller to extract the maximum energy from the WEC array. The Collective Controller will be applied in this paper which has the form of:

$$\vec{u} = -\mathbf{K}_p \vec{z} - \mathbf{K}_d \vec{v} \tag{35}$$

where the controller applies the Proportional-Derivative (PD) control law. $\mathbf{K}_p$ and $\mathbf{K}_d$ are the feedback gains of the controller. The PD control gains matrices take the form of:

$$\mathbf{K}_p = \begin{bmatrix} K_{p,11} & 0 & 0 \\ 0 & K_{p,22} & 0 \\ 0 & 0 & K_{p,33} \end{bmatrix} \tag{36}$$

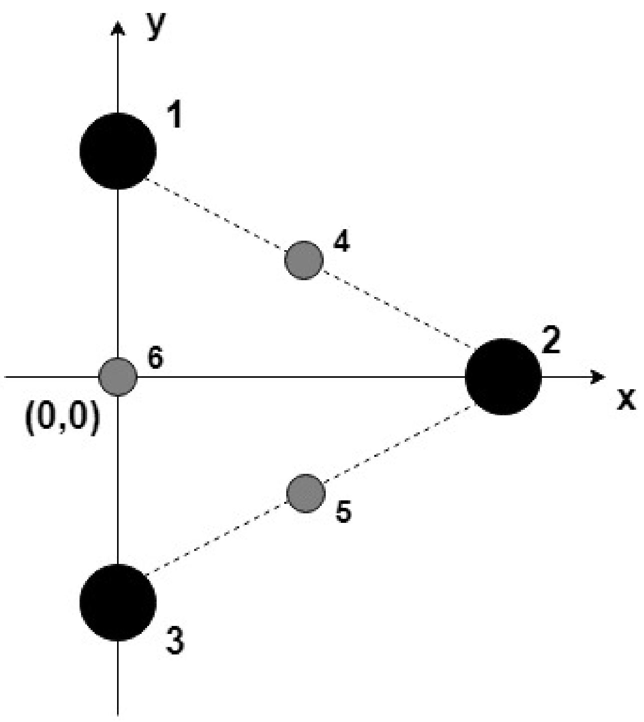

Fig. 2. The layout of the WEC array surrogate model

$$\mathbf{K}_d = \begin{bmatrix} K_{d,11} & K_{d,12} & K_{d,13} \\ K_{d,21} & K_{d,22} & K_{d,23} \\ K_{d,31} & K_{d,32} & K_{d,33} \end{bmatrix} \quad (37)$$

To have the maximum energy absorption and satisfy the constraints, the control feedback gains need to be optimized. The objective function can be expressed as:

$$Min: \quad J = \sum_i \int_0^T u_i v_i dt \quad (38)$$

$$Sub\ to: \quad |\, z_i \,| - z_{max} \leqslant 0$$

$$|\, u_i \,| - u_{max} \leqslant 0$$

$$i = 1,\, 2,\, 3 \quad (39)$$

where the variables of the optimization are:

$$X = [K_{p,11},\, K_{p,22},\, K_{p,33},\, K_{d,11},\, K_{d,12},\, K_{d,13} \\ ,\, K_{d,21},\, K_{d,22},\, K_{d,23},\, K_{d,31},\, K_{d,32},\, K_{d,33}] \quad (40)$$

The z_{max} is the maximum displacement of the floaters in the wave farm and u_{max} is the maximum control capacity.

IV. Numerical Test Case

The numerical simulation results are presented in this section. The simulations are executed in both Matlab®and Ansys AQWA. The performance of the controller computed based on the Surrogate model is compared with the controller computed based on a hydrodynamic model (using WAMIT). The maximum displacement is selected to be $z_{max} = 0.8$ m to keep the hydrostatic force linear. The maximum control capacity is selected to be $u_{max} = 50$ kN which is approximately the same level as the wave excitation force. The parameters of the surrogate model are identified using an optimization process as described in the previous sections.

The two different controllers are evaluated with AQWA simulations. The wave applied in the simulation has the PM spectrum with a zeros cross period of 7.857 s and a significant height of 1 m. The energy extraction is shown in Fig. 3. The energy extracted by the two controllers are very close. Hence we can claim the Collective Controller designed by the Surrogate model is identical to the controller designed using the hydrodynamic model in terms of the energy extraction.

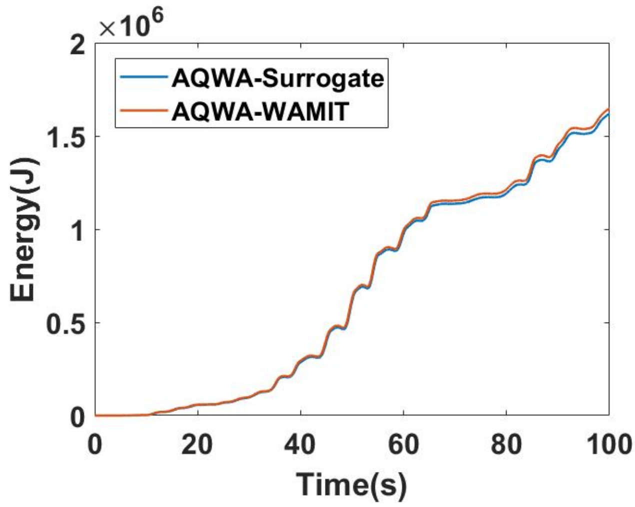

Fig. 3. The comparison of the extracted energy simulated by AQWA with the PD control gains optimized from the Surrogate model and the WAMIT model

V. Conclusion

An approximate surrogate model for arrays of wave energy converters was presented. The main advantage of this surrogate model is its suitability for control systems analysis and design since it is a mechanical system of masses, springs, and dampers. The validity of the surrogate model was tested through numerical simulations that demonstrated good of fitness. A collective control was designed for the array using the proposed surrogate model and the results demonstrate that the controller performances matches that of a controller designed using the classical hydrodynamic model of a WEC array, and hence the proposed model can be used for control design.

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