Effect of Air Compressibility on Primary Energy Conversion Performance of OWC device

Yukitaka Yasuzawa[#1], Takatomo Setoguchi[*2]

[#]*Department of Marine Systems Engineering, Kyushu University*

744 Motooka, Nishi-ku, Fukuoka City, Fukuoka, Japan

[1]yasuzawa@nams.kyushu-u.ac.jp

**Department of Maritime Engineering, Graduate School, Kyushu University*

744 Motooka, Nishi-ku, Fukuoka City, Fukuoka, Japan

***Abstract*— The authors have been developing a floating WEC (wave energy conversion) system with multiple cylindrical OWC columns. This structure has multiple OWC devices for electric power generation and buoyancy columns under the deck structure. In the present study, we have examined the scale effect of OWC device, focusing on the air compressibility of air chamber in OWC device, and have constructed mechanical modelling for the air chamber mechanics based on the theory of air damper with a nozzle. The relation between the water elevation motion and dynamic pressure in the air chamber is derived. We have applied it to our numerical program to estimate the primary energy conversion efficiency of OWC devices for various sizes. It is shown that the air compressibility has to be considered in case of realistic large size OWC device.**

***Keywords*— Wave energy, OWC, WEC, Air compressibility, Scale effect, Primary energy conversion efficiency**

I. INTRODUCTION

The authors have been investigating the performance of MC-OWC system we proposed[1,2,3]. The floating structure has multiple OWC (oscillating water column) devices and buoyancy columns for keeping buoyancy under a large deck structure. Each OWC devices is composed of a submerged vertical cylinder and an air turbine installed at the top of the cylinder to generate electric power.

In the previous conference(AWTEC2016), we showed the comparison between experimental results obtained in the wave tank test and numerical results obtained by using developed by the authors, and good agreement was shown for dynamic pressure, water elevation in OWC devices for MC-OWC, and primary energy efficiency. And their frequency response curves are shown. But we did not considered compressibility of air in the OWC device in the numerical analysis.

Recently scale effect for various devices of ocean renewable energy(ORE) has been discussed in the conferences as well as AWTEC2016. Problems of extracting ORE often involves nonlinearity mainly coming from fluid dynamics including hydrodynamic, aerodynamics in or out of PTO(power take off) devices. The nonlineality makes estimation difficult for large structure used for real ocean field.

In the case of the cylindrical OWC, the cross section area of airflow around the turbine is much smaller than the horizontal water plane area in the cylinder. Vertically reciprocal motion of water column in the OWC column induces the rapid airflow to rotate a turbine. This system is similar to a piston and cylinder with nozzle system. This system has a nonlinear spring and damper effects, and it is used to controlling or suppressing mechanical vibration as an ‘air damper’ for some structures.

In the present study, the authors have examined the scale effect of OWC device, focusing on the air compressibility of air chamber in OWC device, and have constructed mechanical modelling for the air chamber mechanics based on the theory of air damper with a nozzle. The relation between the water elevation motion and dynamic pressure in the air chamber is derived, and we have applied it to our numerical program to estimate the primary energy conversion efficiency of OWC devices for various sizes. It is shown that the air compressibility has to be considered in case of realistic large size OWC device.

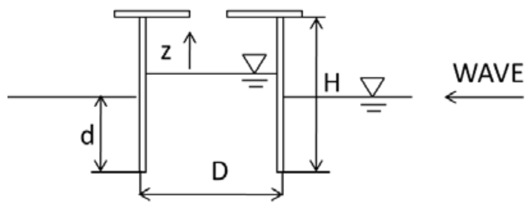

Fig. 1 An cylindrical OWC device(side view)

II. PREVIOUS MODEL OF AIR CHAMBER FOR OWC DEVICE

Relation between dynamic air pressure and the water surface elevation velocity in each chamber is expressed by linearizing Bernoulli’s theorem or pressure equation. No compressibility of air is assumed in the present report. Then,

$$p = \frac{c_A}{A_w}\dot{z} \tag{3}$$

where A_w is the water surface area in OWC devise, c_{Am} is the equivalent damping coefficient of the m-th OWC device. c_{Am} can be expressed by the following equation considering dissipation energy during a period.

$$c_A = \frac{4}{3\pi}\rho_a \left(\frac{A_W}{A_N}\right)^2 A_w |\dot{z}| \tag{4}$$

where ρ_a : density of air, A_N: horizontal area of nozzle.

III. Theory of Present model for OWC device

A. Mechanical modelling and definition of non-dimensional parameters

An air damper model as shown in Fig.2 and its theory are applied for modelling of the air chamber in a cylindrical OWC device to consider the compressibility of air. Air damper is generally used as a device to control or suppress vibration of a structure by setting between an oscillating body and a unit to be controlled. In case of the present OWC device, a rigid piston of the air damper unit is replace by water column in an OWC device. Theory of air damper is presented by Asami[4,5] and equivalent nonlinear spring and damper models of Maxwell and Voigt models are formulated. In the present study, we selected the equivalent Voigt model as shown in Fig.3.

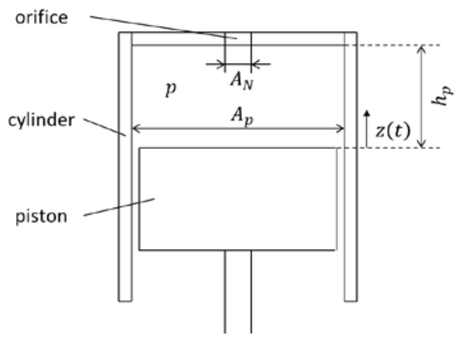

Fig.2 Air damper model

In Fig.8, A_p , A_N are horizontal area of the piston and orifice respectively. The cross section area of the piston is replaced by a water plane area in OWC device. Therefore nozzle ratio of an OWC device is expressed by A_N/A_p . In Fig.3, k_a and c are the equivalent spring stiffness of air and damping coefficient, c respectively. z(t) means the displacement of water surface elevation with air pressure, p. In the present paper, the effect of these equivalent stiffness k and damping coefficient c_a which are frequency dependent and are not constant are focussed.

As the results of this modelling for air chamber, Mechanical modelling for a OWC device may be considered as the system shown in Fig.4 where k_b means restoring force spring due to buoyancy and c_w is equivalent damping coefficient due to energy dissipation of wave radiation. M_w and M^* are mass of oscillating water column and its added mass respectively.

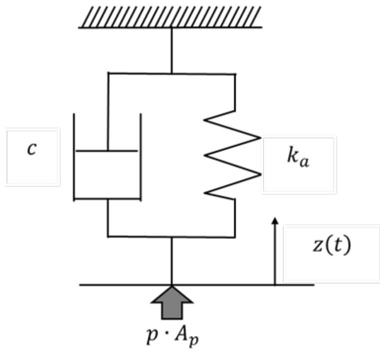

Fig.3 Equivalent Voight model

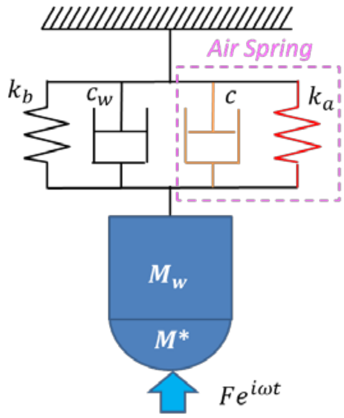

Fig.4 Equivalent mechanical model for a cylindrical OWC device

B. Theory of air damper model for a cylindrical OWC

Mechanical model of air damper is formulated based on the paper presented by Asami[4,5]. In the formulation for the relation between dynamic pressure of the air chamber and the heaving motion of water plane, the followings are assumed.

a) The cylinder is stationary, and the piston or OWC oscillates at a constant frequency
b) The state of air in the OWC changes under adiabatic process.
c) Frictional resistance acting on the sidewall due to heaving motion of OWC can be neglected.
d) Viscous resistance of air can be neglected.
e) Pressure fluctuation in the cylinder can be approximated by sinusoidal function.
f) Air pressure fluctuation in the cylinder is much smaller than atmospheric pressure.

From Poisson’s law, state equation of air under adiabatic process is expressed by the following equation.

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma} \tag{1}$$

where p and p_0 are air pressure in air chamber and atmospheric pressure respectively, and ρ and ρ_0 are the corresponding densities of air. γ is the ratio of specific heat of air and is 1.4.

Equation of continuity considering mass conservation is expressed as follows,

$$\rho_0 A_p h_p = \rho A_p \left(h_p - z\right) + \int_0^t Q_m \, dt \tag{2}$$

where h_p is height of air chamber, Q_m is mass flow rate passing through the orifice at the top of OWC device, that is,

$$Q_m = \rho_0 C_S A_N V \tag{3}$$

where C_s is the coefficient of contraction of air flow at the orifice and V is the flow velocity of air at the orifice.

Bernoulli's theorem gives the relation between pressure and velocity as follows.

$$\frac{1}{2}\rho_0 V^2 + p_0 = \frac{1}{2}\rho_0 \dot{z}^2 + p \tag{4}$$

Considering that airflow velocity, V is much larger than the vertical velocity of water plane in OWC, the velocity of air flow at orifice and the mass flow rate can be expressed approximately as follows respectively.

$$V = \mathrm{sgn}\left(p - p_0\right)\sqrt{\frac{2}{\rho_0}\left|p - p_0\right|} \tag{5}$$

$$Q_m = \mathrm{sgn}\left(p - p_0\right) C_S A_N \sqrt{2\rho_0 \left|p - p_0\right|} \tag{6}$$

From Eqs.(1) to (6), the relation between dynamic pressure fluctuation in air chamber and motion of water plane elevation are expressed by the following non-dimensioned equation.

$$\frac{1}{\gamma}\left(1 - Z\right)\left(1 + P\right)^{1/\gamma - 1}\frac{dP}{dT} + \mathrm{sgn}\left(P\right)\frac{2\pi}{\gamma N_0}\sqrt{\left|P\right|} = \left(1 + P\right)^{1/\gamma}\frac{dZ}{dT} \tag{7}$$

where P, Z, and T are non-dimensioned pressure change, water plane elevation displacement, and time respectively defined as the followings,

$$P = \frac{p - p_0}{p_0} \tag{8}$$

$$Z = \frac{z}{h_p} \tag{9}$$

$$T = \frac{\omega t}{2\pi} \tag{10}$$

where ω is oscillating angular frequency corresponding to wave frequency. And non-dimensional parameter N_0 is defined as follows.

$$N_0 = \frac{\omega A_p h_p}{\gamma C_S A_N}\sqrt{\frac{\rho_0}{2 p_0}} \tag{11}$$

C. Equivalent damping and spring coefficients in Voigt model for the present OWC device

By linearizing Eq (7), governing equation becomes,

$$\left(1 - Z\right)\frac{dP}{dT} + \frac{2\pi}{N} P = \left(\gamma + P\right)\frac{dZ}{dT} \tag{12}$$

Furthermore, by using perturbation technique, relation between air pressure fluctuation and water plane elevation can be finally expressed as follows.

$$(p - p_0) A_p = c\dot{z} + k_a z \tag{13}$$

where

$$c = \frac{N}{1 + N^2}\frac{\gamma p_0 A_p}{\omega h_p} \tag{14}$$

$$k_a = \frac{N^2}{1 + N^2}\frac{\gamma p_0 A_p}{h_p} \tag{15}$$

$$N = \sqrt{\frac{\sqrt{1 + N_1^4} - 1}{2}} \tag{16}$$

$$N_1 = \frac{\omega A_p}{b_1 C_s A_N}\sqrt{\frac{\rho_0 z_0 h_p}{\gamma p_0}} \tag{17}$$

where z_0 is the amplitude of water line elevation in an OWC cylinder.

IV. Discussion of Scale Effect by Calculation

A. Cylindrical OWC devices of small and large scales

A small OWC device that may be used in the wave tank test and a large full scale OWC device that may be installed in the real ocean are selected to investigate scale effect due to air damper model generated in the previous section as shown in Table I and parameters of the dimensions are shown Fig.5. Scale ratio is defined by the ratio of the test tank size OWC and the size of actual OWC device. In the case of Table I, scale ratio is 1/30. Experiments and numerical analysis were performed for regular waves with amplitude of 0.015m in the small size for tank test and 0.45m in the large size for actual device where the wavelength range is roughly between 5 and 50.

TABLE I
DIMENSIONS OF SMALL AND LARGE OWC DEVICES

	External Diameter D [m]	Thickness t[m]	Height H [m]	draft d [m]	Nozzle ratio
tank size OWC	0.2	0.0030	0.4	0.15	1/200
actual size OWC	6.0	0.090	12.0	4.5	1/200

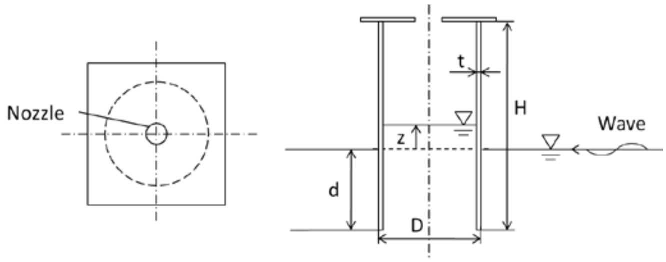

Fig.5 Dimensions of a cylindrical OWC device

B. Scale effect for equivalent spring and damping for a OWC

Equivalent spring stiffness k_a and damping c were calculated for various sizes or various scale ratios by using Eqs.(14) and (15) selecting water elevation in OWC, z_0 =0.45m x (scale ratio). Both the added stiffness and damping due to the air behaviour in OWC cylinder are normalized into non-dimensional parameters.

Fig.6 shows the equivalent ratio of air spring stiffness to buoyancy spring constant k_b which is expressed by,

$$k_b = \rho_w g A_p \tag{18}$$

where ρ_w is the density of water.

The spring stiffness ratio means added spring stiffness ratio due to air compressibility in OWC cylinder to buoyancy spring. Generally the larger the size of device is, the larger this added stiffness becomes. In case of tank test size (scale ratio is generally less than 0.1), the added stiffness can be neglected. But in case of real large OWC device (scale ratio=1), the added stiffness cannot be neglected. This is an important scale effect. The added stiffness effect is larger for higher wave frequency as well.

The damping ratio of OWC device defined by,

$$\zeta = \frac{c}{2\sqrt{M\left(k_a + k_b\right)}} \tag{19}$$

where the equivalent mass of OWC is defined by

$$M = M_w + M^* = \rho_w \pi \left(\frac{D_{in}}{2}\right)^2 \left(d + \frac{1}{3} D_{in}\right) \tag{20}$$

referring to Fig. (4). Here D_{in} is the inner diameter of the cylinder. Fig.7 is the calculated results of equivalent damping ratio of an OWC device. The equivalent damping of tank test size is different from that of large real size OWC device and smaller. This tendency is remarkable for high wave frequency.

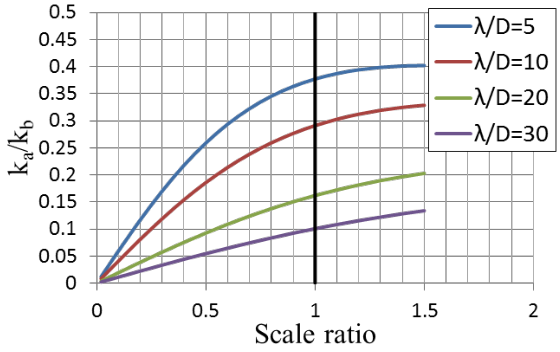

Fig.6 Equivalent spring stiffness ratio of air chamber of an OWC device (Z_0=0.45m x scale ratio)

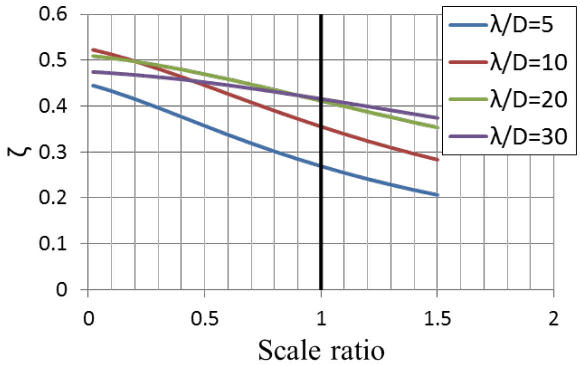

Fig.7 Equivalent damping ratio of an OWC device (Z_0=0.45m x scale ratio)

C. Scale effect of air compressibility on performance of OWC device in regular waves

The added stiffness effect and damping effect of air derived in the Section III have been introduced into the present numerical program to calculate the performance of energy conversion in regular waves. Newly introduced relation between dynamic air pressure in OWC cylinder and the elevation of inner water surface by Eq.(13) are used instead using the previous relation shown in Eq.(3).

Frequency response curves for water elevation amplitude in OWC device, dynamic air pressure amplitude, phase difference between the pressure and inner water elevation, and primary energy conversion. Wave amplitude of incident regular wave is selected as a=0.45m x (scale ratio). The scale effect is small for inner water surface elevation. However, the scale effect is remarkable for dynamic air pressure and primary energy conversion efficiency. Peak frequency moves left and the efficiency globally reduced as the size of OWC becomes larger.

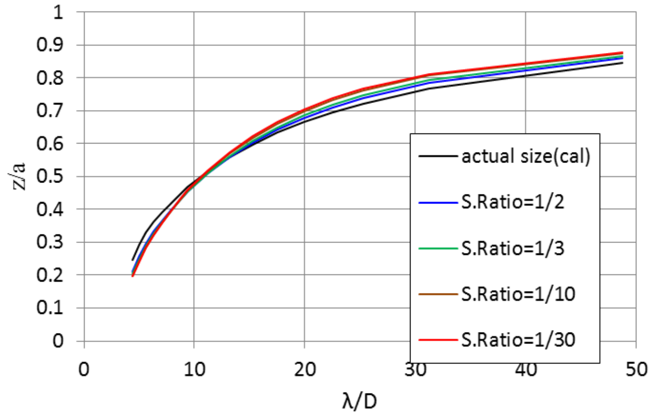

Fig.8 Numerical result of water surface elevation amplitude
(a: wave amplitude=0.45m * S.Ratio)

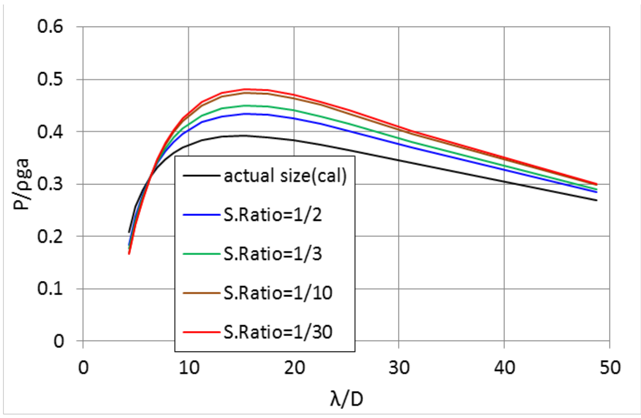

Fig.9 Numerical results of dynamic air pressure amplitude of inner air
(a: wave amplitude=0.45m * S.Ratio)

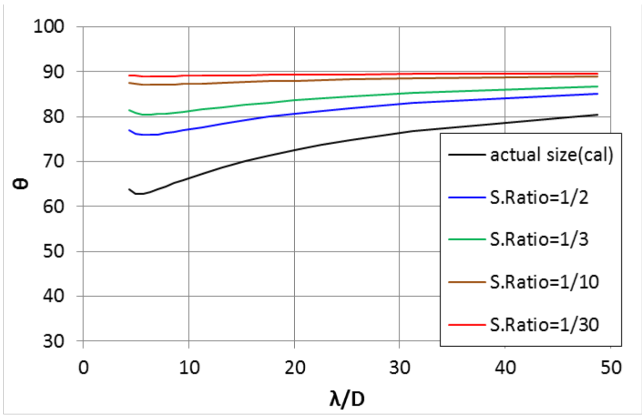

Fig.10 Numerical results of phase difference between dynamic pressure and water surface elevation (a: wave amplitude=0.45m * S.Ratio)

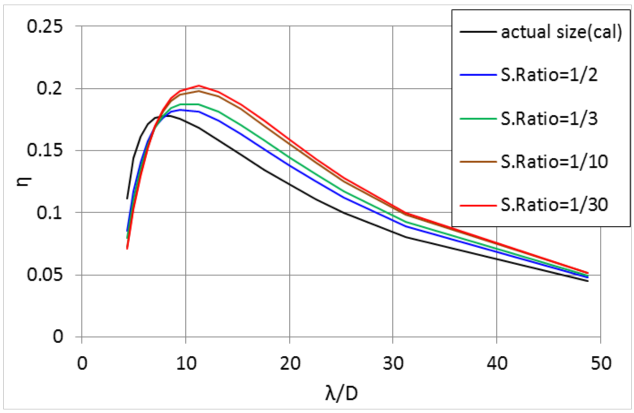

Fig.11 Numerical result of primary energy conversion efficiency
(a: wave amplitude=0.45m * S.Ratio)

V. COMPARISON OF NUMERICAL RESULTS AND EXPERIMENTS

To compare the present numerical results with experiment, wave tank test for medium scale (scale ratio= 1/12) has newly executed in the deep towing tank in RIAM by the same procedure and same facility as the wave tank test of small scale OWC device. Each scale and dimension of OWC specimen are shown in Table 2.

Table 2 Dimensions and scale ratios of experimental specimen of OWC devices

Scale	Scale ratio	External Diameter D [m]	Height H [m]	Draft d [m]
Small	1/30	0.2	0.4	0.15
Medium	1/12	0.5	1.0	0.375
Real size	1	6.0	12.0	4.50

Experimental results of inner water elevation amplitude, dynamic air pressure amplitude, phase, and primary energy conversion efficiency are shown in Figs.12, 13, 14, and 15 by small circles for various wave periods. Black circles are the results of small scale OWC (scale ratio=1/30) and red circles are newly obtained results for medium scale OWC device (scale ratio=1/12). Red and black solid lines shows the numerical results considering the air damper mechanics. However, the differences between the red and black lines are very small because medium scale OWC is too small to indicate the effect of air spring and damper effect. Further investigation of the scale effect of equivalent spring and damper effect is needed.

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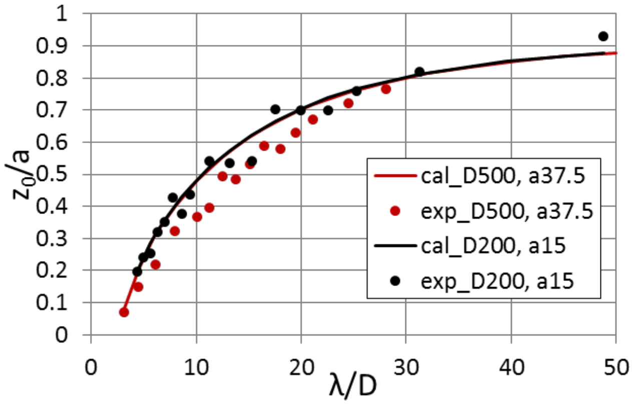

Fig.12 Experimental and calculation results of water surface elevation amplitude (a: wave amplitude=0.45m * S.Ratio)

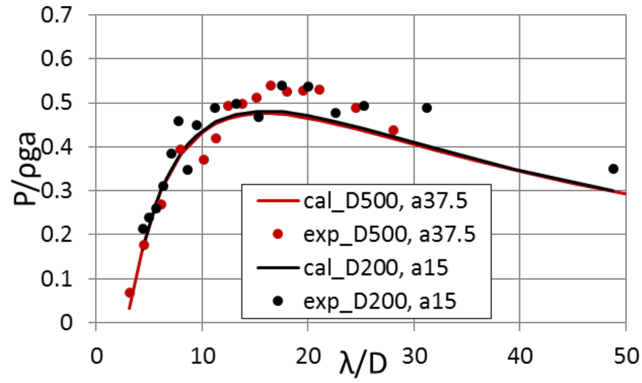

Fig.9 Experimental and calculation results of dynamic air pressure amplitude of inner air (a: wave amplitude=0.45m * S.Ratio)

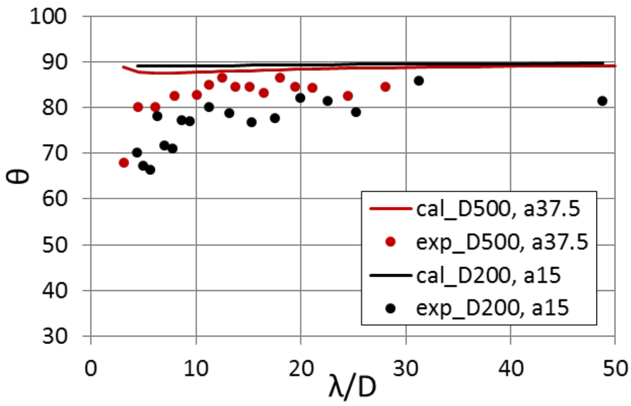

Fig.10 Experimental and calculation results of phase difference between dynamic pressure and water surface elevation (a: wave amplitude=0.45m * S.Ratio)

VI. CONCLUSIONS

In the present study, we have examined the scale effect of OWC device, focusing on the air compressibility of air chamber in OWC device, and have constructed mechanical modelling for the air chamber mechanics based on the theory of air damper with an orifice. The relation between the water elevation motion and dynamic pressure in the air chamber is derived. We have applied it to our numerical program to estimate the primary energy conversion efficiency of OWC devices for various sizes. It is shown that the air compressibility has to be considered in case of realistic large size OWC device judging from calculation results. Medium scale OWC device are tested in tank test in regular waves but the scale is still too small to prove the scale effect we showed in the numerical analysis results. Further experimental research to prove the scale effect is needed.

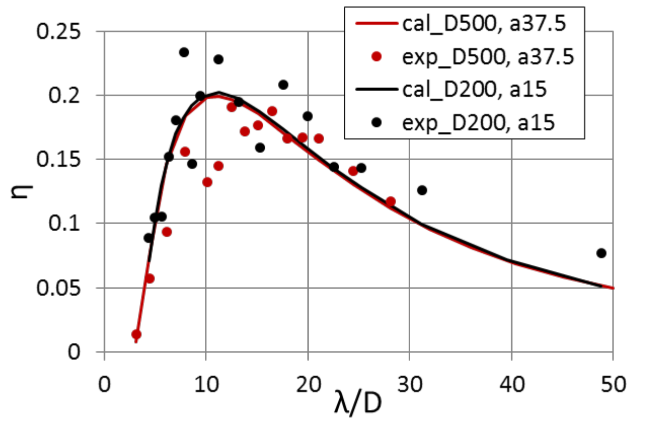

Fig.11 Experimental and calculation results of primary energy conversion efficiency (a: wave amplitude=0.45m * S.Ratio)

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