

# Optimization of resistive load for a wave energy converter with linear generator power take off

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**Abstract**— The concept of the wave energy converter, developed at Uppsala University is based on a linear generator. Direct power take off mechanism is applied in the system. Optimization of wave energy converter by maximizing its average output power is carried out. In this paper, the passive control by a resistive load is applied in the model. The generator damping force is modelled using the equivalent electrical circuit representation of the generator with the resistive load connected. The resistive load of the outer circuit  $R_{load}$  is the optimized parameter. Matlab function pattern search is used in the optimization algorithm to obtain the optimal  $R_{load}^{opt}$  for the maximum value of the average output power. The problem is solved for regular plane-parallel waves and the annual average power output is estimated for the wave climate at the Lysekil test site. Cummins equation and State Space Method are utilized to calculate the body motion in time domain.

**Keywords**— Wave energy converter, point absorber, optimization, resistive load, linear generator, state space method

## I. INTRODUCTION

Sea wave is a promising source of renewable energy, available to harvest by countries having a shoreline. Wave energy converter (WEC) of a point absorbing type (Fig. 1) is a concept, developed at Uppsala University since 2002 [1]. The buoy on the water surface absorbs kinetic and potential wave energy that is directly translated to the linear generator via a stiff wire. The generator consists of a stator and a translator. The translator is mounted with permanent magnets that provide a fluctuating magnetic flux through the stator windings when the translator moves relative to the stator, driven by the incoming wave. As a result, the voltage is induced in the stator windings. The WEC design is robust due to reduced number of mechanical parts and, therefore, has less need for maintenance.

Control of the WEC is one of the essential stages in the development of the device and improvement of its operation. Reviews on different approaches towards implementation of control strategies for WEC equipped with the linear generator are presented in [2, 3]. Passive control application by resistive load was considered in [4] and by resonant rectification in [5]. Latching control strategy for a WEC was suggested by Budal and Falnes in [6]. Latching is also known as phase control, aimed to adjust phases of the excitation force and the translator velocity by locking and unlocking the translator at certain time instants. Alternatively, declutching control is investigated

in [7] and [8]. Declutching assumes assigning damping force of the generator to be zero for a certain time intervals. The successful application of these two controls are limited due to a necessity of wave prediction at WEC's location. Economical aspects of latching control has been investigated in [9] for a point absorber with a linear generator direct drive power take off. An overview of various numerical methods is presented in [10] together with a comparison of two modelling approaches: boundary element method (BEM) and numerical wave tank method (NWT) to calculate hydrodynamic forces.

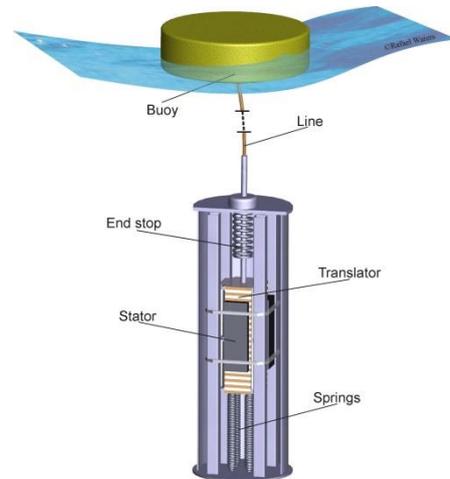


Fig. 1. Schematic illustration of Uppsala's wave energy converter connected to the buoy.

In this work, the WEC concept developed at Uppsala University is studied. The work focuses on resistive load optimization to maximize the average output power, extracted by the WEC from an incoming wave. The linear generator is modelled via the equivalent electric circuit. The wave climate at the Lysekil test site obtained from the waves measured during one year are used to produce optimum loads and calculate power matrix. The optimization is performed using non-linear optimization functions of Matlab such as pattern search and minimization function with constraints, their computational performance and precision is compared for each sea state as well as for the annual average absorbed power estimation. The same initial guess is used for all wave conditions.

The rest of the paper is organized as follows. In Section II, methods, used to solve the problem, are presented. In Sec-

tion III, the obtained results are presented and discussed and conclusions of the work come in Section IV.

## II. STATEMENT OF THE PROBLEM

Modelling of the WEC is carried out, taking into account all steps of energy conversion from the incoming wave to the output power, so called ‘wave-to-wire’ approach. The cylindrical buoy is partially submerged and floats on the water surface. The direct drive linear generator PTO is mounted on a concrete foundation and stands on the seabed (see Fig. 1). One of the degrees of freedom, the heave, is the most essential for linear generator operation and, therefore, is considered in the calculations. Motion of the buoy and translator in the linear generator are caused by forces, illustrated in Fig. 2.

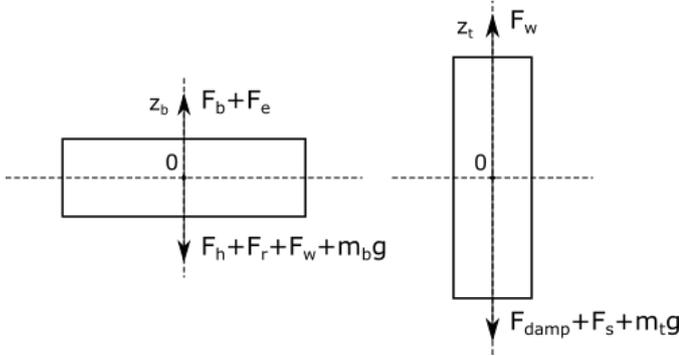


Fig.2. Forces acting on the buoy (left) and translator (right).

The dynamics of the WEC is described by the following equations:

$$m_b a_b = -m_b g + F_b + F_e - F_w - F_h - F_r \quad (1)$$

$$m_t a_t = -m_t g + F_w - F_{damp} - F_s \quad (2)$$

where  $m_b$ ,  $m_t$  are the masses of buoy and translator correspondingly;  $g$  is the gravitational acceleration;  $a_b$ ,  $a_t$  are the accelerations of the buoy and translator;  $F_w$  is the stiff wire force;  $F_h$  is the hydrostatic stiffness force;  $F_r$  is the radiation force;  $F_{damp}$  is the damping force of the linear generator;  $F_b$  is the buoyancy force;  $F_e$  is the excitation wave force;  $F_s$  is the end stop spring force.

In the linear generator, the upper spring acts as an energy storage, as well as securing the ends of the WEC hull from destructive influence of extreme forces, decelerating the translator. End stop spring force can be found as:

$$F_s = \begin{cases} k_s(z_t \pm l_f), & \text{for } l_f < |z_t| \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

where  $l_f$  is the distance till the upper/lower end stop spring;  $z_t$  is translator displacement.

The connecting wire between the buoy and translator is assumed to be tight due to the downward forces acting on the translator. Assuming the buoy transfers the motion directly to the translator in the linear generator, and the frictional losses

in the connection wire are neglected. The stiff wire force is given by:

$$F_w = \begin{cases} k_w(z_b - z_t), & \text{for } z_b \geq z_t \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where  $k_w$  is the spring stiffness coefficient.

The damping force of the linear generator is modelled using the equivalent electric circuit representation of the generator and connected load [9].

### A. Hydrodynamic modelling

Interaction of the buoy with the fluid is the first step of wave energy conversion. The hydrodynamic force  $F_{hd}$  acting on the buoy includes excitation, radiation and hydrostatic stiffness forces:

$$F_{hd} = F_e - F_r - F_h \quad (5)$$

Added mass, radiation damping, hydrostatic stiffness and excitation force coefficients are obtained using WAMIT, a Boundary Element Method (BEM) based software, where the fluid is assumed irrotational, inviscid and incompressible.

The radiation force from the radiation problem of the wave-body interaction is proportional to buoy's oscillating velocity  $V_b(\omega)$  and is given by:

$$F_r(\omega) = -Z_r(\omega)V_b(\omega) \quad (6)$$

where  $\omega$  is the wave angular frequency;  $Z_r(\omega)$  is the complex radiation impedance whose real part is the radiation resistance  $R$  and the imaginary part contains the added mass  $M_a$ :

$$Z_r(\omega) = R + i\omega M_a \quad (7)$$

The hydrostatic stiffness force [11] can be found as follows:

$$F_h(\omega) = -\rho g S Z_b(\omega) \quad (8)$$

where  $Z_b(\omega)$  is the displacement of the buoy;  $\rho$  is the water density;  $S$  is the water plane area of the floating buoy.

WAMIT calculates the hydrodynamic coefficients in the frequency domain. The hydrodynamic coefficients are transferred from the frequency domain to the time domain using the inverse Fourier transform allowing for the state space method (SSM) approximation of the radiation term. In the time domain, excitation force is given by:

$$F_e(t) = f_e(t) * \eta(t) \quad (9)$$

where  $\eta(t)$  is the water surface elevation with respect to the mean water level;  $f_e(t)$  is the impulse response function of the excitation force; and the asterisk denotes the convolution.

Taking into account the above, we can find the radiation force in time domain by:

$$F_r(t) = M_a a_b(t) + \int_0^t K(t - \tau) v_b(\tau) d\tau \quad (10)$$

where  $K$  is the inverse Fourier transform of radiation impedance or impulse response function of the retardation damping force in the heave motion.

### B. State space method

SSM is used to approximate the convolution term of the impulse response function and the translator velocity in (10), since SSM allows reducing the computational time compared to direct calculation of the convolution integral. By SSM we obtain the buoy and translator position and velocity in time domain. In the state space realization, the input  $u(t)$  and output  $y(t)$ , providing approximation to the integral term, are given by [12]:

$$\dot{X}(t) = AX(t) + Bu(t) \quad (11)$$

$$y(t) = CX(t) \approx \int_0^t K(t-\tau)v_b(\tau)d\tau \quad (12)$$

where  $X(t)$  is the state vector;  $A$ ,  $B$  and  $C$  are the system matrices of the state space coefficients that can be found in [12]. The initial condition for the model is  $X(0)=0$  for  $t=0$ .

Matlab function ode45 was used to find the solution for the differential equations (1), (2) and (11).

It is worth noting that SSM is a time domain simulation method that is suitable for optimization problems in wave energy field due to its ability to combine hydrodynamic modelling with non-linear behaviour of the total system in the energy conversion [12].

### C. Calculation of the power output

Equivalent electric circuit of the linear generator [13] is illustrated in Fig. 3, where  $R_{load}$  is the passive control by resistive load. The resistive load value is optimized with relation to the average output power as the objective function. Some parameters of the WEC can be found in Table I.

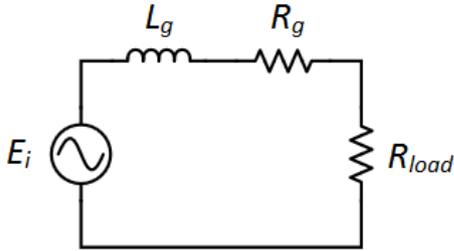


Fig.3. Equivalent electric circuit of the linear generator, where  $L_g$  and  $R_g$  – generator inductance and resistance;  $E_i$  – voltage source;  $R_{load}$  – passive control by resistive load.

The active electrical power  $P$  that is extracted by the WEC from the incoming wave is found by [10]:

$$P = F_{damp}v_t \quad (13)$$

where  $v_t$  is the translator velocity.

The damping force of the linear generator is given as follows:

$$F_{damp} = \frac{3E_i^2 R}{R^2 + X^2} \frac{1}{v_t} \quad (14)$$

where  $R = R_g + R_{load}$  is the total resistance of the system;  $v_t$  is the translator velocity.

The induced voltage  $E_i$  in the stator windings is as follows:

$$E_i = \frac{2\pi N\Phi_0}{\tau_p} v_t \quad (15)$$

where  $N$  is number of turns in the stator windings;  $\Phi_0$  is the amplitude of the magnetic flux;  $\tau_p$  is the pole pair width.

The inductive reactance  $X$  of the generator is given by:

$$X = \frac{2\pi L_g v_t}{\tau_p} \quad (16)$$

where  $L_g$  is generator inductance.

The average power at the time interval from 0 to  $t$  is given by:

$$P_{av}(t) = \frac{1}{t} \int_0^t F_{damp} v_t dt \quad (17)$$

When calculating the output power, the situation of a partial overlap of the stator and translator is taken into account (see Table I) by making the induced voltage proportional to the active area of the generator.

TABLE I  
PARAMETERS OF THE WEC

Parameter (dimension)	Value
Water depth (m)	25
Buoy inner radius (m)	1.9
Length of translator (m)	2.1
Length of stator	1.2
Length of end stop spring (m)	0.62
Spring coefficient (kN/m)	270
Stiffness coefficient of the wire (kN/m)	833
Maximum upper stroke length (m)	1.6
Maximum lower stroke length (m)	1.1
Buoy mass (kg)	3000
Translator mass (kg)	5000
Nominal voltage (V)	148
Generator design speed (m/s)	0.7
Pole pair width (m)	0.08
Electric frequency (rad/m)	79
Generator resistance ( $\Omega$ )	0.64
Generator inductance (H)	0.02

### D. Input wave

Regular plane-parallel wave is used as an input to the model. A sinusoidal harmonic wave can be expressed as follows:

$$\eta(t) = A \cos(\omega t) \quad (18)$$

where  $A = H/2$  is the wave amplitude equal to half wave height  $H$  and  $\omega$  is the wave angular period equal to  $2\pi/T$ ,  $T$  is the wave period.

Wave climate at the Lysekil test site of Swedish west coast [14] is taken as a basis for the modelling. Wave height  $H$ , wave period  $T$  and occurrence frequency during one year time period are shown in Fig.4.

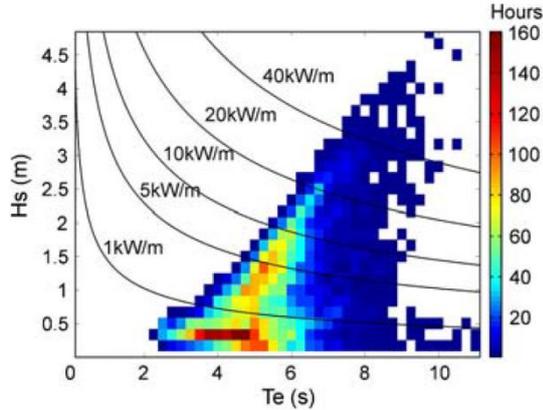


Fig.4. Wave climate at Lysekil test site during one year [14].

### E. Optimization method

Optimization of the passive control by resistive load is aimed to find the fitting value of the resistive load corresponding to the maximum output power in various wave conditions.

The optimization is carried out using the Matlab function patternsearch. It is based on the polling method with the adaptive mesh, aligned with the coordinate directions. The search begins with the provided initial point. The algorithm computes the objective function at the mesh points, by adding to the coordinates of the mesh point the coordinates of the initial value. The objective function is calculated for all points of the mesh until the maximum value is found. Then the next point is set for the sequence. The size of the mesh is being adapted during the search, for example, in the case of unsuccessful polling, the mesh size decreases and increases in case of successful polling. In comparison with fmincon and fminunc the function patternsearch gives precise result and find the global minimum [15].

The resistive load is optimized relatively to the objective function of the average output power, eq. (17). The constraint to the problem is  $R_{load} > 0$ . An initial value of the resistive load of  $0.001 \Omega$  is assumed.

## III. RESULTS AND DISCUSSION

For harmonic waves and for each wave condition, the solution is obtained by pattern search solver the number of iterations varies from 22 through 48 with the mean of 34 iterations. Number of iterations here defines how many points were polled before obtaining the fitting one. Beside this, one of important optimization parameters is the number of function evaluations (NFE). For each value of the optimum output power, the NFE varies from 41 through 83 with the mean of

62 evaluations. In all simulations, the optimization was terminated when the mesh size tolerance limit was reached, and the mesh size tolerance level was set to  $10^{-6}$ .

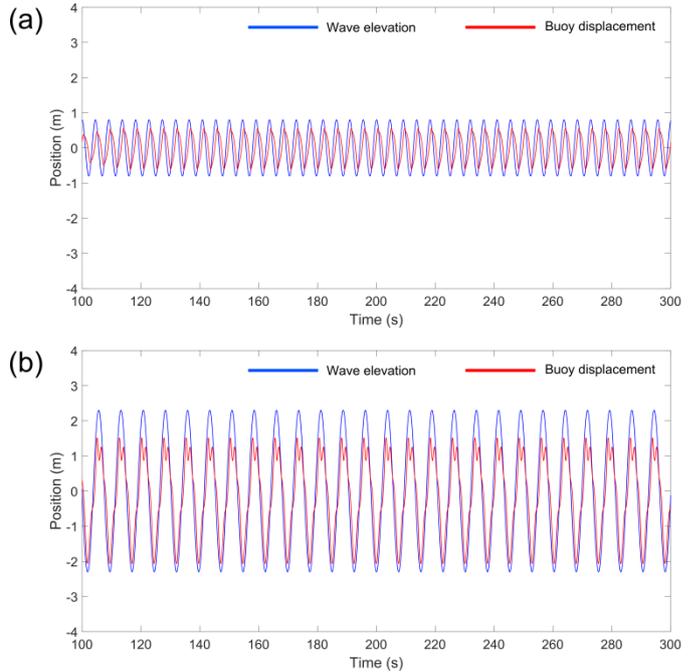


Fig. 5. Wave elevation (blue) and buoy displacement (red) for a period of 200 s with regular waves: (a) wave condition of  $H=1.5$  m,  $T=5$  s; (b) wave condition of  $H=4.5$  m,  $T=9$  s.

Non-linear optimization algorithm is one of the accessible variants to find the optimal damping force. Compare with the sweep parameter optimization, it provides precise solution and can speed up the searching procedure. For example, sweep parameter optimization can lead up to 100 iterations to obtain a solution.

Obtained results on wave elevation and buoy displacement are shown in Fig. 5. Fig.5a shows data for  $H=1.5$  m,  $T=5$  s and Fig.5b presents the wave condition of  $H=4.5$  m,  $T=9$  s.

Optimal value of passive resistance  $R_{load}^{opt}$  was found for the wave climate parameters, corresponding to the Lysekil wave data from [14] and the results are presented in Fig. 6.

From the Fig. 6, one can notice that the increasing wave height leads to an increased power absorption.

For shorter waves, larger resistance is required. This, in turn, means that the lower damping force needs to be applied in the generator. Moreover, for the same wave height, shorter waves contain less wave power (wave power flux per meter of wave crest length is proportional to the square of the wave height). Therefore, less damping force can be provided by short-circuited generator.

For the wave heights of 2 m and higher, there is no significant increase in the absorbed power (the maximum of average power output for the WEC parameters is calculated to be about 25kW). Perhaps, that is due to the limited stroke length of the particular device, designed for the Lysekil wave climate.

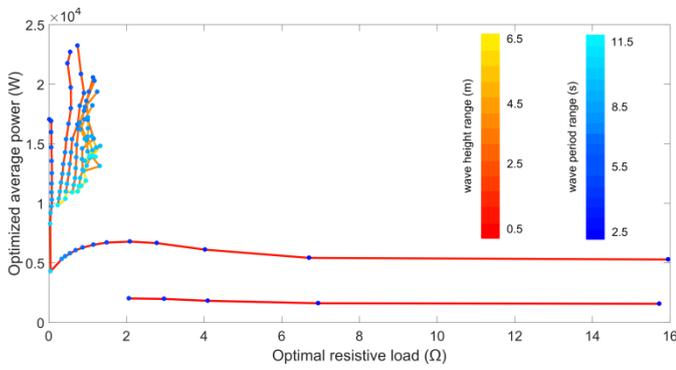


Fig. 6.  $R_{load}^{opt}$  to the average power output for the different wave conditions with regular waves. Here, equal wave heights are connected by representative coloured line, wave periods are indicated by the data point colours.

Considering the future research, it would be interesting to compare other non-linear optimization methods, as well as the sensitivity to initial conditions and time efficiency investigation.

#### IV. CONCLUSIONS

In this paper, the problem of optimization of wave energy converter with passive control by resistive load was stated and solved. The goal of the optimization was to achieve the maximum output power for the optimal values of the resistive load as a passive control. Behaviour of the system with nonlinear characteristics of the power conversion was analysed. The generator damping force was modelled using the equivalent electric circuit representation of the generator and connected load.

Matlab function patternsearch on the basis of the polling method with the adaptive mesh, aligned with the coordinate directions was used to find the solution for maximization of power output of WEC with constraint  $R_{load} > 0$ . Damping force as a function of the translator velocity was calculated for the regular waves, provided for the Lysekil test site wave climate.

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