A Numerical Calculation for Hydrodynamic Response Analysis of a Multi-buoy WEC Platform

Sanghwan Heo[#1], Weoncheol Koo[#2], Min-Su Park [*3]

[#]*Department of Naval Architecture and Ocean Engineering, Inha University*
Incheon, Republic of Korea

[1]shheo@inha.edu

[2]wckoo@inha.ac.kr

[*]*Korea Institute of Civil Engineering and Building Technology*
Gyeonggi, Republic of Korea

[3]mspark@kict.re.kr

***Abstract*— In this study, numerical analysis was performed to obtain the hydrodynamic responses of a multi-buoy WEC platform. The hydrodynamic forces acting on the submerged slender-type structure members and the buoys were calculated using the modified Morison equation and commercial software. The natural frequencies and mode shapes of the platform were determined by using the modal analysis. The Newmark-beta time-integration method was employed to produce time series of displacement and bending stresses..**

***Keywords*— Dynamic response, Modal analysis, Morison equation, Newmark-beta method, WEC platform**

I. INTRODUCTION

Nowadays, fossil fuels are the primary energy sources, which are non-renewable and limited resources. Thus, the studies on renewable energy development have been carried out actively throughout the world. Among renewable energy sources, ocean wave energy is one of the viable resources, and its amount of power generation is gradually increasing [1]. In other words, wave power could be considered as one of the promising marine renewable energy resources.

Wave Energy Converter (WEC) transforms ocean waves into electricity. A typical point absorber type WEC is Wave Star [2]. It has multiple buoys connected to a bridge structure. Generally, the platform structure is fixed to the sea bottom by steel piles or lifted by jack-up legs. During the operation of energy take-off, the deck load and the buoy motions directly affect the legs of platform. Therefore, it is essential to evaluate the hydrodynamic responses of the structure for safety in working conditions.

In this study, numerical analysis was performed to evaluate the hydrodynamic responses of a multi-buoy type WEC platform. The mathematical formulation for a fixed-type ocean structure has been reported by various researchers [3-6]. The bridge structure was composed of cylindrical beam elements which diameter is relatively smaller than the incident wavelength. The governing equation of motion of each beam member can be expressed in matrix form and assembled by direct stiffness method [3], [4]. The submerged elements of the platform and buoys are subjected to wave forces. These hydrodynamic forces can be calculated by using the modified Morison equation and commercial software WAMIT [6]-[8]. The natural frequencies and mode shapes of the platform were determined by using the modal analysis. The Newmark-beta step-by-step time-integration scheme was applied to obtain the time series of hydrodynamic responses of the platform [9].

II. MATHEMATICAL FORMULATION

The governing equation of motion for entire members in the matrix form can be expressed as follows [3]-[5]

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F_{Leg}\}+\{F_{Buoy}\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are the mass matrix, damping matrix and stiffness matrix, respectively; $\{u\}$ is the displacement vector of the platform; $\{F_{Leg}\}$ and $\{F_{Buoy}\}$ are the hydrodynamic force vector acting on the legs and buoys, respectively.

Incident ocean wave was assumed as linear wave (Airy wave). The hydrodynamic forces acting on the submerged elements were calculated by using the modified Morison equation [6], [7]. The hydrodynamic force vector can be written as

$$\{F_{Leg}\}=[C_{Inertia}]\{\ddot{v}\}-[C_{Added}]\{\ddot{u}\}+[C_{Drag}]\{|\dot{v}-\dot{u}|(\dot{v}-\dot{u})\} \tag{2}$$

where $[C_{Inertia}]=[\ddots\rho C_M V\ddots]$, $[C_{Added}]=[\ddots\rho(C_M-1)V\ddots]$ and $[C_{Drag}]=[\ddots 0.5\rho C_D A\ddots]$ are the matrix form of the inertia coefficient, added mass coefficient and drag coefficient, respectively. C_M and C_D are inertia and drag coefficients. $\{v\}$ is the displacement vector of water particles. ρ is the density of water, V is the enclosed volume of the member and A is the projected area of the member in the direction of flow. In this study, the inertia coefficient of 2.0 and drag coefficient of 1.0 were applied. These values are the general value of a circular cylindrical and a slender element, respectively.

The hydrodynamic forces acting on the buoys were calculated by using a commercial program WAMIT. This program has been known for hydrodynamic analysis of non-slender body. The hydrodynamic force vector can be calculated by [8]

$$\left\{F_{Buoy}\right\} = \frac{\rho g H}{2}\left\{\overline{F}_{Buoy}\right\} \tag{3}$$

where, g is the gravitational acceleration, H is the incident wave height and $\left\{\overline{F}_{Buoy}\right\}$ is the normalized hydrodynamic force vector calculated by WAMIT. It provides not only the magnitude of the force but also the phase angle.

In general, the natural frequencies of a structure affect the dynamic characteristics of the structure. The natural frequencies and mode shapes of the WEC platform were determined by using the modal analysis. The equation of free vibration of the structure can be written as

$$[M]\{\ddot{u}\} + [K]\{u\} = 0 \tag{4}$$

The general solution of the displacement vector can be expressed by

$$\{u\} = \{\phi\} e^{i\omega t} \tag{5}$$

where $\{\phi\}$ is the modal vector and ω is the frequency of external force. By substituting Eq. (5) into Eq. (4), we can obtain the modal matrix and natural frequencies of the structure.

The equation of motion (Eq. (1)) can be decoupled into N single-degree-of-freedom (SDOF) equations.

$$\begin{aligned}[I]\{\ddot{q}\} + \left[\ddots 2\zeta_j\omega_j \ddots\right]\{\dot{q}\} + \left[\ddots \omega_j^2 \ddots\right]\{q\} \\ = [\Phi]^T\left\{F_{Leg}\right\} + [\Phi]^T\left\{F_{Buoy}\right\} \qquad j = 1,\cdots N\end{aligned} \tag{6}$$

where $[I]$, ζ_j and ω_j are the identity matrix, *j*-th mode of damping coefficient and *j*-th mode of natural frequency of the structure, respectively. $\{q\}$ is the standardized displacement vector and $[\Phi]^T$ is the transpose of the modal matrix.

The Newmark-beta step-by-step time-integration scheme was applied to calculate the standardized responses of each SDOF equation in the time domain [9]. The standardized responses can be obtained by following equations.

$$q_{n+1} = q_n + \delta\dot{q}_n + (0.5-\beta)\delta^2\ddot{q}_n + \gamma\delta^2\ddot{q}_{n+1}n \tag{7}$$

$$\dot{q}_{n+1} = \dot{q}_n + (1-\gamma)\delta\ddot{q}_n + \gamma\delta^2\ddot{q}_{n+1} \tag{8}$$

$$\ddot{q}_{n+1} = \overline{F} - 2\zeta_j\omega_j\dot{q}_{n+1} - \omega_j^2 q_{n+1} \tag{9}$$

where $\overline{F} = [\Phi]^T\left\{F_{Leg}\right\} + [\Phi]^T\left\{F_{Buoy}\right\}$, n is time step, δ is time increment, β and γ are Newmark coefficients. In this study, the constant average acceleration method was applied ($\beta = 0.25$, $\gamma = 0.5$).

III. Numerical Results and Discussion

Fig. 1 shows the model of a simplified multi-buoy WEC platform. This platform was modelled with 112 nodal points and 114 three-dimensional beam elements. The length of each leg was 25 m. The external loads were exerted in the x-direction only. The deck of the platform was connected with legs at 16 m above the sea bottom. The water depth was 10 m and the air gap was 6 m. Six buoys were connected to the platform by arms. In this study, it is assumed that the arm was fixed to the structure so that the forces acting on the buoy could be transmitted directly to the structure. Fig. 2 shows the model of a buoy. It was hemispherical in shape with a diameter of 3 m and a draft of 1.5 m. Each buoy was modelled with 1728 nodal points and 432 panels. These nodal points can describe the submerged buoys with smooth surfaces. The structural properties of the platform and buoy are in Table 1.

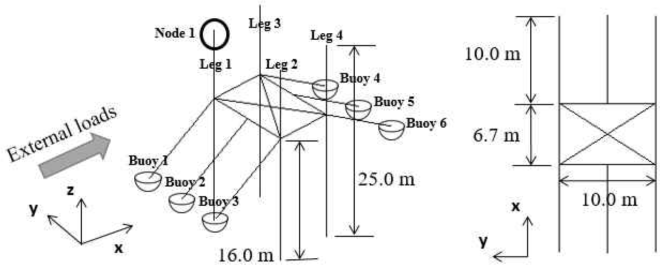

Fig. 1 Finite element model of the simplified WEC platform with multiple buoys

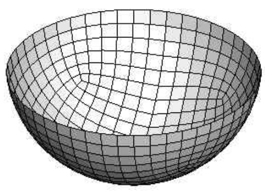

Fig. 2 Model of each buoy

TABLE I
STRUCTURAL PROPERTIES OF THE SIMPLIFIED WEC PLATFORM

Description	Value
Deck size	15 × 10 × 2 m
Diameter of a buoy	3 m
Draft of a buoy	1.5 m
Diameter of leg member	0.6 m
Thickness of leg member	15 mm
Deck weight	980 kN
Structural weight per unit volume	78 kN/m^3
Modulus of elasticity	2.1 × 10^8 kN/m^2
Modulus of rigidity	8.3 × 10^7 kN/m^2

TABLE II
MAGNITUDE AND PHASE ANGLE OF HYDRODYNAMIC HEAVE FORCES ACTING ON THE BUOYS

Quantity	Buoy 1	2	3	4	5	6
Magnitude (kN)	113.4	112.8	113.4	111.6	110.9	111.6
Phase angle (degree)	31.69	31.74	31.69	-68.3	-68.2	-68.3

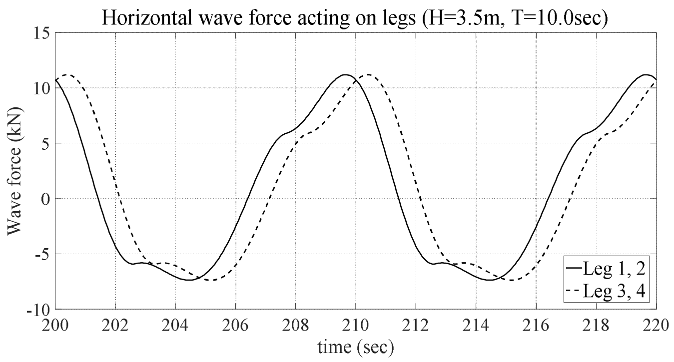

Fig. 3 Time history of horizontal wave force acting on legs (H = 3.5 m, T = 10.0 sec)

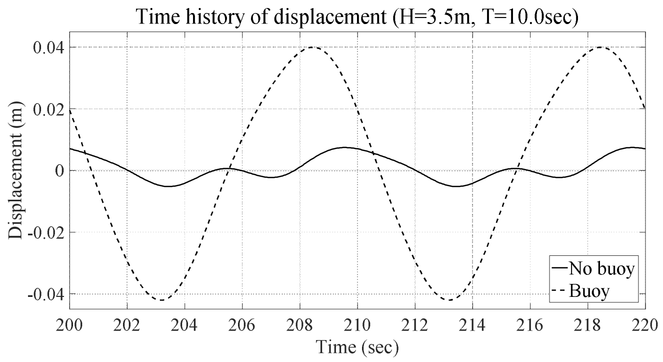

Fig. 4 Time history of displacement at nodal point 1 (H = 3.5 m, T = 10.0 sec)

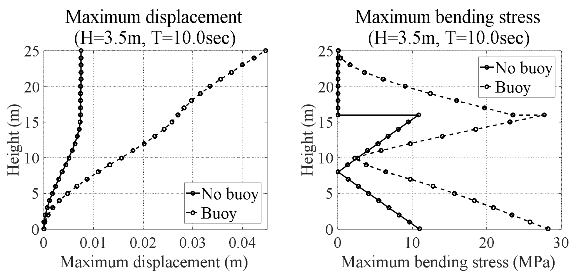

Fig. 5 Maximum displacement and maximum bending stress of the platform

In this study, we assume that the platform was connected to the centre of buoy. The connection point had rotation-free condition. Therefore, the translational forces (surge, sway, heave) were only considered for each buoy. As an incident wave condition, wave height of 3.5 m and period of 10 sec were applied. Table 2 shows the magnitude and the phase angle of hydrodynamic heave forces on the buoys, which forces are larger than other translational forces. Because of the interaction between each buoy, the magnitude of force on the buoys in the same line was slightly different (front 1-3, rear 4-6).

For time domain analysis, time increment of 0.05 sec and total simulation time of 250 sec were adopted. Fig. 3 shows the time history of horizontal wave forces acting on each leg. The shape of horizontal wave force had the same value in all legs. However, due to the distance between the legs, a phase difference occurred. The difference was about 0.73 sec, which value can be calculated by the ratio of the gap between legs (6.7 m) to the wave celerity (9.24 m/s). In addition, it can be seen that the maximum horizontal force was only about 10% of the heave force acting on the buoys. Because the submerged beam elements of the platform were slender member, the horizontal wave loads were relatively small. Therefore, it is inferred that the platform response can be greatly dependent of the presence or absence of hydrodynamic forces on the buoys. Fig. 4 shows the time history of platform displacement at nodal point 1. As predicted earlier, the hydrodynamic forces acting on the buoys give a great impact on the platform displacement.

Fig. 5 shows the maximum displacement and the bending stress of the platform along the vertical direction. Since the platform was fixed at the sea bottom, the displacement is zero at the zero point (sea bottom). The maximum displacement increased with the distance from the sea bottom in both cases. At the top of the platform (25 m), the maximum displacement was about 6 times difference between buoy inclusion and no buoy (only platform). The maximum bending stresses were observed at critical points such as 16.0 m and the sea bottom (0 m). In the middle of the leg, the bending stress was relatively small because of small moments act at the area. Since the forces on the buoy were transmitted to the platform through the arm, the point of minimum moment moved up and the resultant bending stress also occurred at the point.

IV. CONCLUSIONS

In this study, numerical analysis was performed to calculate the hydrodynamic responses of a multi-buoy WEC platform in the time domain. The bridge structure was composed of cylindrical beam elements which diameter is relatively smaller than the incident wavelength. The hydrodynamic forces acting on the slender members were calculated by using the modified Morison equation. The diffracted wave loads acting on the buoy were calculated by a commercial software. The Newmark-beta step-by-step time-integration scheme was applied to obtain the time series of hydrodynamic responses of the platform

The maximum displacement and bending stress of the platform were calculated and compared their results according to the effects of buoy existence. It was found that the maximum displacement had about 6 times difference between buoy inclusion and no buoy (only platform) at the top of the platform. Therefore, the hydrodynamic behaviour of the present type of ocean structure can be greatly affected by the buoys, because wave loads on the submerged slender beam elements are relatively small.

Finally, the developed numerical scheme can be a useful tool for analysing the hydrodynamic performance of multi-purpose ocean structure connected to other equipment (e.g., wind turbine, current turbine, or solar panel).

ACKNOWLEDGMENT

This research was a part of the project titled, “Manpower training program for ocean energy” funded by the Ministry of Oceans and Fisheries, Korea. This research was also financially supported by the “Korea-UK Global Engineer Education Program for Offshore Plant” through the Ministry of Trade, Industry&Energy(MOTIE) and Korea Institute for Advancement of Technology(KIAT)

REFERENCES

[1] EU-OEA, “Oceans of Energy - European Ocean Energy Roadmap 2010-2050”, 3rd ICOE, 2010.

[2] M. Kramer, L. Marquis, and P. Frigaard, “Performance evaluation of the wavestar prototype”, *The 9th European Wave and Tidal Energy Conference: EWTEC 2011*, 2011.

[3] K. Kawano, K. Venkatarama, and T. Hashimoto, “Seismic response effects on large offshore platform”, in *Proceeding of the Ninth International Offshore and Polar Engineering Conference*, 1999, p. 528-535.

[4] Hamrit, F., Necib, B. and Driss, Z, 2015, Analysis of Mechanical Structures Using Beam Finite Element Method, International Journal of Mechanics and Applications, 5(1), 23-30.

[5] M. S. Park, “Dynamic Response Evaluations of Offshore Platforms due to Wave Force Interactions and Seismic Force”, M. Sc. Thesis, Kagoshima University, Korimoto, Kagoshima, Japan, 2009.

[6] S. K. Coakrabarti, W. A. Tam, and A. L. Wolbert, “Wave Forces on a Randomly Oriented Tube”, in *Offshore Technology Conference*, 1975, pp. 433-441.

[7] J. R. Morison, J. W. Johnson, and S. A. Schaaf, “The Force Exerted by Surface Waves on Piles”, *Journal of Petroleum Technology*, vol. 2(05), pp. 149-154, May. 1950.

[8] C. H. Lee, “WAMIT theory manual”, Massachusetts Institute of Technology, Department of Ocean Engineering, 1995.

[9] N. M. Newmark, “A Method of Computation for Structural Dynamics”, *Journal of the Engineering Mechanics Division*, vol. 85(3), pp. 67-94, Jul. 1959.