Dynamic stability of a surfaced current turbine system

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Abstract-Taiwan is with the presence of Kuroshio Current in the east part of the island. This ocean energy source is stable and rich so that it is potential to be developed and utilized. Due to maintain and water-proof of system, the design of surfaced ocean turbine system is made. However, the ocean velocity and direction will vary. The wave will change with the wind. These factors significantly affect the stability of a surfaced turbine. The investigation about the instability of a surfaced turbine is helpful for the practical design of current power plant. In this study, the system is composed of turbine, buoyance platform, traction rope and mooring foundation as shown in Figure 1. In addition to the current velocity and wave, the gravity, buoyance, drag force of turbine structure on the stability are also great. The mathematical model of the system is presented. The coordinate of the ocean current turbine system is shown in Figure 2. The analytical solution of the general system is proposed. The effects of current velocity, wave, geometry parameters of structure on the pitching motion and stability are investigated.

Keywords- stability; ocean current power system; surfaced type; buoyance platform; mooring foundation.

1 Introduction

The environmental pollution due to the application of traditional energy resources (e.g., fossil fuels) is serious. The renewable energy resources such as wind, sun, ocean waves, and tidal currents are interesting. There are several different forms of ocean energy that are being investigated as potential sources for power generation [1-3]. Taiwan is with the presence of Kuroshio Current in the east part of the island. The current has a mean velocity of about 1.2~1.53 m/s near the surface. The potential electricity capacity in the Taiwan current is about 4GW. This ocean energy source is stable and rich so that it is potential to be developed and utilized. Due to maintain and water-proof of system, the design of surfaced ocean turbine system is made.

The ocean current turbine is tethered to the sea floor and uses sustained ocean currents to produce electricity. The system is composed of turbine, buoyance platform, traction rope and mooring foundation. The stability of the system is important for the commercial operation. So far, no literature is devoted to investigate the stability of the ocean current turbine system.

2 Governing equations

Considering the vertical vibration of the system subjected to the surface wave, the equations of motion are

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix}+\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} T_{Tur}\dfrac{X_0}{\sqrt{L_1^2-X_0^2}}+A_1\rho g H_0 \sin\Omega t \\ A_2\rho g H_0 \sin(\Omega t+\Phi) \end{bmatrix} \tag{1}$$

where

$$K_{11}=A_1\rho g+T_{Tur}\frac{1}{L_2}, K_{12}=K_{21}=\frac{-T_{Tur}}{L_2}, \quad K_{22}=A_2\rho g-\frac{T_{Tyr}}{L_2} \tag{2}$$

in which A_1 is the floater area, A_2 the turbine area, X_0 the depth of bed, $\{X_1, X_2\}$ the vertical displacements, T_{Tur} the tension of wire between the turbine and the floater, H_0 the wave amplitude, Ω the angular frequency of wave, Φ the phase angle, $\{M_1, M_2\}$ the masses, ρ density, g gravity.

The equations in the horizontal and pitching motion are

$$M_2\ddot{y}+C_D\rho AV\dot{y}+C_D\rho AVl\dot{\psi}=0 \tag{3}$$

$$I\ddot{\psi}-C_D\rho AVle_D\dot{\psi}+\left[We_{GB}+T_{Tur}e_D\left(\frac{X_1-X_2}{L_2}\right)\right]\psi - C_D\rho AVe_D\dot{y}=0 \tag{4}$$

where y is the horizontal displacement, ψ the pitch angle of the turbine, A is the area of drag, C_D the drag coefficient, e_{GB} the distance between the centers of gravity and buoyance, V the current velocity, *l* the radius of rotation.

3 Solution method

3-1 Vertical vibration

The solution of Eq. (1) is expressed as

$$X_1(t)=X_{10}+X_{11}(t), \quad X_2(t)=X_{20}+X_{21}(t) \tag{5}$$

Sub. (5) into (1), one obtains

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} X_{10} \\ X_{20} \end{bmatrix}=\begin{bmatrix} T_{Tur}\dfrac{X_0}{\sqrt{L_1^2-X_0^2}} \\ 0 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{bmatrix} \ddot{X}_{11} \\ \ddot{X}_{21} \end{bmatrix}+\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix} = \begin{bmatrix} A_1\rho g H_0 \sin\Omega t \\ A_2\rho g H_0 \sin(\Omega t+\Phi) \end{bmatrix} \tag{7}$$

Further, the solution of Eq. (6) is obtained

$$X_{10}=\frac{T_{Tur}\frac{X_0}{\sqrt{L_1^2-X_0^2}}}{\left(K_{11}-K_{12}\frac{K_{21}}{K_{22}}\right)},\quad X_{20}=-\frac{K_{21}}{K_{22}}X_{10} \tag{8}$$

The solution of (7) is expressed as

$$\begin{bmatrix}X_{11}\\X_{21}\end{bmatrix}=\begin{bmatrix}X_{11c}\\X_{21c}\end{bmatrix}\cos\Omega t+\begin{bmatrix}X_{11s}\\X_{21s}\end{bmatrix}\sin\Omega t \tag{9}$$

Substituting Eq. (9) into (7), the solution is derived

$$\begin{bmatrix}X_{11c}\\X_{21c}\end{bmatrix}=\frac{\beta_c}{|\boldsymbol{A}|}\begin{bmatrix}-K_{21}\\K_{11}-\Omega^2M_1\end{bmatrix},$$

$$\begin{bmatrix}X_{11s}\\X_{21s}\end{bmatrix}=\frac{1}{|\boldsymbol{A}|}\begin{bmatrix}\alpha\left(K_{22}-\Omega^2M_2\right)-\beta_sK_{21}\\-\alpha K_{12}+\beta_s\left(K_{11}-\Omega^2M_1\right)\end{bmatrix}, \tag{10}$$

where

$$|\boldsymbol{A}|=\begin{vmatrix}K_{11}-\Omega^2M_1 & K_{12}\\K_{21} & K_{22}-\Omega^2M_2\end{vmatrix}.$$

$$\begin{aligned}\alpha&=A_1\rho gH_0,\\ \beta_c&=A_2\rho gH_0\sin\Phi,\\ \beta_s&=A_2\rho gH_0\cos\Phi.\end{aligned} \tag{11}$$

Moreover, the frequency equation is

$$|\boldsymbol{A}|=0 \tag{12}$$

The natural frequencies are

$$\Omega_{1,2}^2=\frac{1}{2}\left[\left(\frac{K_{22}}{M_2}+\frac{K_{11}}{M_1}\right)\pm\sqrt{\left(\frac{K_{22}}{M_2}+\frac{K_{11}}{M_1}\right)^2-4\left(\frac{K_{11}}{M_1}\frac{K_{22}}{M_2}-\frac{K_{12}K_{21}}{M_1M_2}\right)}\right] \tag{13}$$

3-2 Horizontal and pitching vibration

Assume the solution of Eqs. (3,4)

$$\begin{aligned}y&=y_0+y_c\cos\Omega t+y_s\sin\Omega t\\ \psi&=\psi_0+\psi_c\cos\Omega t+\psi_s\sin\Omega t\end{aligned} \tag{14}$$

Sub. Eq. (14) into Eqs. (3) and using the harmonic balance method, one obtains

$$o_1y_c+o_2y_s+o_3\psi_s=0 \tag{15}$$

$$p_1y_s+p_2y_c+p_3\psi_c=0 \tag{16}$$

where

$$p_1=M_2\Omega^2,\ p_2=\Omega C_D\rho AV,\ p_3=\Omega C_D\rho AVl$$

$$o_1=-M_2\Omega^2,o_2=\Omega C_D\rho AV,o_3=\Omega C_D\rho AVl$$

Sub. Eq. (14) into Eq. (4) and using the harmonic balance method, one obtains

$$q_0\psi_0+q_1\psi_c+q_2\psi_s=0 \tag{17}$$

$$r_0\psi_0+r_1\psi_c+r_2\psi_s+r_3y_s=0 \tag{18}$$

$$s_0\psi_0+s_1\psi_c+s_2\psi_s+s_3y_c=0 \tag{19}$$

where

$$\begin{aligned}q_0&=We_{GB}\\ q_1&=\frac{1}{2}T_{Tur}e_D\left(\frac{X_{11c}-X_{21c}}{L_2}\right)\\ q_2&=\frac{1}{2}T_{Tur}e_D\left(\frac{X_{11s}-X_{21s}}{L_2}\right)\\ r_0&=-T_{Tur}e_D\left(\frac{X_{11c}-X_{21c}}{L_2}\right)\\ r_1&=-\left(-\Omega^2I+We_{GB}+T_{Tur}e_D\frac{X_{10}-X_{20}}{L_2}\right)\\ r_2&=\Omega C_D\rho AVle_D,r_3=\Omega C_D\rho AVe_D\\ s_0&=T_{Tur}e_D\left(\frac{X_{11s}-X_{21s}}{L_2}\right)\\ s_1&=\Omega C_D\rho AVle_D\\ s_2&=\left(-\Omega^2I+We_{GB}+T_{Tur}e_D\frac{X_{10}-X_{20}}{L_2}\right)\\ s_3&=\Omega C_D\rho AVe_D\end{aligned} \tag{20}$$

Based on Eqs. (15-19), the frequency equation is derived

$$\begin{aligned}&o_3\left(q_1r_2s_4p_5\right)-o_4\left(q_1r_2s_3p_5-s_1r_2q_3p_5\right)\\&+o_5\left(q_1r_2s_3p_4-s_1r_2q_3p_4-r_1q_2p_3s_4\right)=0\end{aligned} \tag{21}$$

The effects of parameters on the resonance of the system will be investigated via the frequency equation (21),

4 Numerical results

Figures 4-7 show that the effects of the wave frequency *f*, the drag force T_{Tur} and the turbine area A_2 on the amplitudes $\{X_{11},X_{21}\}$ of vertical vibration.

Figure 4 demonstrates that the effect of the wave frequency on the amplitudes $\{X_{11},X_{21}\}$ of vertical vibration of the system with $\{T_{Tur}=40\text{ tons},A_1=29.9\text{ m}^2,A_2=80\text{ m}^2\}$. It is found that the two natural frequencies are {0.751 Hz, 1.22 Hz}.

Figure 5 demonstrates that the effect of the wave frequency on the amplitudes $\{X_{11},X_{21}\}$ of vertical vibration of the system with $\{T_{Tur}=100\text{ tons},A_1=29.9\text{ m}^2,\ A_2=80\text{ m}^2\}$. It is found that the two natural frequencies are {0.761 Hz, 1.21 Hz}.

Figure 6 demonstrates that the effect of the wave frequency on the amplitudes $\{X_{11},X_{21}\}$ of vertical vibration of the system with { $T_{Tur}=40\text{ tons},A_1=29.9\text{ m}^2,A_2=120\text{ m}^2$ }. It is found that the two natural frequencies are {0.751 Hz, 1.49Hz}.

The effect of the turbine area on the second natural frequency is significant. However, its effect on the first frequency is negligible.

Figure 7 demonstrates that the effect of the wave frequency on the amplitudes $\{X_{11}, X_{21}\}$ of vertical vibration of the system with { T_{Tur} = 40 tons, $A_1 = 40$ m^2, $A_2 = 80$ m^2 }. It is found that the two natural frequencies are {0.871 Hz, 1.49Hz}. The effect of the floater area on the two natural frequencies is significant.

It is concluded that the larger the areas of the floater and turbine are, the higher the natural frequencies are. The effect of the floater area is more significantly greater than that of the turbine area.

Figure 8 shows that the wave frequency f and the drag force T_{Tur} on the critical distance e_{GB} between the gravity and the buoyance centre for the moment of inertial $I = 8.33\times10^5\left(tons-m^2\right)$. It means that at the critical distance $e_{GB,critical}$ the resonance will happen.

It is well known that considering the static stability, the higher distance e_{GB} is preferred. For the wave frequency over 0.02 Hz, the critical distance $e_{GB,critical}$ is less than one, except at frequency f=1.9 Hz. It is concluded that if $e_{GB} > 1m$ the resonance will not occur for the wave frequency over wave frequency over 0.02 Hz. However, for the wave frequency under 0.02 Hz, the critical distance $e_{GB,critical}$ is larger than two. It is easily controversial to the static stability.

Figure 9 demonstrates that considering the moment inertial $I = 1.67\times10^3\left(tons-m^2\right)$, it is demonstrated that for the wave frequency under 1 Hz, the critical distance $e_{GB,critical}$ is larger than two. It is easily controversial to the static stability. It is concluded that the effects of wave frequency f and the moment inertial I on the resonance is significant.

5 Conclusion

In this study, the mathematical model of the system composed of turbine, buoyance platform, traction rope and mooring foundation is derived. The analytical solution of the general system is proposed. It is found that the effect of the areas of the floater and turbine on the natural frequencies is significant. The larger the areas of the floater and turbine are, the higher the natural frequencies are. The effect of the floater area is more significantly greater than that of the turbine area. Moreover, the effects of wave frequency f and the moment inertial I on the resonance is significant.

Acknowledgment

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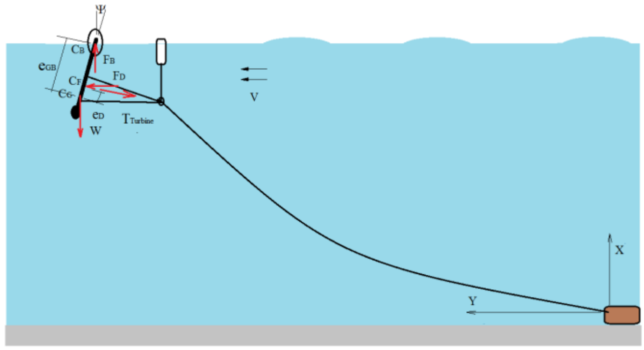

Fig. 1 The current turbine system composed of turbine, buoyance platform, traction rope and mooring foundation.

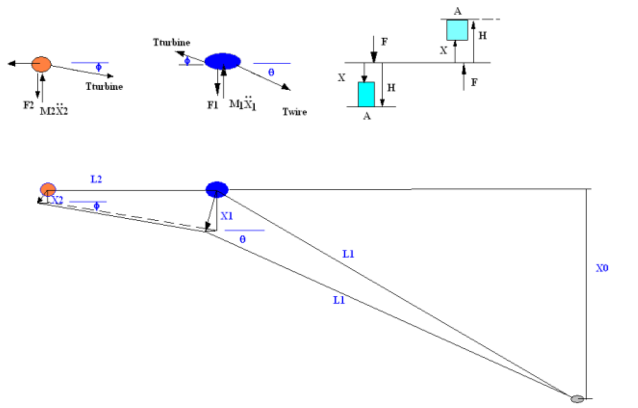

Fig. 2 Concentrated mass model of the ocean current turbine system subjected to wave

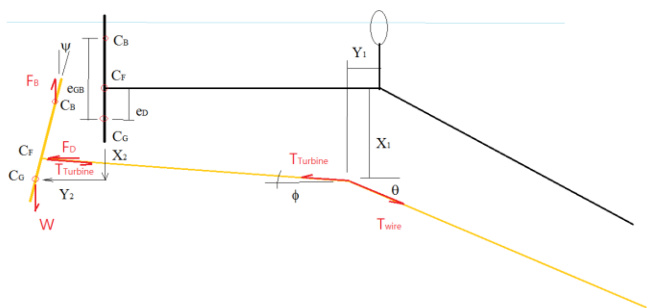

Fig. 3 Coordinate and force distribution of the system

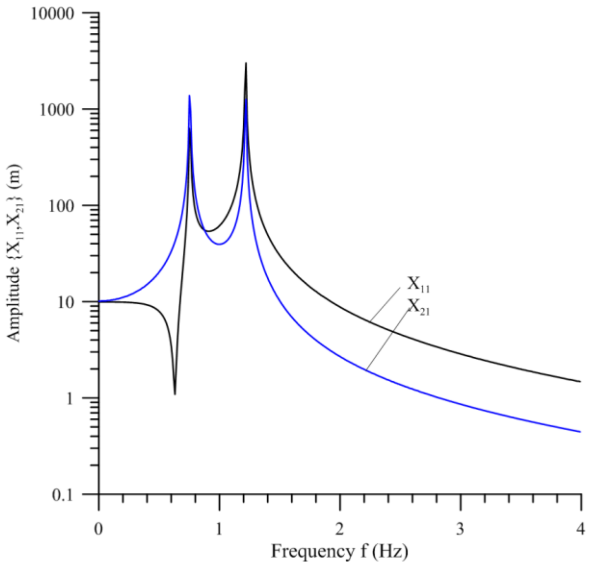

Fig. 4 the effect of the wave frequency f on the amplitudes $\{X_{11}, X_{21}\}$ of vertical vibration. [$A_1 = 29.9\ \mathrm{m}^2, A_2 = 80\ \mathrm{m}^2$, $M_1 = 13.26 tons$, $M_2 = 100 tons$, $L_1 = 2780m$, $L_2 = 50m$, $X_0 = 850m$, $H_0 = 10m$ $T_{Tur} = 40$ tons]

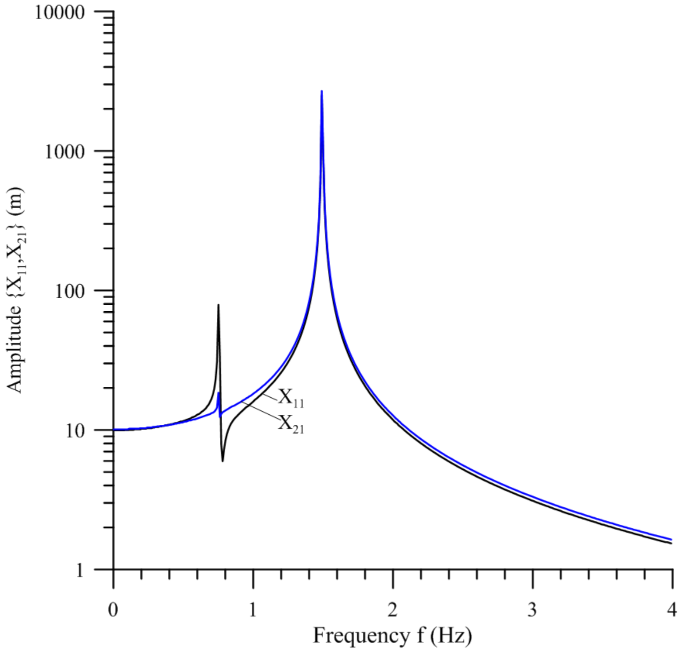

Figure 6 the effect of the wave frequency f on the amplitudes $\{X_{11}, X_{21}\}$ of vertical vibration. [$A_2 = 120\ \mathrm{m}^2$, other parameters are the same as those in Figure 4]

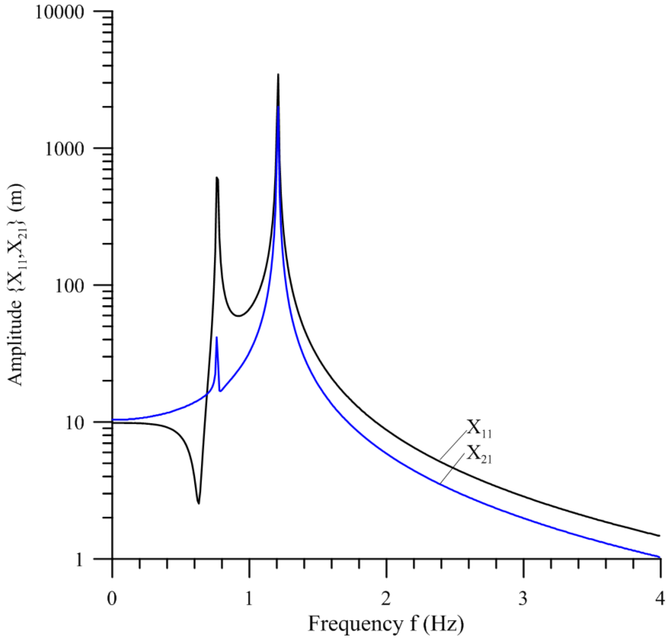

Figure 5 the effect of the wave frequency f on the amplitudes $\{X_{11}, X_{21}\}$ of vertical vibration. [$T_{Tur} = 100$ tons , other parameters are the same as those in Figure 4]

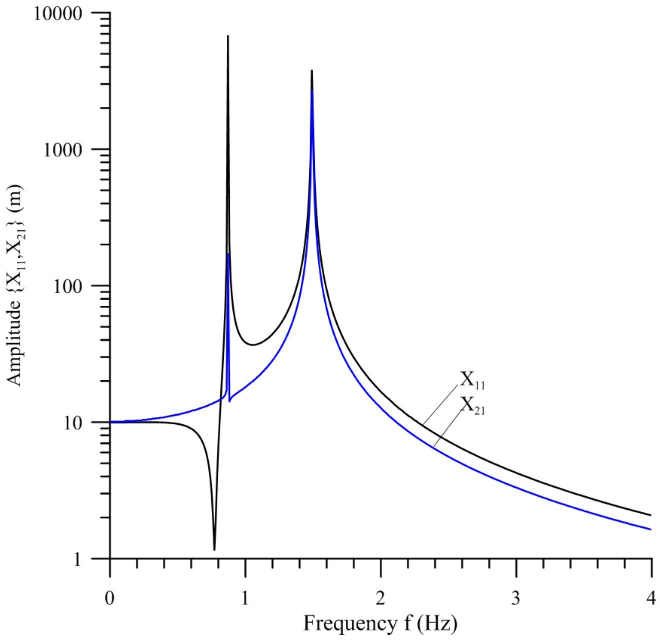

Figure 7 the effect of the wave frequency f on the amplitudes $\{X_{11}, X_{21}\}$ of vertical vibration. [$A_1 = 40\ \mathrm{m}^2$, other parameters are the same as those in Figure 4]

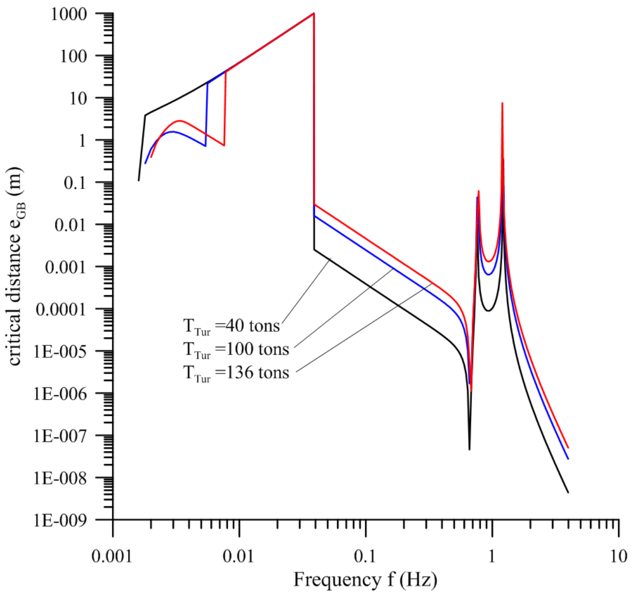

Fig. 8 the effect of the wave frequency *f* and the drag force T_{Tur} on the critical distance between the centers of gravity and buoyance for resonance. [$C_D = 1.0$, $V = 1m/s$, $e_D = 1m$, $l = 1m$, $I = 8.33\times10^5$ $\left(tons - m^2\right)$, other parameters are the same as those in Figure 4]

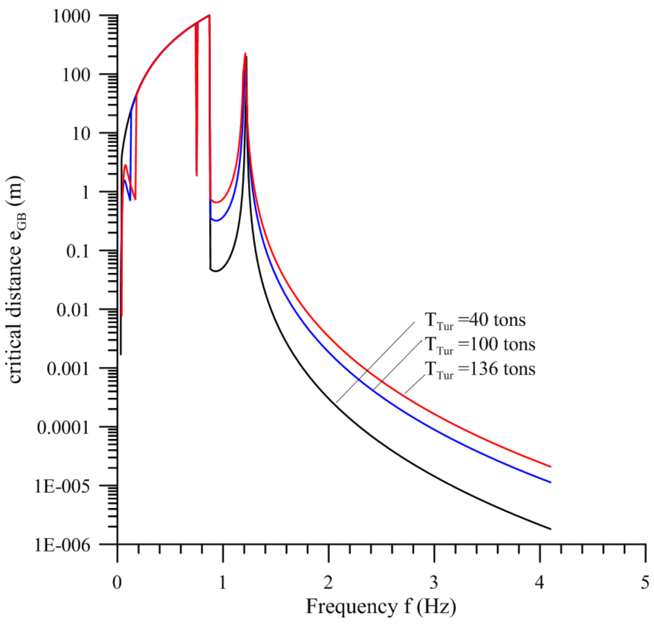

Fig. 9 the effect of the wave frequency *f* and the drag force T_{Tur} on the critical distance between the centers of gravity and buoyance for resonance. [$I = 1.67\times10^3\left(tons - m^2\right)$, other parameters are the same as those in Figure 8]