Optimization of Heterogeneous Arrays of Wave Energy Converters

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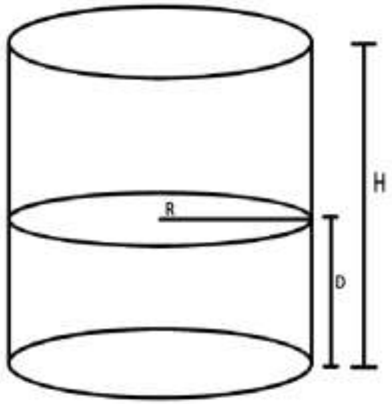
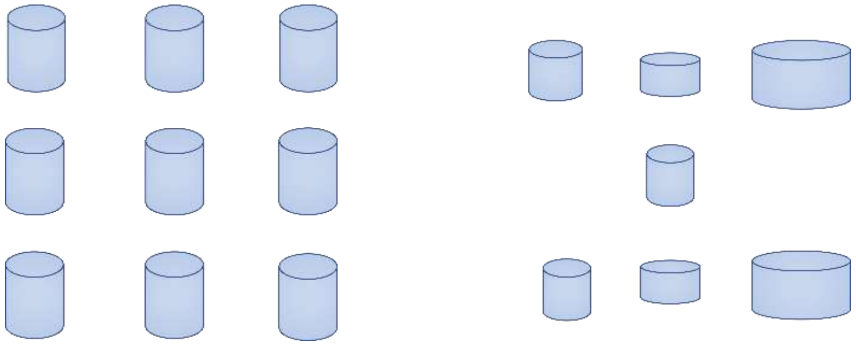
Background/Research Question



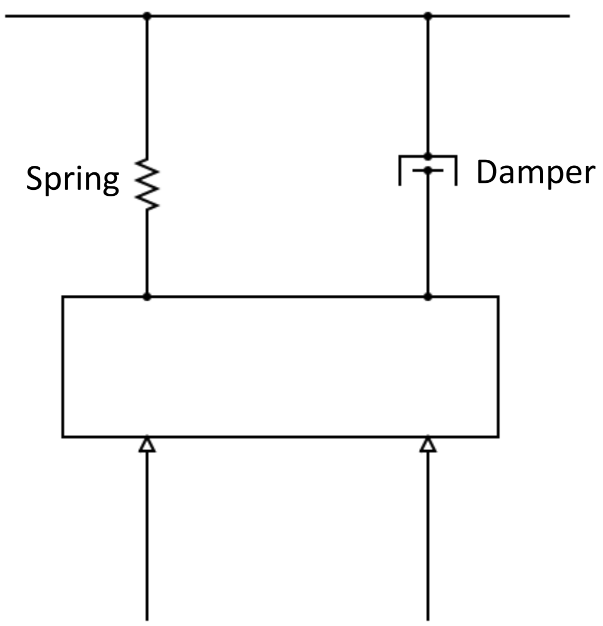
- The WEC dimensions are usually designed for the prevailing sea conditions in a particular location.
- Control co-design is desired.
- WEC arrays usually contain identical devices
- Can we optimize the dimensions and control to maximize the overall power absorption?



Hom. array Vs Het. array.

- How can we further improve performance by optimizing the number of devices in the array?

Dynamic Model: Spring-Mass-Damper Approximation



- A simple point absorber WEC

$$(m + m_\infty)\ddot{x} + C_r\dot{x} + kx = f_e - u$$

- Array dynamics

$$(M + M_\infty)\ddot{\vec{x}} + C_r\dot{\vec{x}} + K\vec{x} = \vec{f}_e - \vec{u}$$

- Hydrodynamic coefficients

$$M_\infty = \begin{bmatrix} m_{\infty 11} & \cdots & m_{\infty 1n} \\ \vdots & \ddots & \vdots \\ m_{\infty n1} & \cdots & m_{\infty nn} \end{bmatrix}$$

$$Cr_\omega = \begin{bmatrix} Cr_{\omega 11} & \cdots & Cr_{\omega 1n} \\ \vdots & \ddots & \vdots \\ Cr_{\omega n1} & \cdots & Cr_{\omega nn} \end{bmatrix}$$

$$\text{q} - \text{factor} = \frac{P_{array}}{N * P_{isolated}}$$

Problem I: Homogenous array optimization

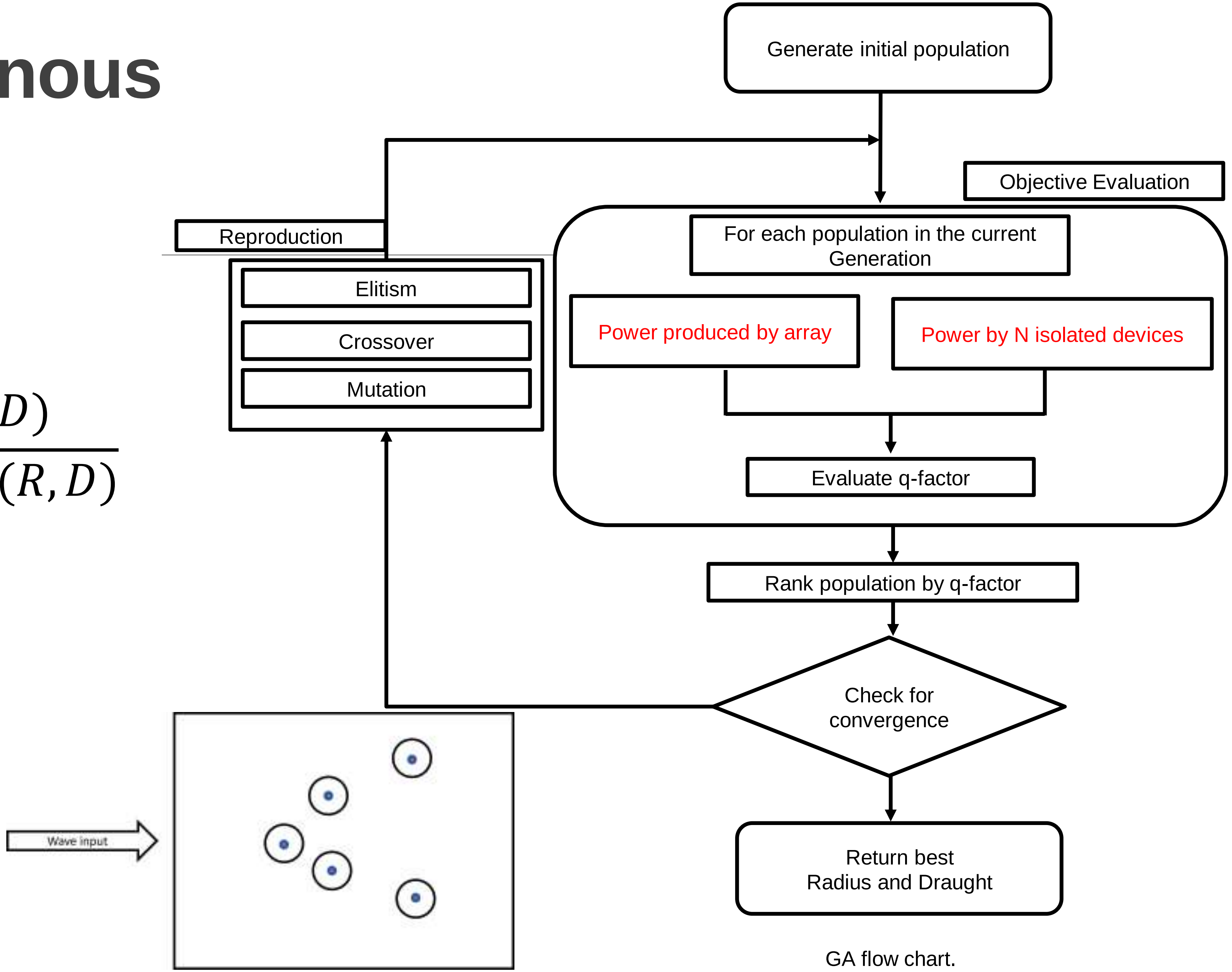
Homogenous array

$$Maximize: q = \frac{P_{array}(R, D)}{N * P_{isollated}(R, D)}$$

$$Subject\ to:$$

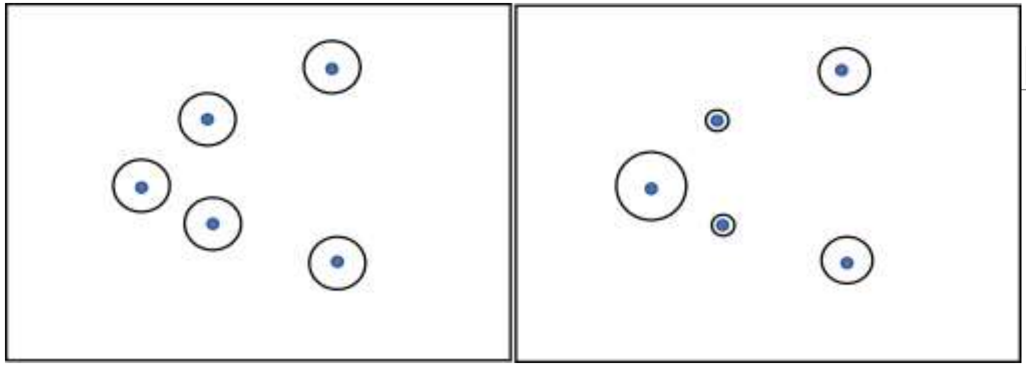
$$R \in [R_{min}, \quad R_{max}]$$

$$D \in [D_{min}, \quad D_{max}]$$



GA flow chart.

Problem II: Heterogenous array optimization



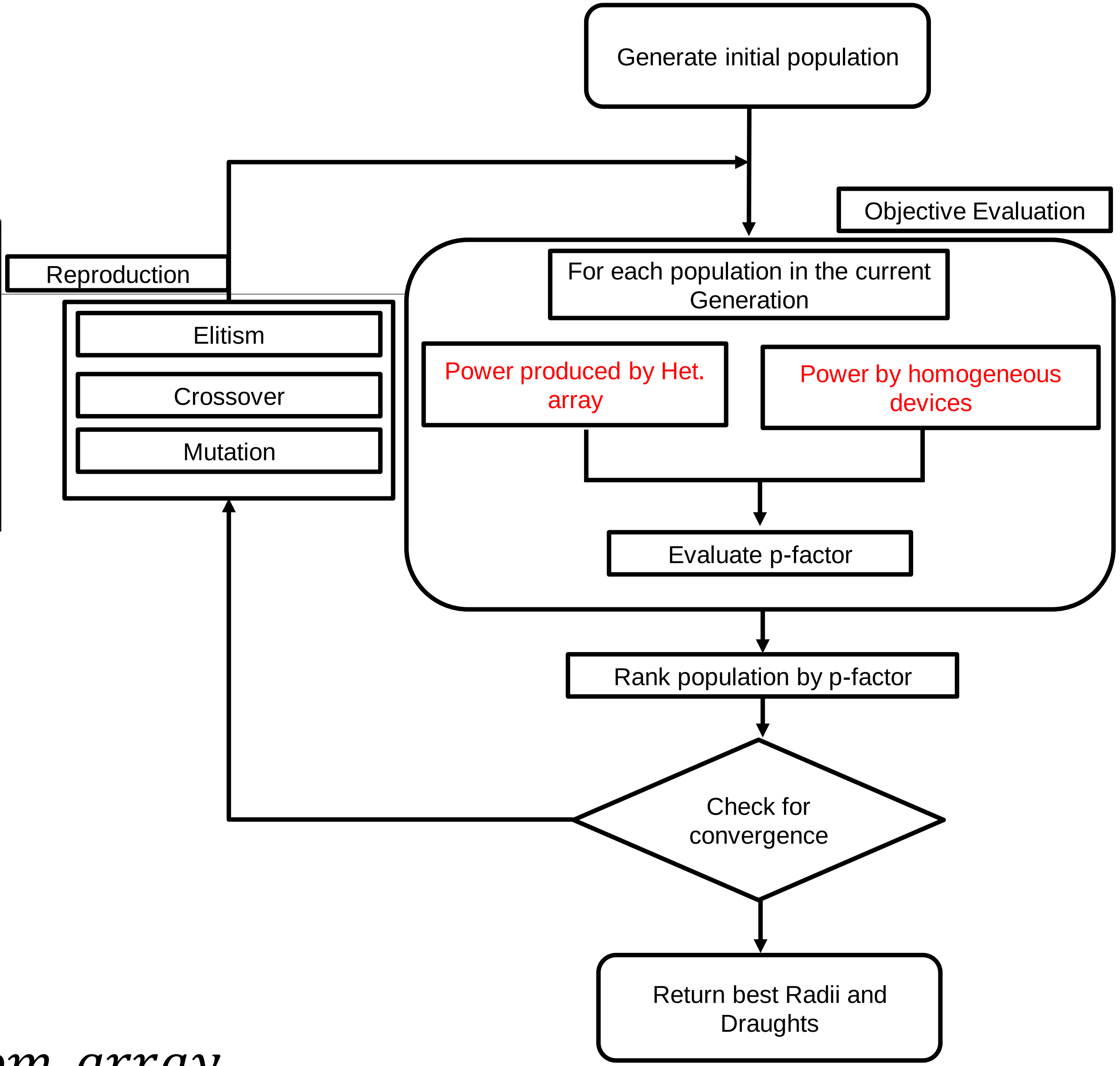
Heterogeneous array

$$Maximize: p = \frac{P_{heterogeneous}}{P_{homogeneous}}$$

$Subject\ to:$

$R_i \in [R_{min}, R_{max}]$, $D_i \in [D_{min}, D_{max}]$

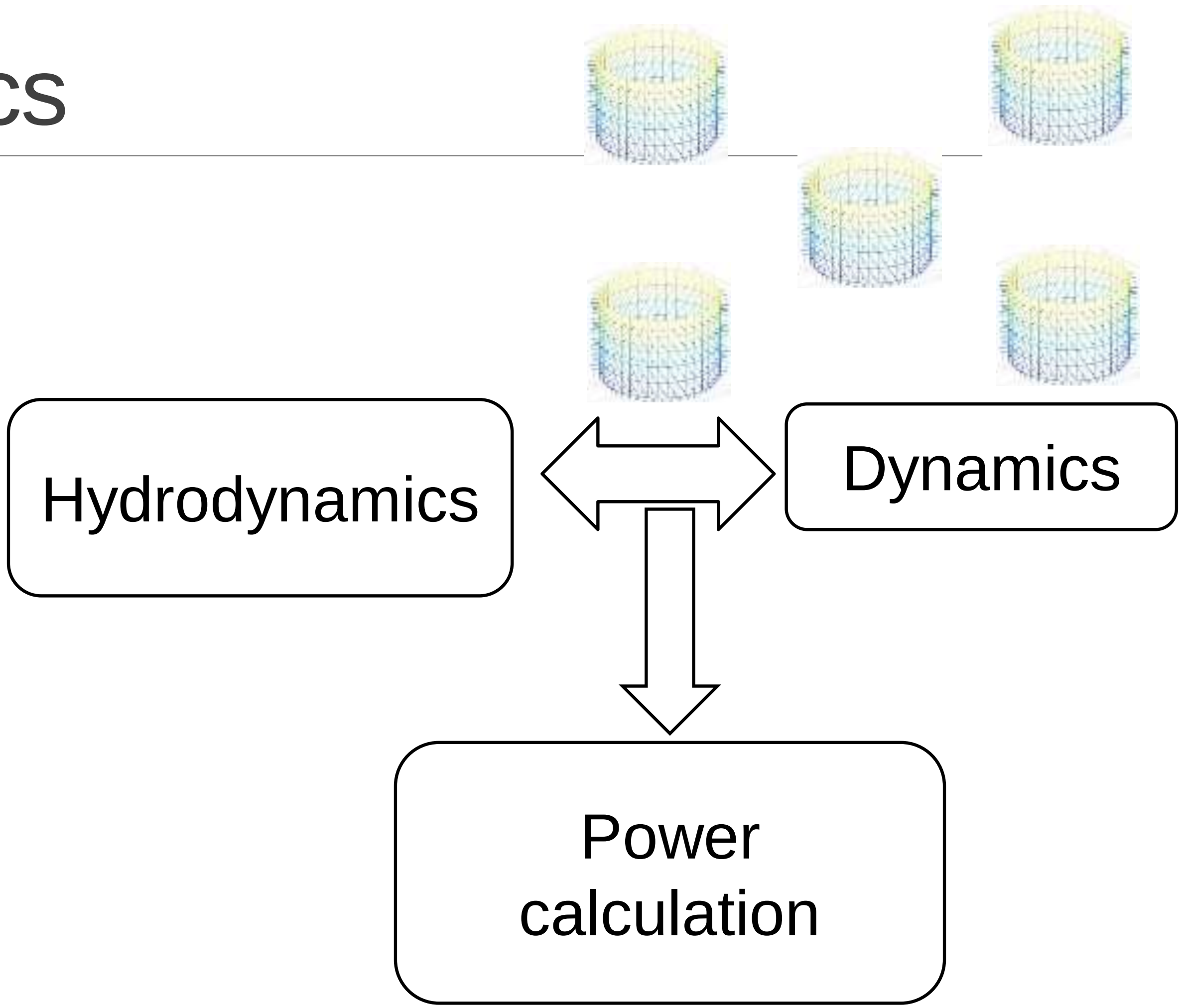
$Total\ Vol.of\ Het\ array \leq Total\ Vol.of\ Hom.array.$



GA flow chart.

WEC Array Hydrodynamics

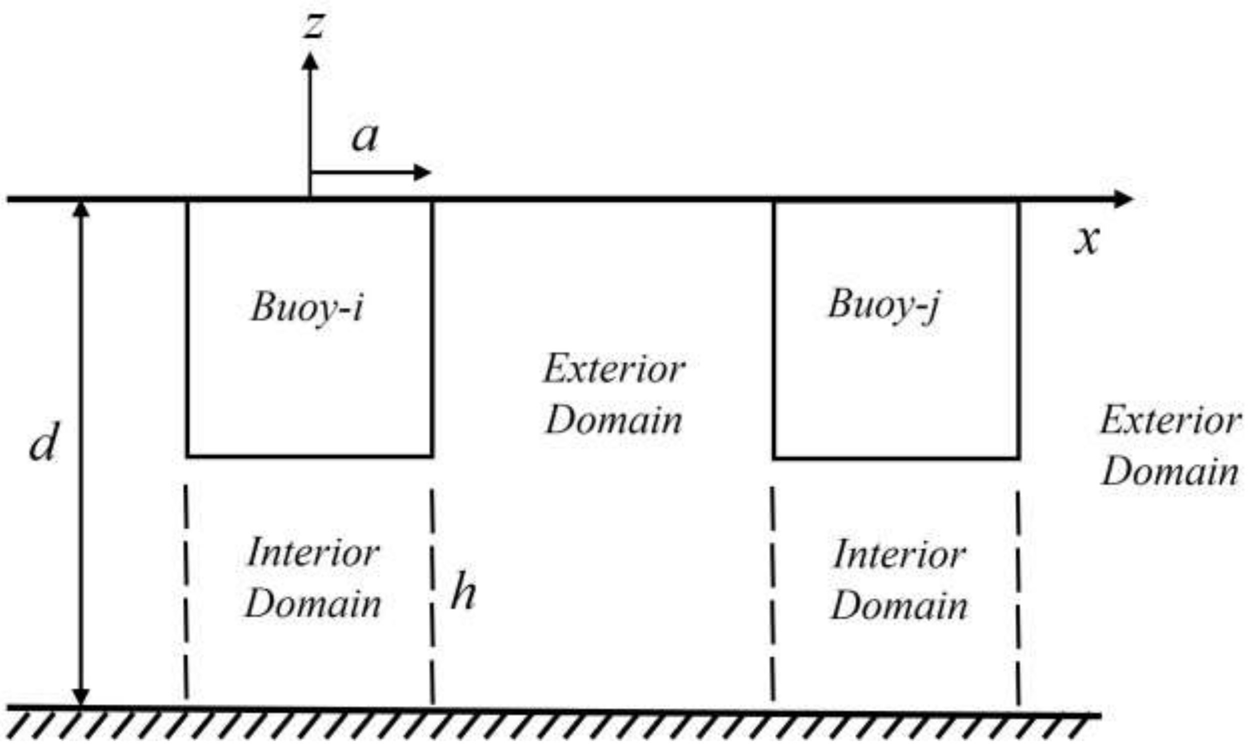
- We need to compute added mass, radiation damping coefficients, and excitation force coefficients.
- Boundary Element Methods tools vs. an approximate analytic method
- GA needs objective values only; qualitative conclusions on the objective values of different solutions are okay



Approximate analytic coefficients computation

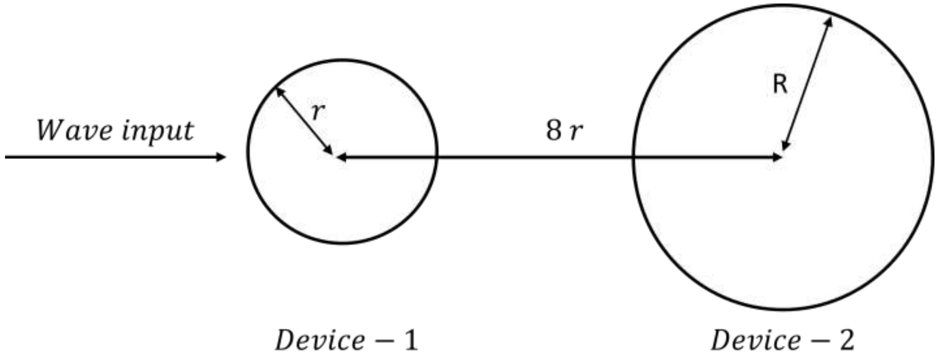
$$\emptyset(r,\theta,z) = \overbrace{\emptyset_0(r,\theta,z) + \emptyset_7(r,\theta,z)}^{Diffraction} + \underbrace{\sum_{q=1}^{6} \emptyset_q(r,\theta,z)}_{Radiation}$$

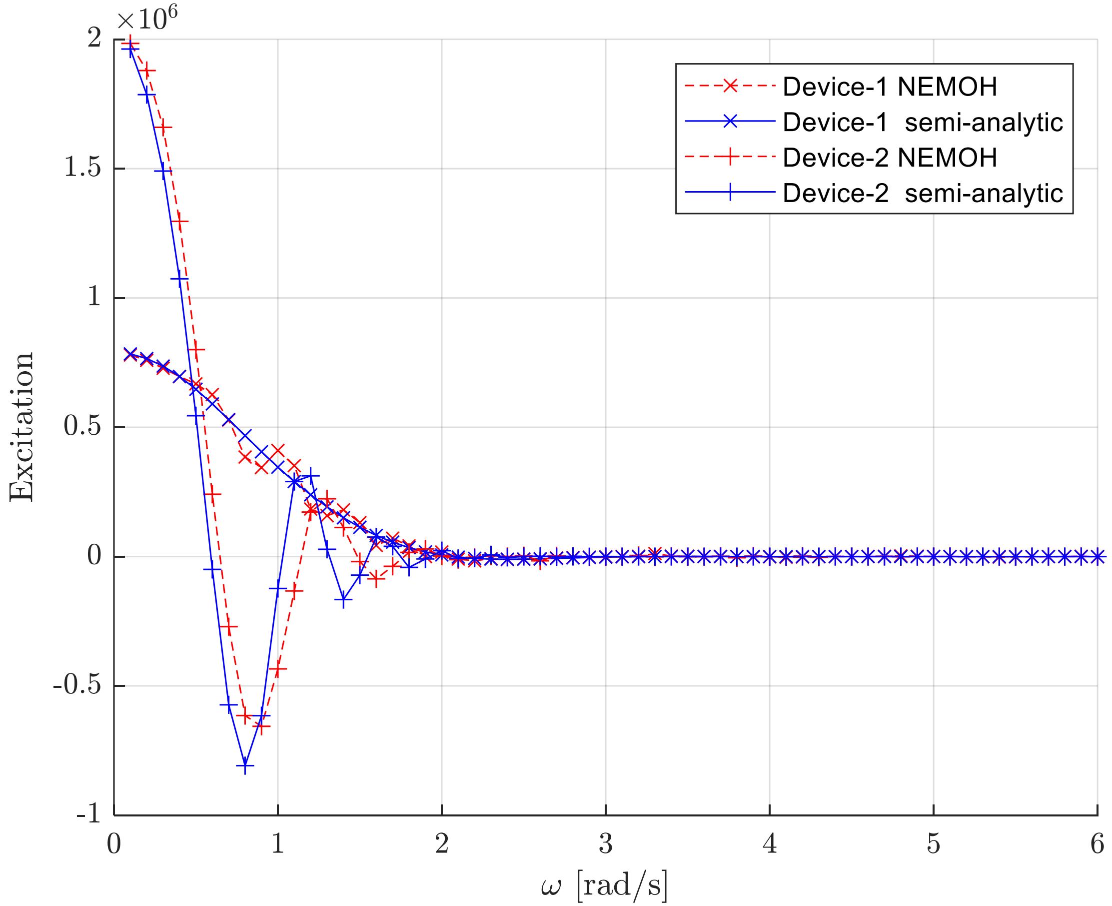
- $\nabla^2\emptyset = 0$



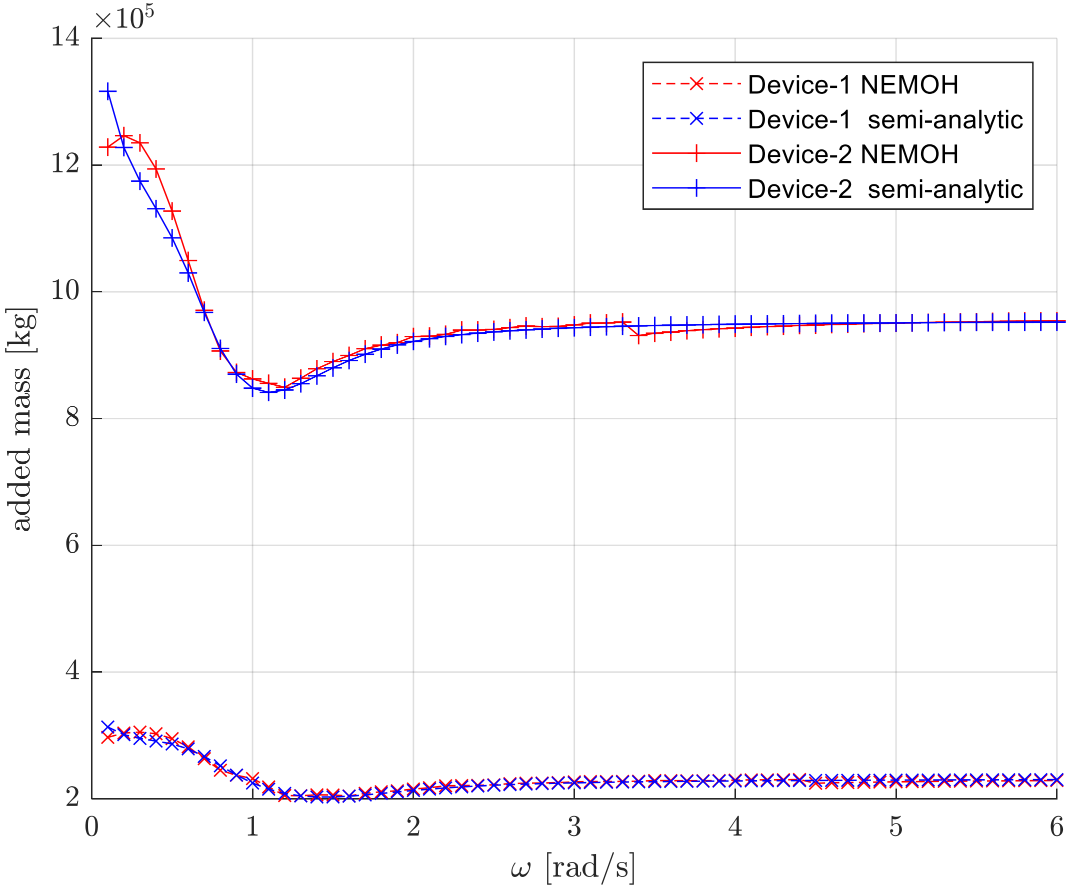
Fluid Domain.

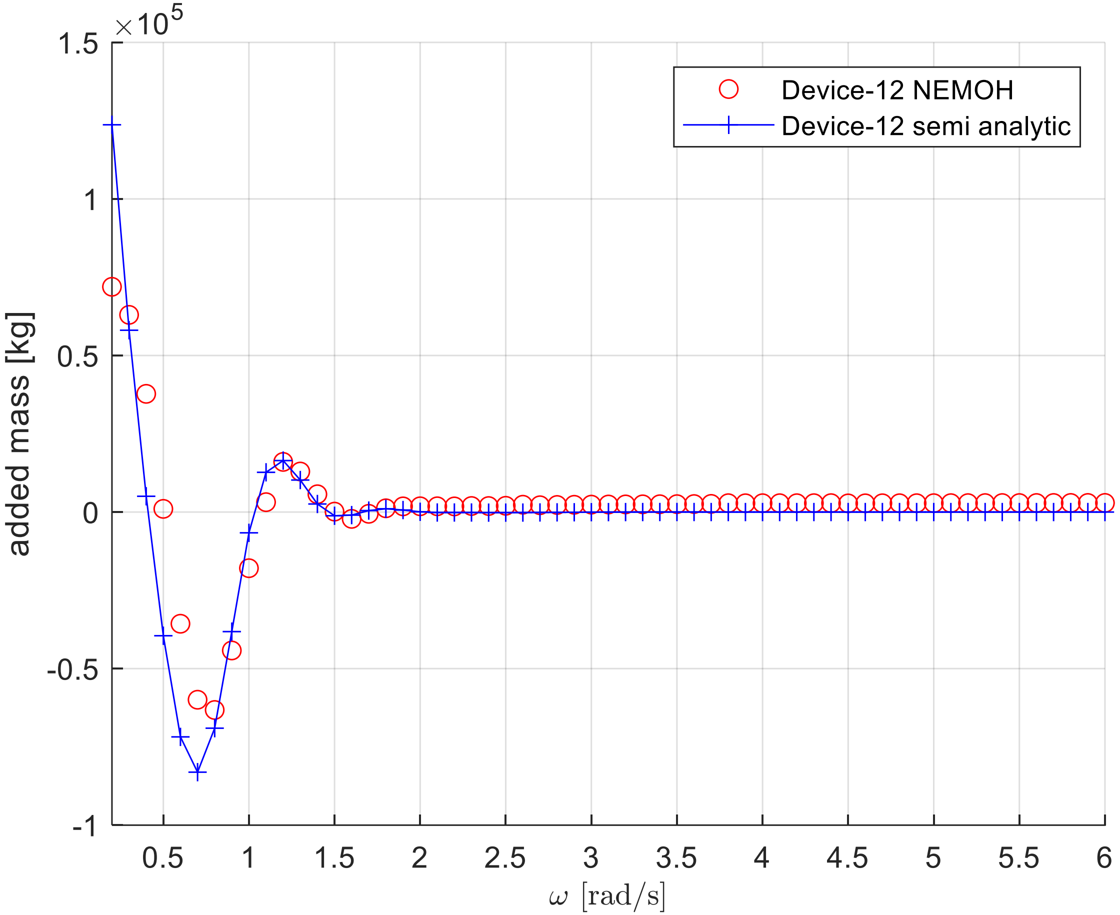
Validation of hydrodynamics



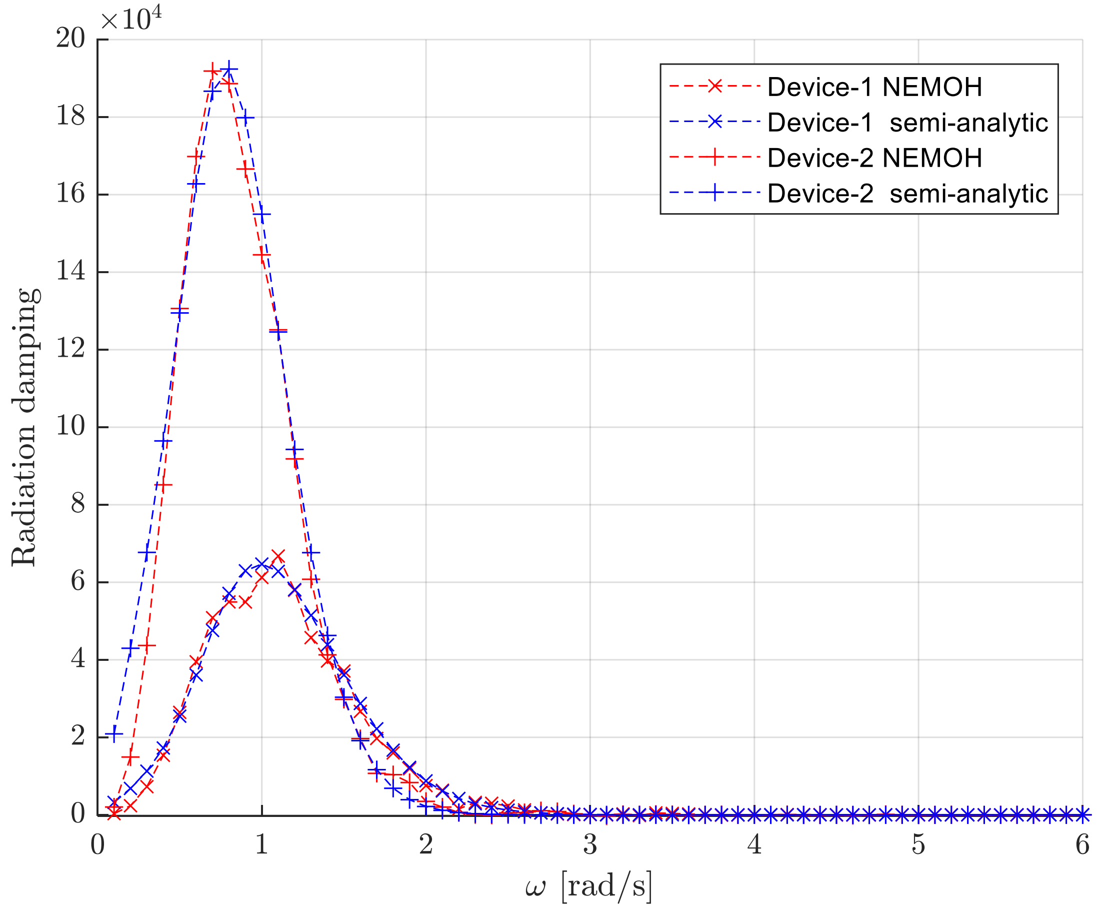


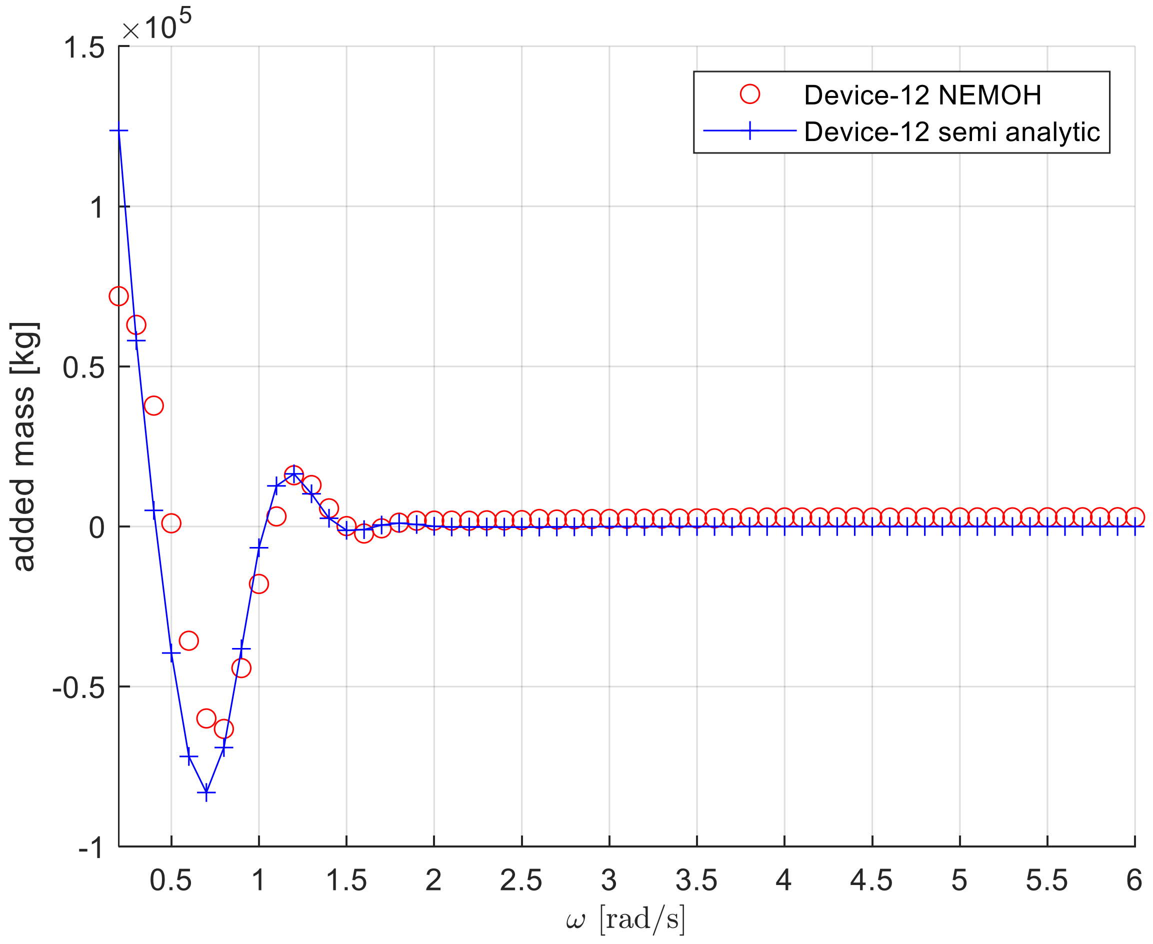
Validation of hydrodynamics





Validation of hydrodynamics





Computational speed

7 device array

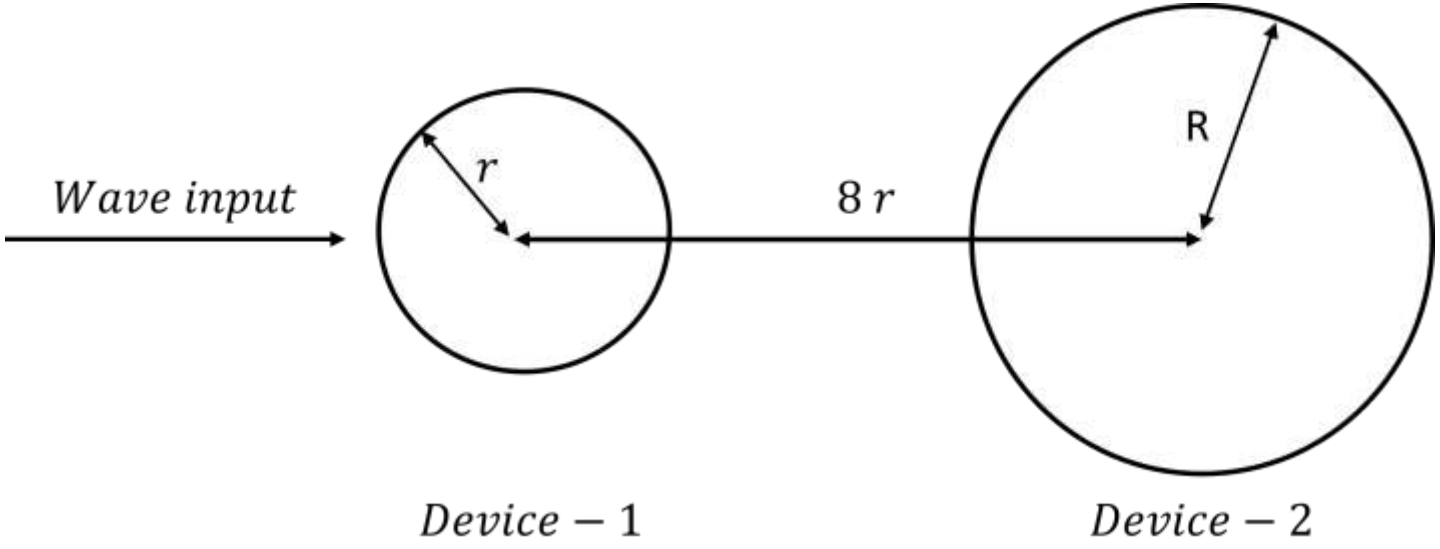
	Semi Analytic	Nemoh
7	13.9748 s	2912.662 s (48 mins)

2 device array

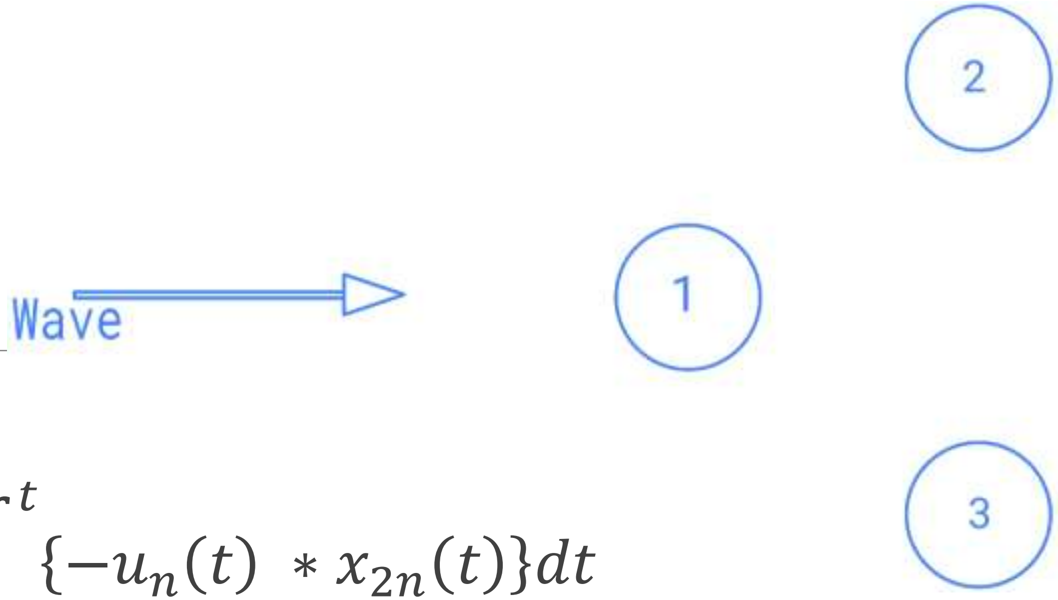
	Semi Analytic	Nemoh
2	4.18226 s	113.814748 s

The radii are, $r = 5\,m, R = 8\,m$,

draughts $\mathrm{h} = 8\,m$, $\mathrm{H} = 7\,m$.



Control Force



- The objective function:

$$Mininimze: \quad J\big(u(t), x_2(t)\big) = \sum_{n=1}^{N} \int_0^t \{-u_n(t) \ * x_{2n}(t)\} dt$$

- Subject to EOM:

$$\dot{\vec{x}}_1 \ = \vec{x}_2$$

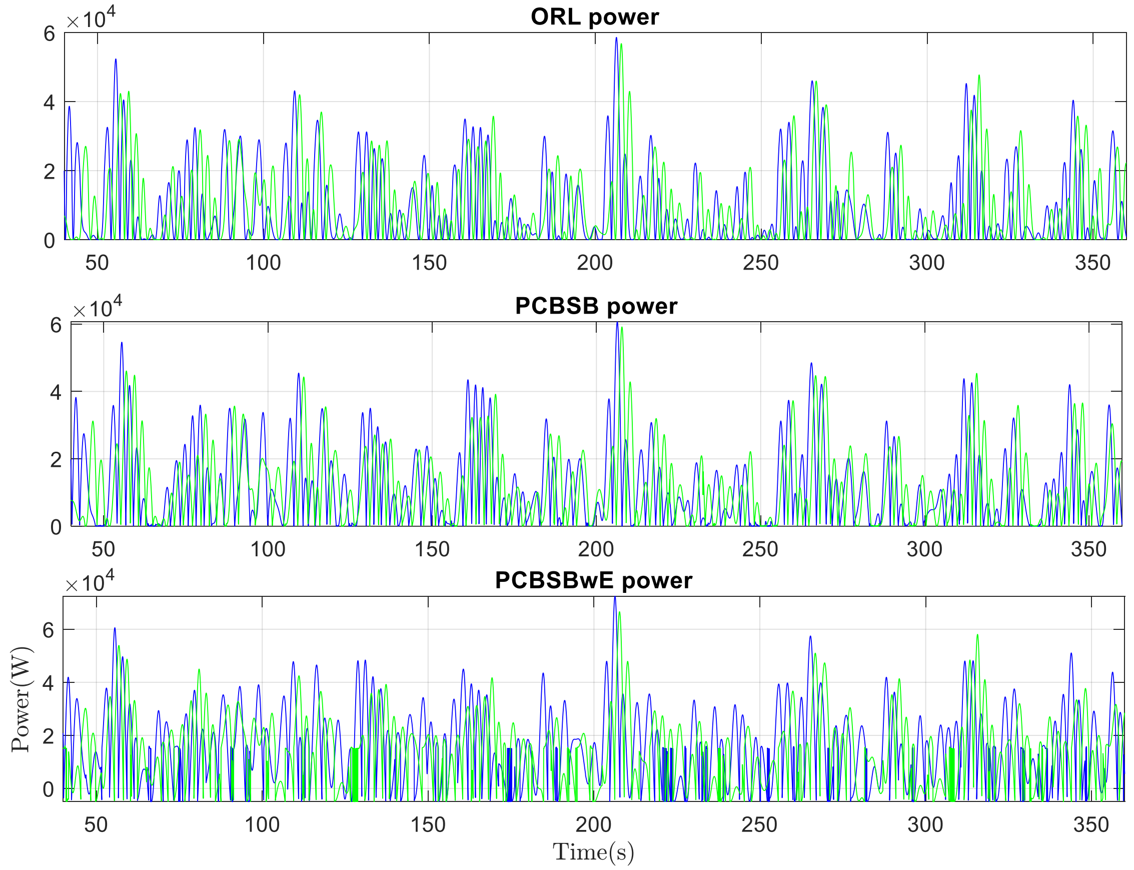
$$\dot{\vec{x}}_2 = \frac{1}{M}\left(\vec{f}_e(x_3) \ - C\vec{x}_2 - K\vec{x}_1 - \vec{u}\right)$$

$$x_3 = 1$$

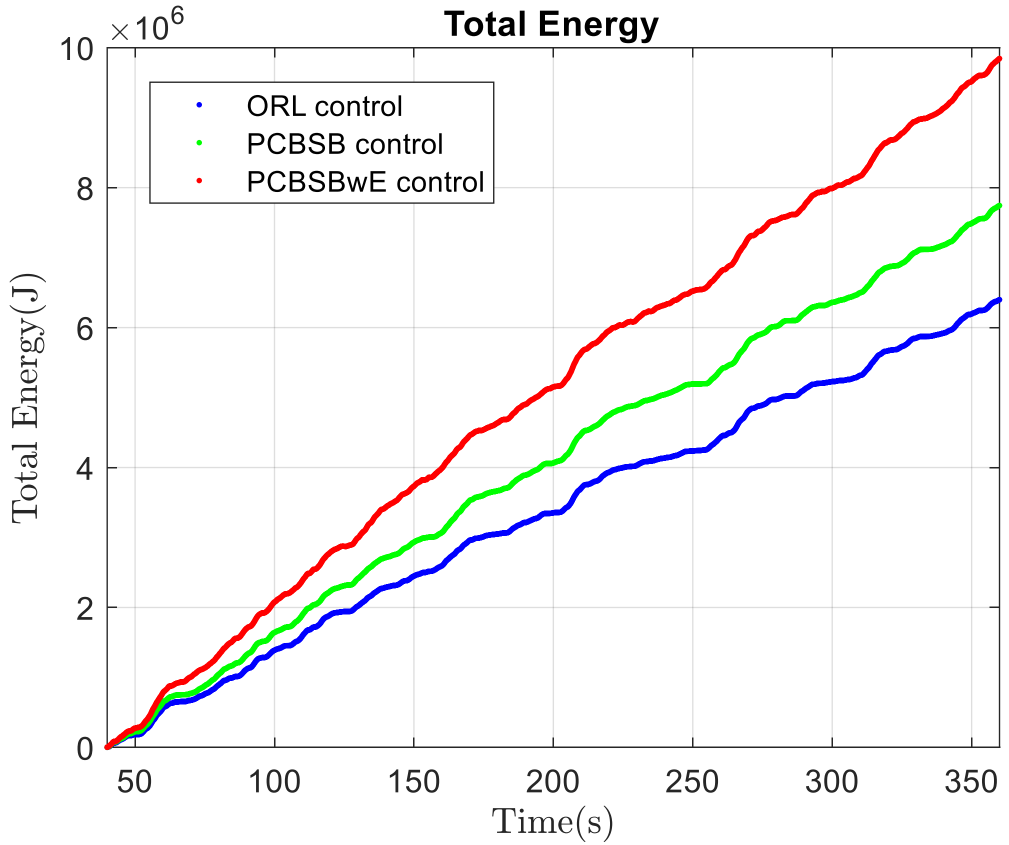
- The power constraints and control constraints:

$$\vec{u}(t) \ * \vec{x}_2(t) \geq -\vec{\epsilon} \ , \qquad\qquad |\vec{u}(t)| \leq \vec{\Gamma}$$

Control Force and Power Computation



Power when using ORL and PCBSB.



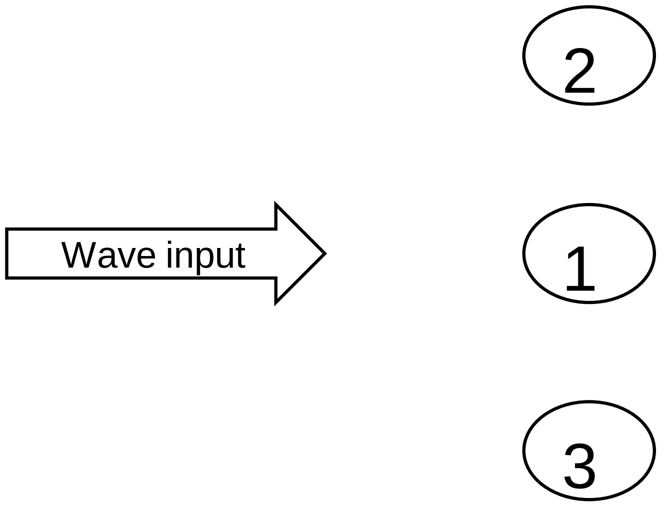
Energy extracted when using ORL and PCBSB.

Simulation

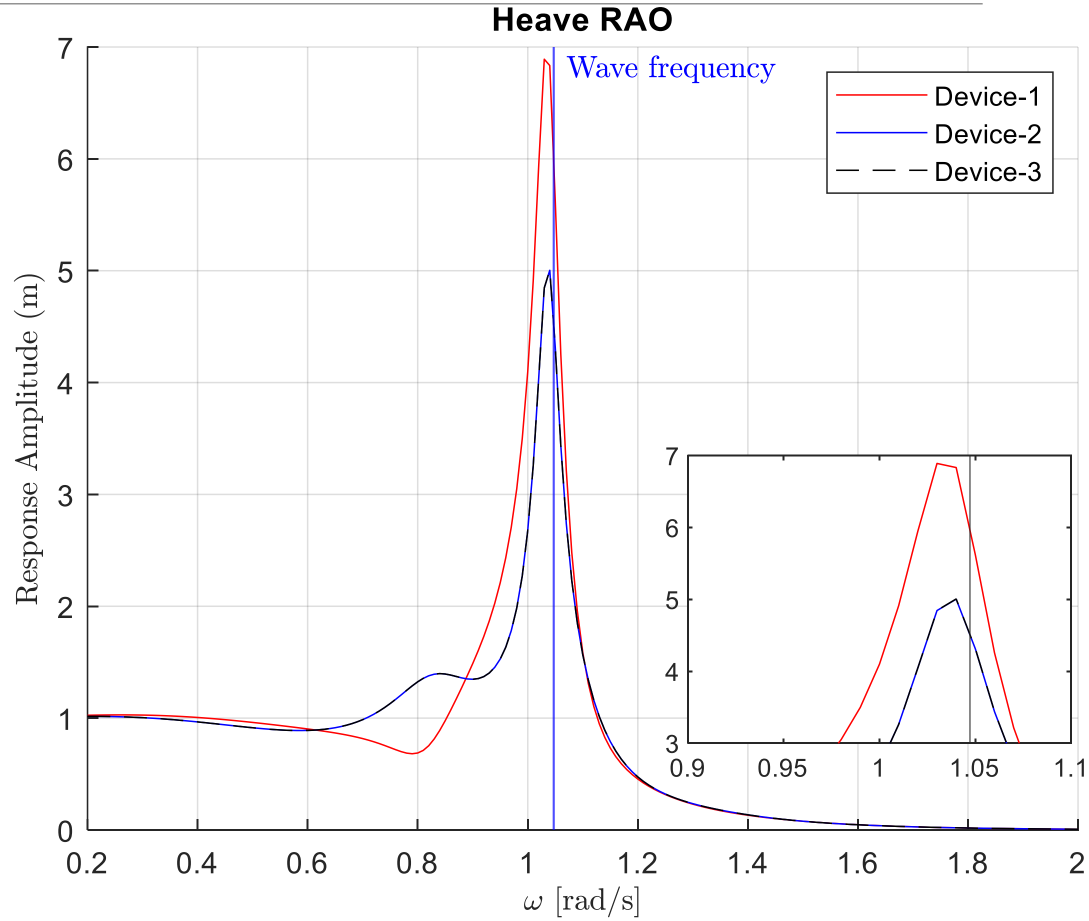
The simulation parameters are as follows:

- Wave condition: $T = 6s, H = 0.8222\ m$. (site: Newport, Oregon)
- $(Radius\ and\ draught)\ Upper\ Bound = 10\ m$.
- $(Radius\ and\ draught)\ Lower\ Bound = 1m$.
- Hydrodynamic parameters are calculated using the approximate analytic method.
- Power from the array is computed using the constrained control PCBSB

Homogenous array result – 3 Devices

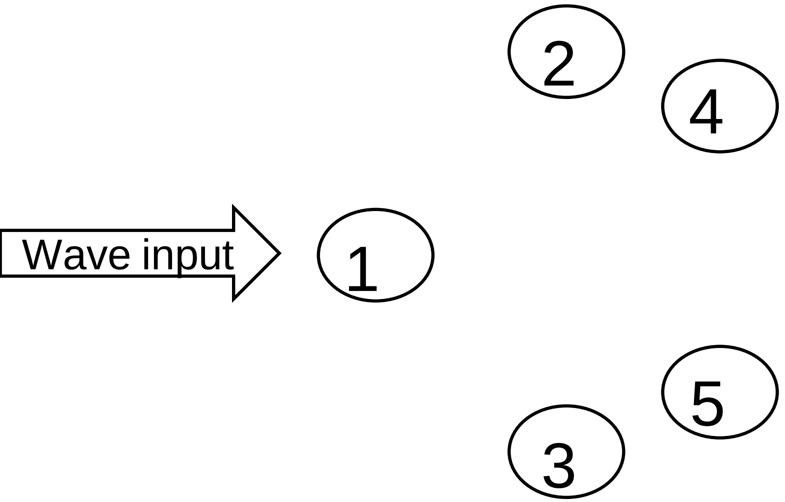


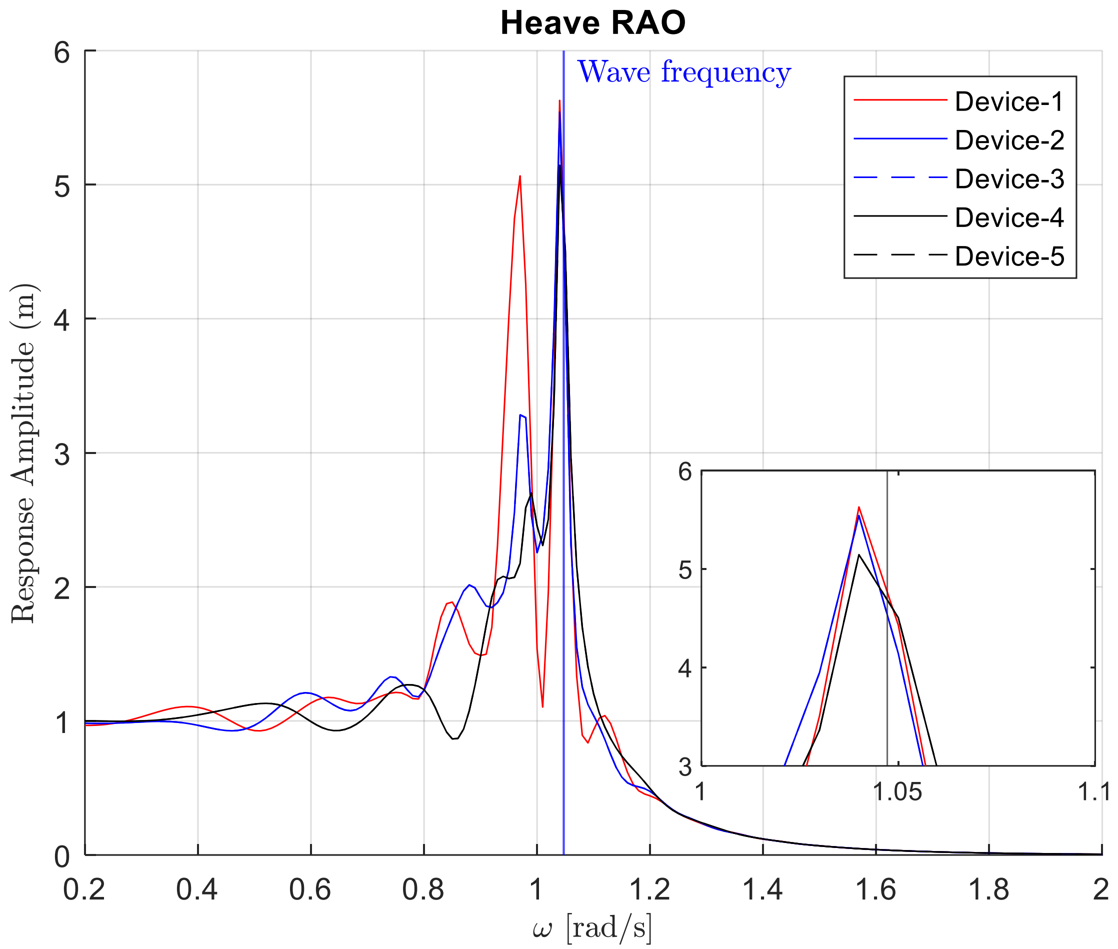
	X (m)	Y (m)
1	0	0
2	0	40.255
3	0	-40.255



- Optimized Radius: $R = 7.2249\, m$

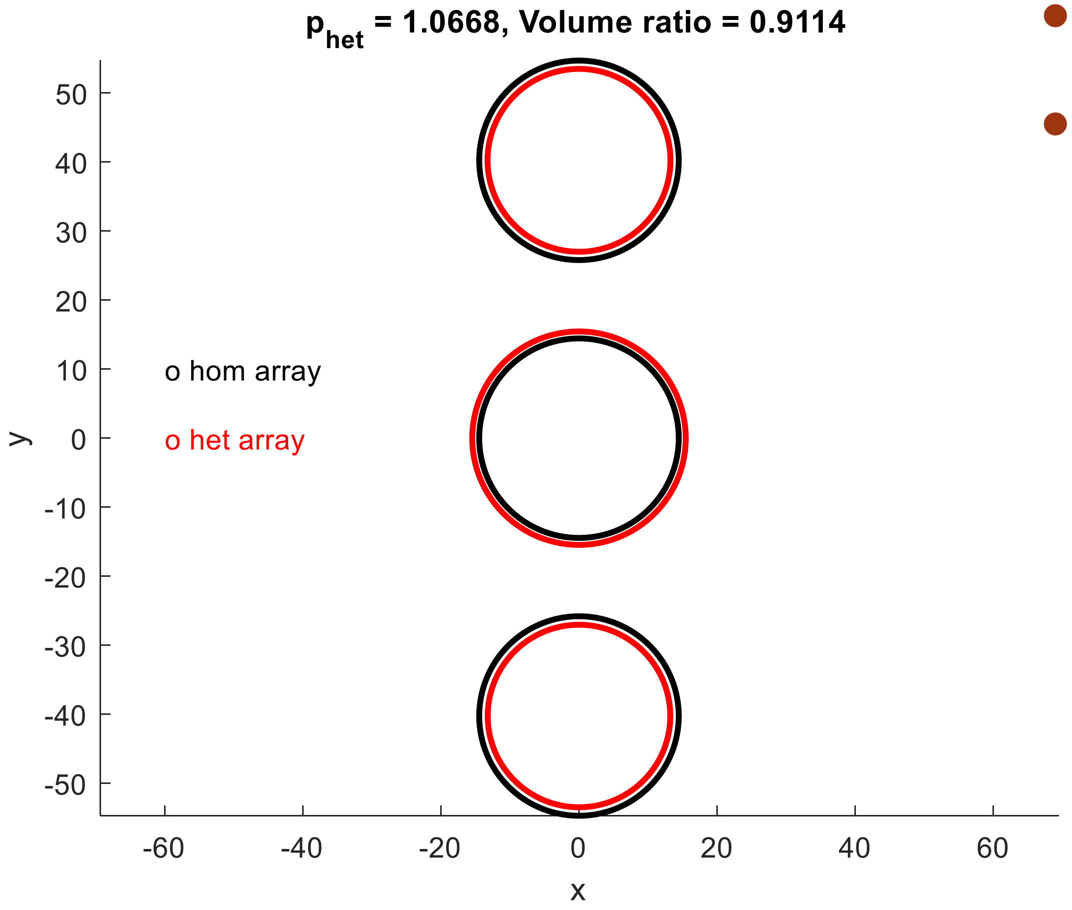
Homogenous array result – 5 Devices





- Optimized Radius: $R = 6.1196\text{m}$

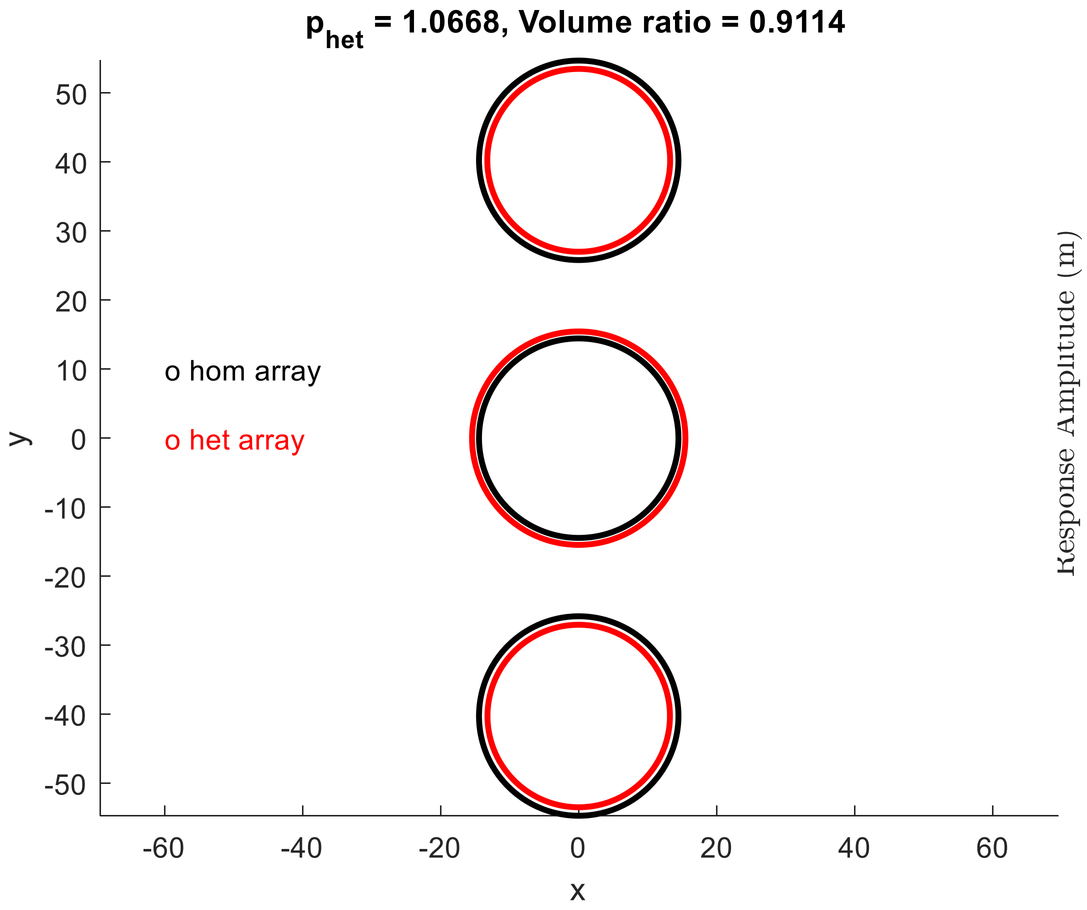
Heterogeneous array result: 3 devices

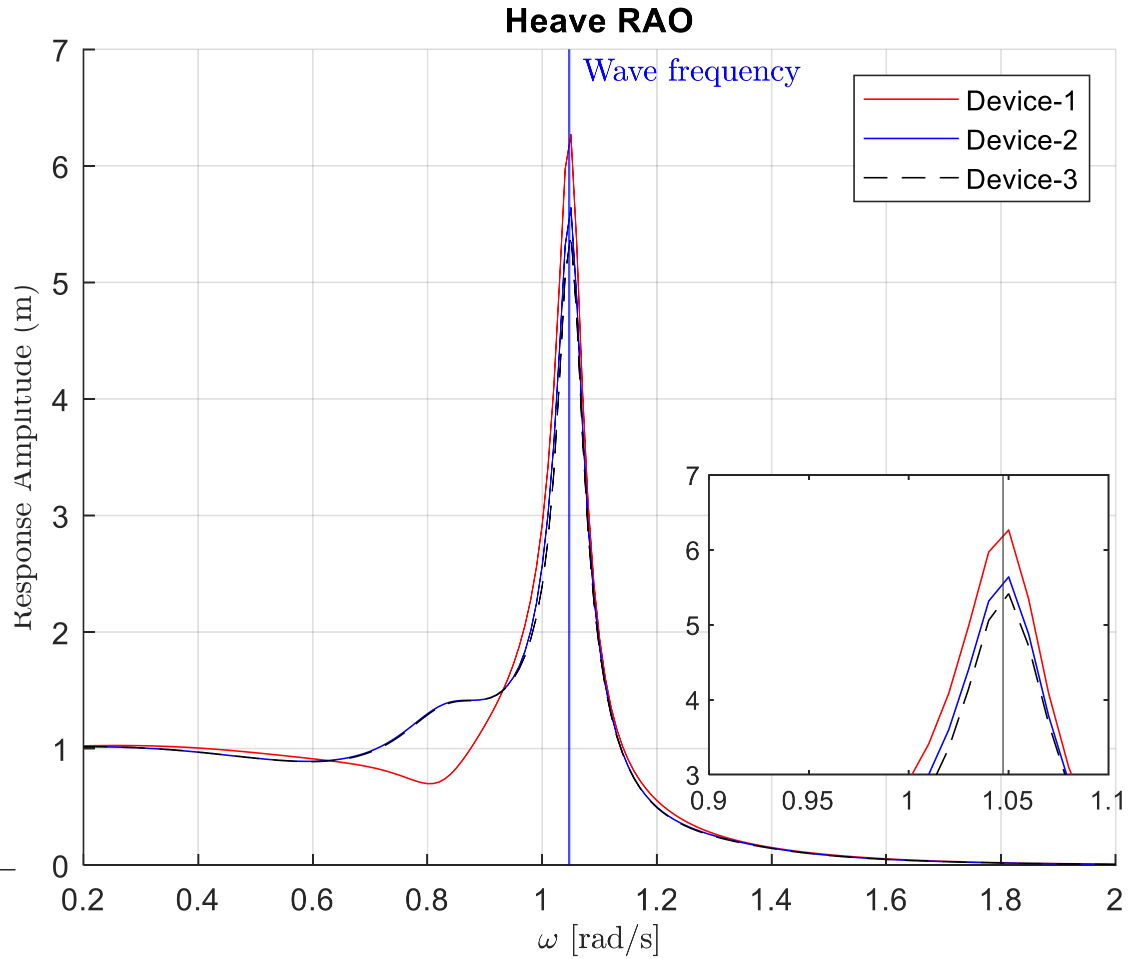


- Optimized dimensions
- $Homogeneous: R = 7.2249\, m$

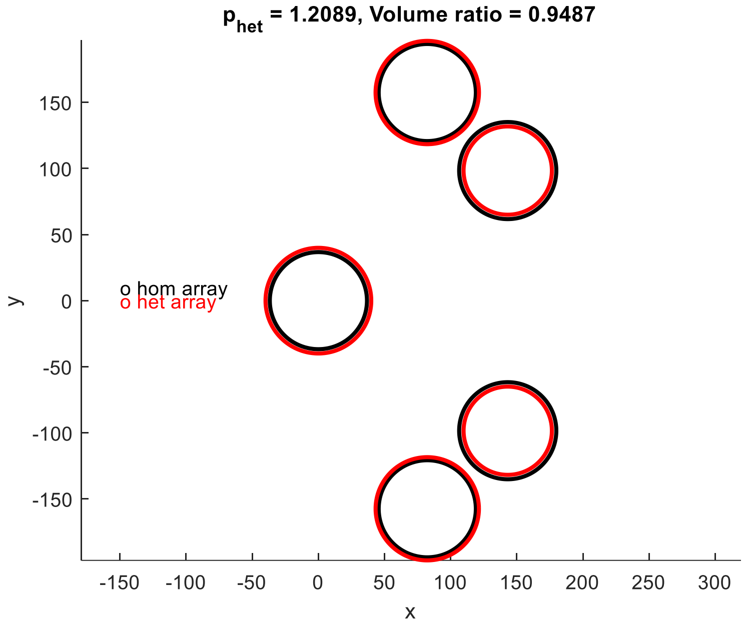
R1	R2	R3	p
7.7297	6.6232	6.6095	1.0668
7.7527	6.6143	6.6214	1.0667

Heterogeneous array result: 3 devices





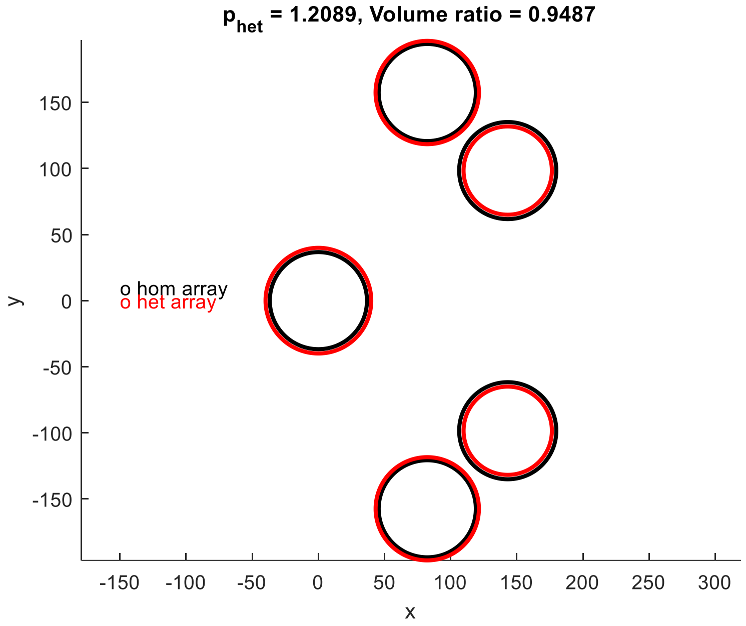
Heterogeneous array result: 5 devices

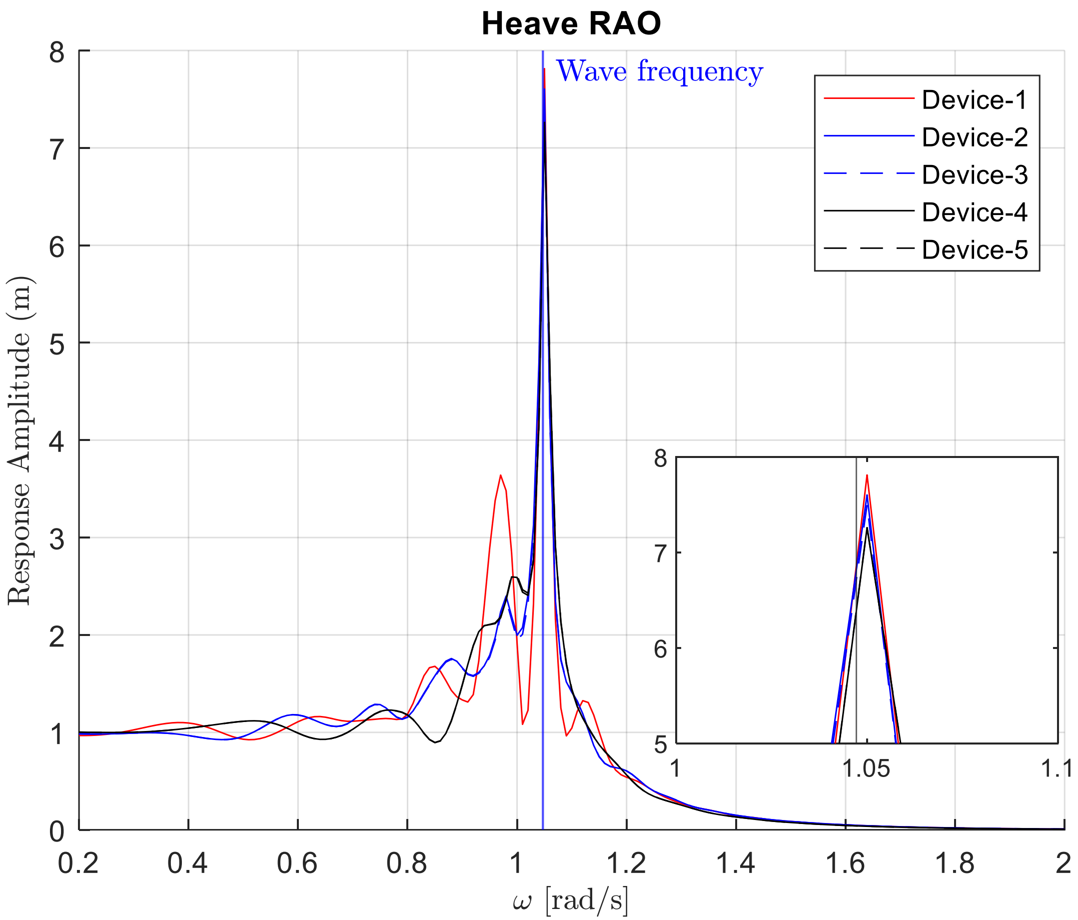


- Optimized dimensions
- Homogeneous: $R = 6.1196\, m$

R1	R2	R3	R4	R5	p
6.6600	6.5131	6.5064	5.5635	5.5623	1.2089
6.6368	6.4822	6.4776	5.5457	5.5254	1.2087

Heterogeneous array result: 5 devices

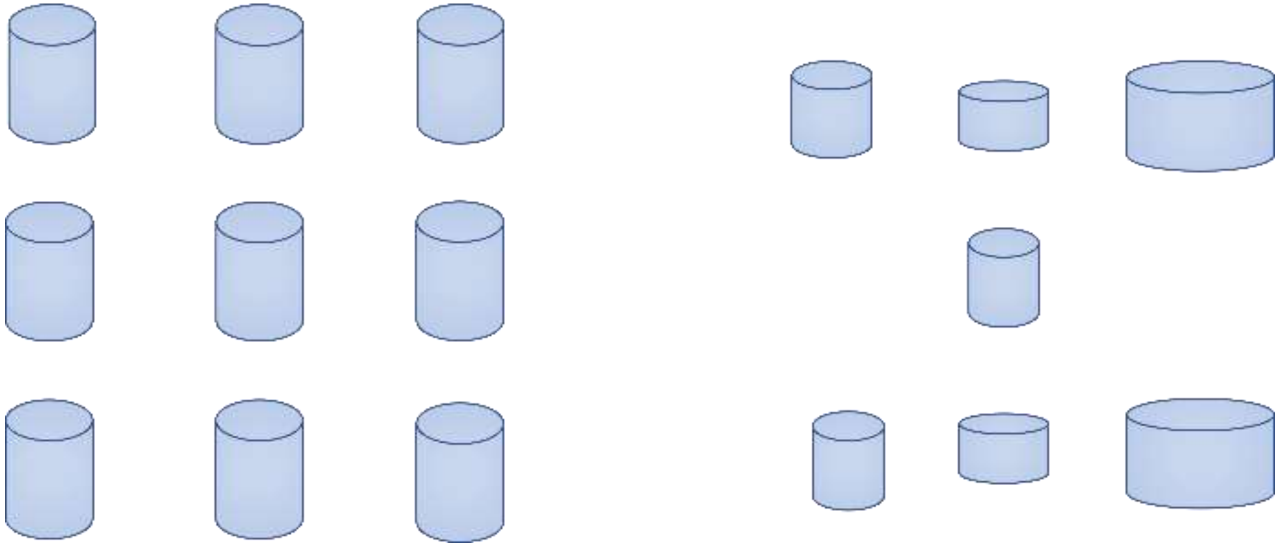




Problem III: HGGA-Heterogenous array optimization

Heterogeneous array

$$Maximize: p = \frac{P_{heterogeneous}}{P_{homogeneous}}$$



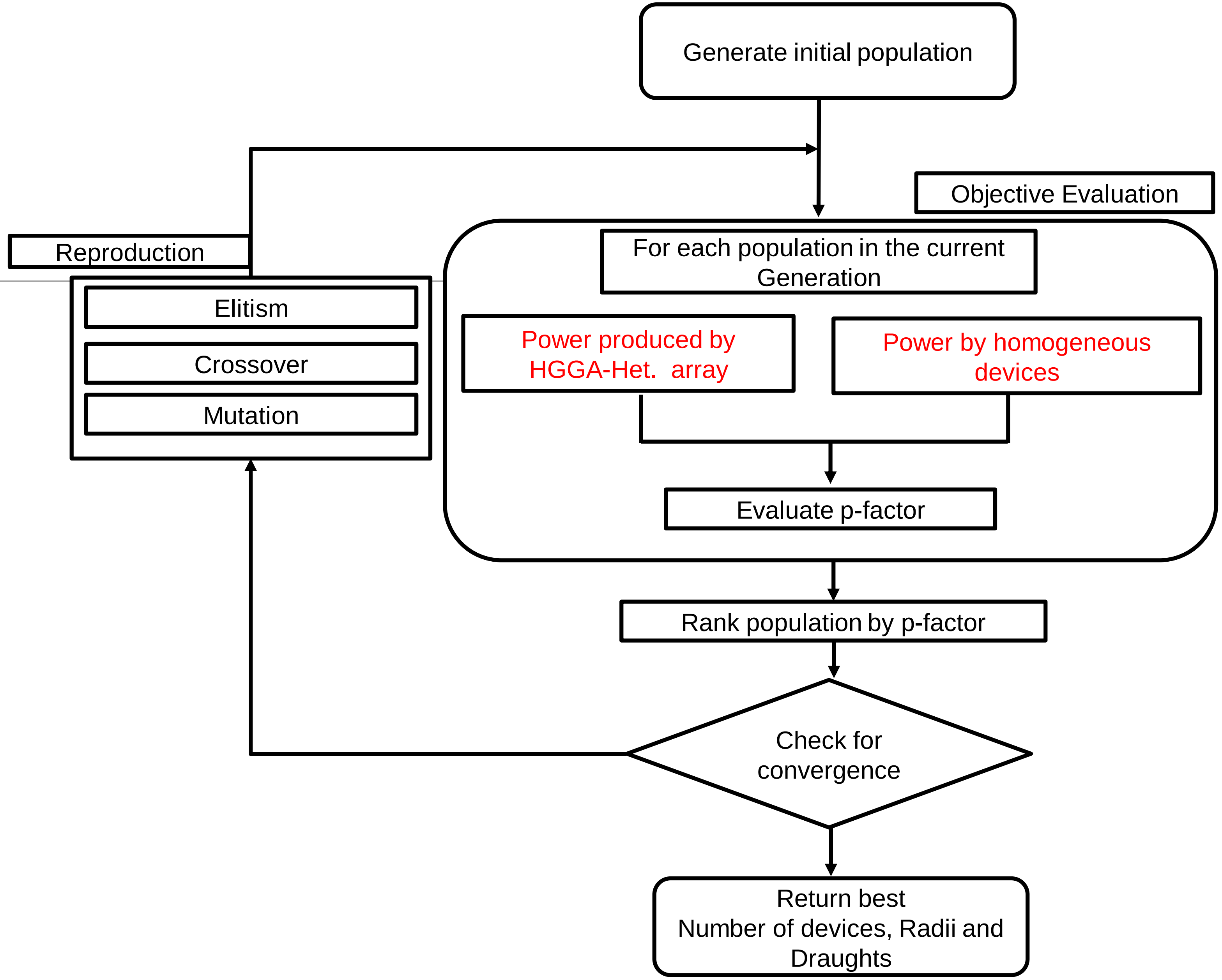
Hom. array Vs Het. array.

$$Subject\ to:$$

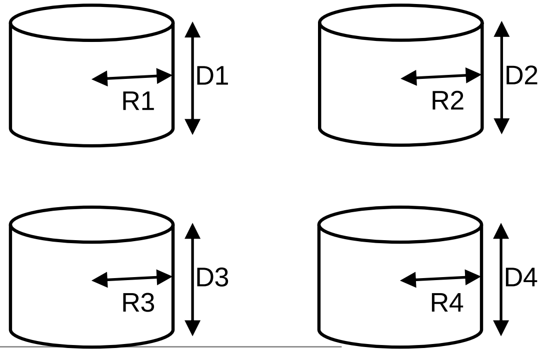
$$R_i \in [R_{min}, R_{max}], D_i \in [D_{min}, D_{max}], N \in [1, N_{homogenous}]$$

$$Total\ Vol.of\ Het\ array \leq Total\ Vol.of\ Hom.array.$$

Problem III: HGGA-Heterogenous array optimization



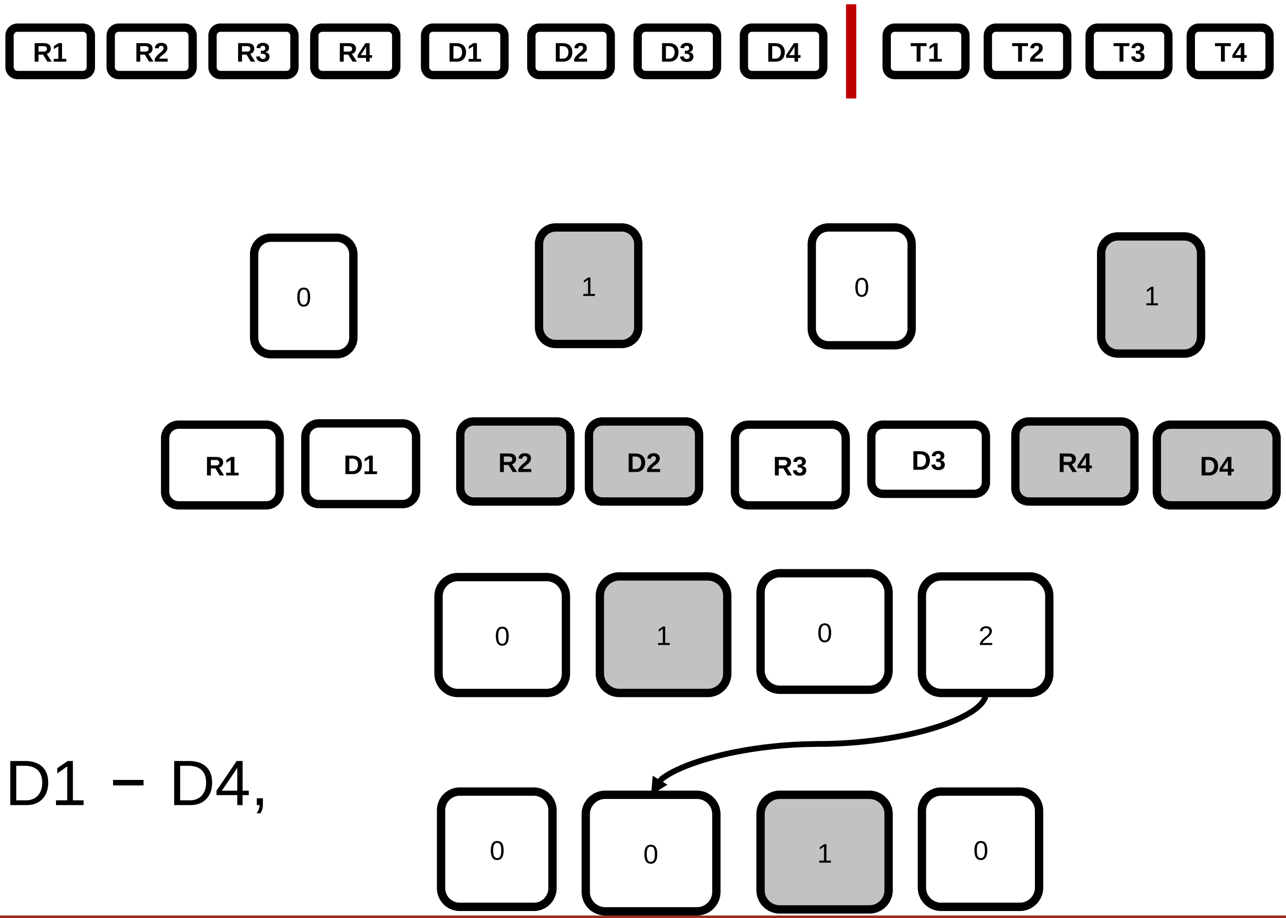
GA flow chart.



Hidden Genes GA (HGGA)

HGGA

- A variation on GA that allows optimizing the number of design variables, simultaneously with the variables.

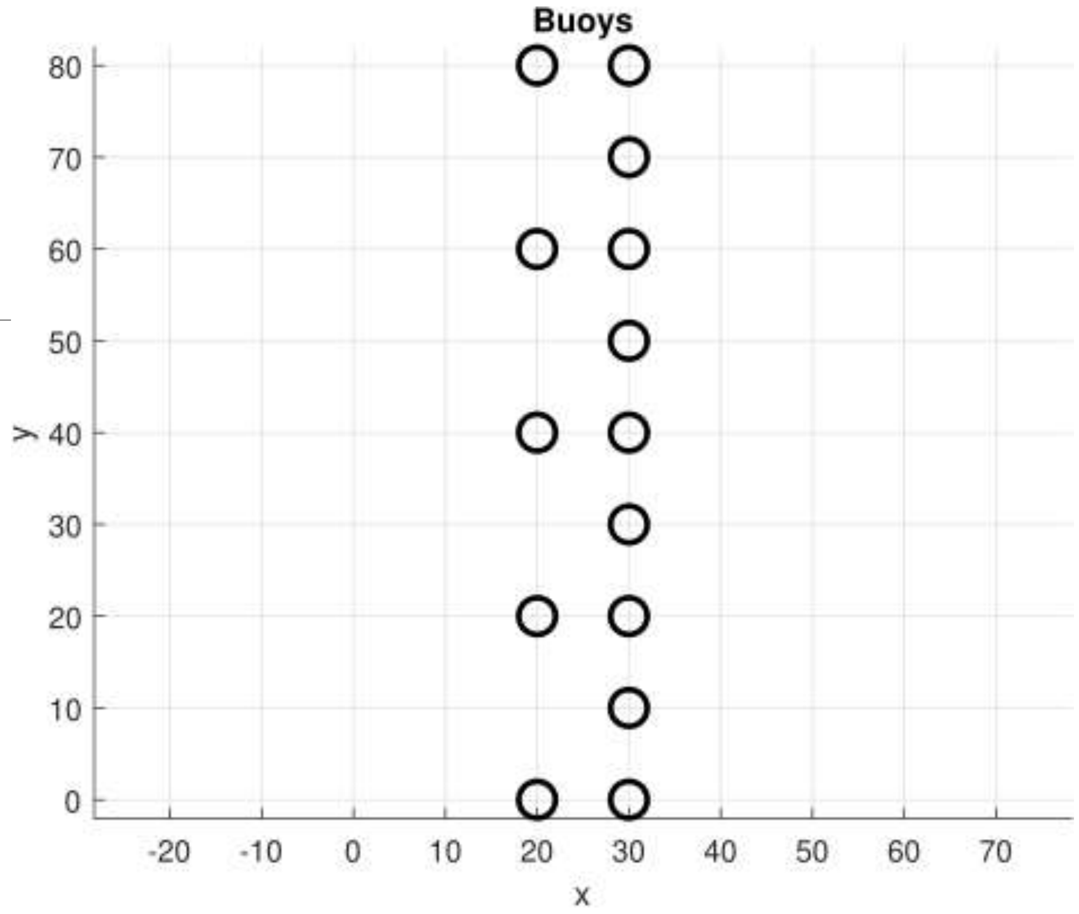


Example of 4 WECs

- A maximum of 4 WECs
- Variables: Radius R1 – R4, Draughts D1 – D4, and tags.

Homogeneous Array from literature

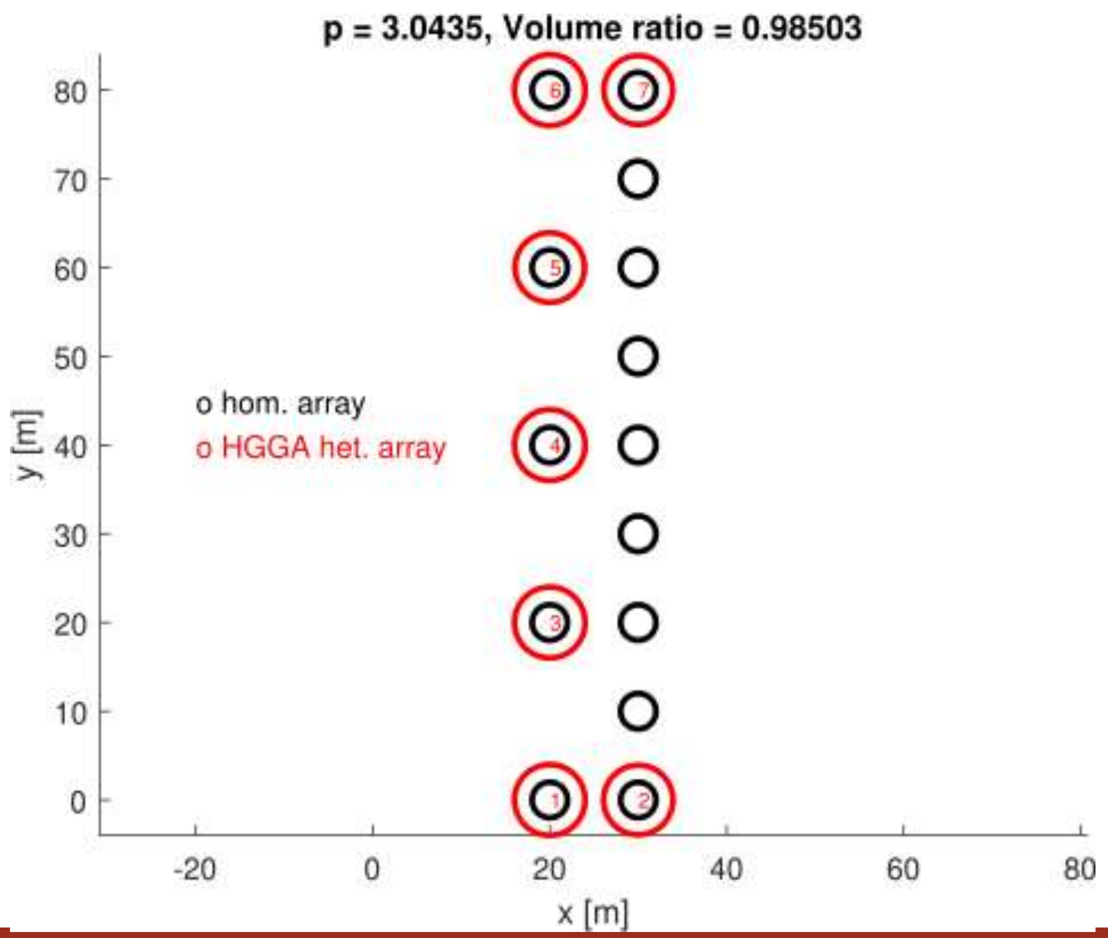
- Test case from Giassi 2018.
- R = 2 m, draft d = 0.5 m, and water depth h = 25 m.
- Wave site in Lysekil on the Swedish west coast.

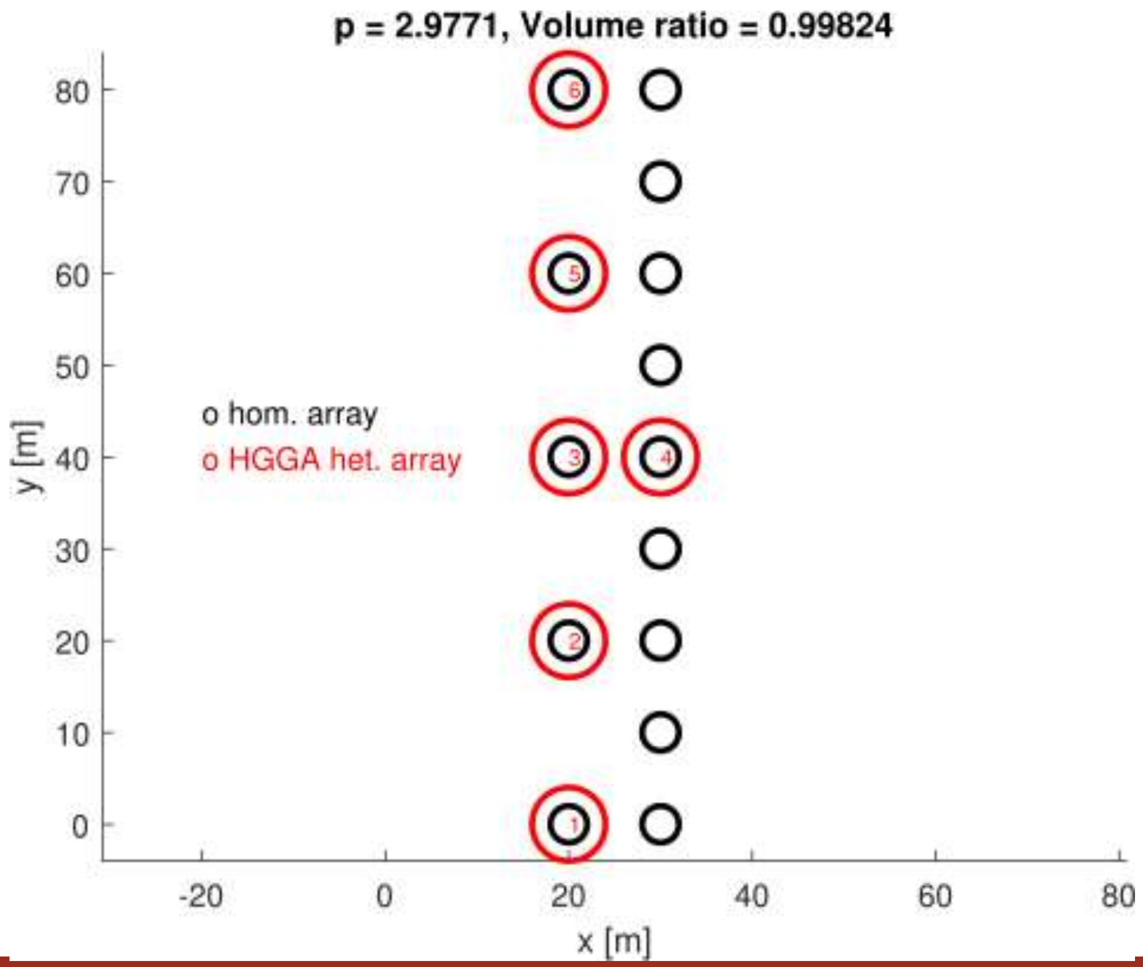


- Regular wave, wave height H = 1.53 m , period T = 5.01s.
- Wave propagating along the x-axis.

$$Maximize: p = \frac{P_{heterogeneous}}{P_{homogeneous}}$$

HGGA-Heterogeneous array result





Conclusion

- Investigated increasing power by allowing devices of different dimensions in the same array.
- Could allow the number of devices to vary during optimization
- Heterogenous arrays can produce more power while reducing total volume of buoys.

Acknowledgements

- This paper is based upon work supported by NSF, Grant Number 2048413.
- Authors would like to thank the collaborators from Maynooth University and Queens University Belfast for their input feedback: John Ringwood, Oliver Mason, Andrei Ermakov, and Pal Schmitt.

Questions

Semi-analytic hydro coefficients

- In the whole fluid domain, the governing equation $\nabla^2\emptyset = 0$
- Boundary conditions:
  - Free surface boundary conditions $\omega^2\emptyset - g\frac{\partial\emptyset}{\partial z}\Big|_{z=0} = 0$
  - Seabed condition $\frac{\partial\emptyset}{\partial z}\Big|_{z=-d} = 0$
  - Impermeable surface condition on the body surface

$$\frac{\partial\emptyset}{\partial r} = 0, (r = a, -h \le z \le 0)\ , \frac{\partial\emptyset}{\partial z} = 0, (0 \le r \le\ a, z = -h)$$

  - Summerfeld radiation condition $\lim_{x\to\infty}\sqrt{r}\left(\frac{\partial\emptyset}{\partial r} - ik_n\emptyset\right) = 0$

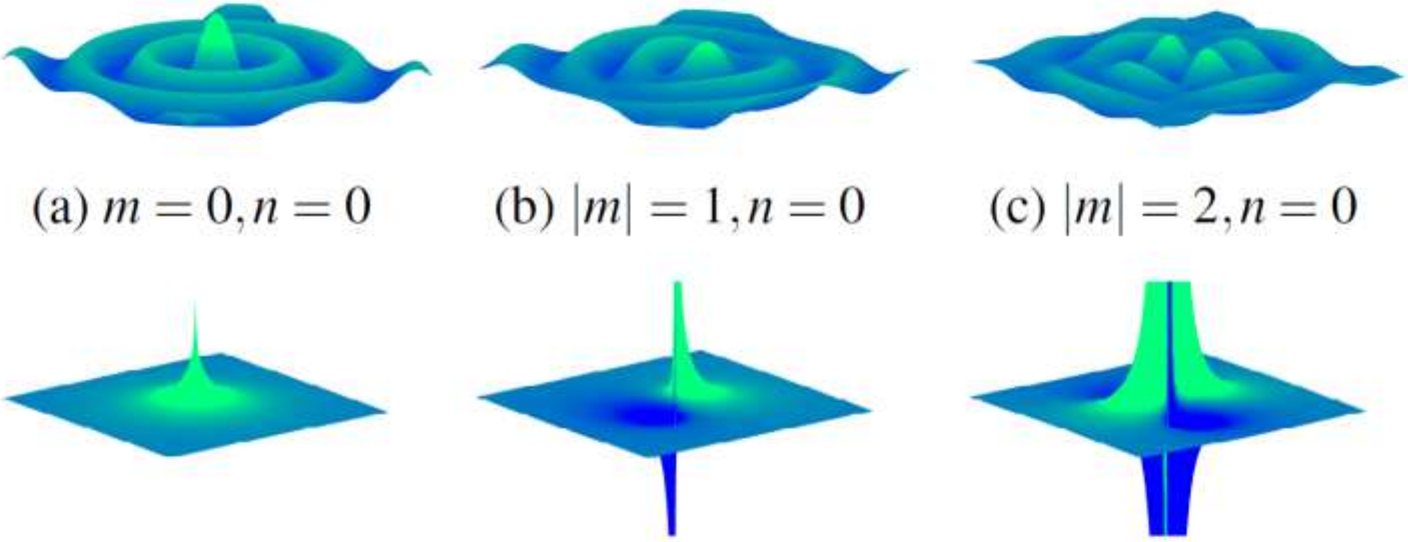
Radiation: exterior potential functions

- The homogenous potential function

$$\emptyset_{3,h}^{E} = D_{R0}\frac{H_m(k_0 r)}{H_m(k_0 a)} + \sum_{q=1}^{\infty} D_{Rq}\frac{K_m(k_q r)}{K_m(k_q a)}$$

Progressive and evanescent waves.

$$\emptyset_{3,p}^{E} = 0$$



(a) $m = 0, n = 0$ (b) $|m| = 1, n = 0$ (c) $|m| = 2, n = 0$

(d) $m = 0, n = 1$ (e) $|m| = 1, n = 1$ (f) $|m| = 2, n = 1$

Radiation: Interior potential functions

- Based on the methods of variable separation and matching eigenfunction expansion for the velocity potential.
- Separation of variables : $\emptyset_{3,m}^{I}(r,z) = \emptyset_{3,h}^{I} + \emptyset_{3,p}^{I}$
- The homogenous potential function

$$\emptyset_{3,h}^{I} = \frac{C_{R0}}{2}\left(\frac{r}{a}\right)^{m} + \sum_{n=1}^{\infty} C_{Rn} \frac{I_m\left(\frac{n\pi r}{(d-h)}\right)}{I_m\left(\frac{n\pi a}{(d-h)}\right)} cos\left(\frac{n\pi z}{(d-h)}\right)$$

- The particular solution

$$\emptyset_{3,p}^{I} = \frac{1}{2(d-h)}\left[(z+d)^2 - \frac{r^2}{2}\right]$$

Continuity conditions

- In both the radiation and diffraction problems, the matching conditions represent the continuity of mass flux, pressure and normal velocity.
- The velocity potentials between interior and exterior domains are matched at the imaginary boundary ($r = a$).

$$\emptyset^I = \emptyset^E, \; \frac{\partial \emptyset^E}{\partial r} = \frac{\partial \emptyset^I}{\partial r} (-h \le z \le -d), \qquad \frac{\partial \emptyset^E}{\partial r} = 0, (-h \le z \le 0),$$

- The unknown Fourier coefficients C_n, D_n, C_{Rn}, D_{Rn} are solved using the matching conditions.
- The hydrodynamic coefficients are found by integrating the potential functions over their corresponding area.