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State estimation for wave energy converters

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1 Introduction

This report gives a brief discussion and examples on the topic of state estimation for wave energy converters (WECs). These methods are intended for use to enable real-time closed loop control of WECs. The algorithm for the optimal estimation of unknown inputs and state requires the system to be expressed using a discrete-time state-space model as [1]

$$x_{k+1} = Ax_k + Bu_k + Gd_k + w_k \quad (1a)$$

$$y_k = Cx_k + Du_k + Hd_k + v_k, \quad (1b)$$

where d_k is the unknown input at time step k (i.e. the excitation force $d_k = F_{e_k}$ or the excitation pressure $d_k = P_{e_k}$), u_k is the known input (i.e. the actuator's force $u_k = F_{a_k}$), y_k is the measurements vector (i.e. acceleration and pressure $y = [\ddot{z}, P]^T$), w_k and v_k are the process and measurements noise, respectively. Two examples are discussed here: using position and acceleration measurements (Section 2) and using pressure and acceleration measurements (Section 3).

2 Using position

This Section provides an example for using position and acceleration to predict the state of a WEC. Sample code for implementing this example is provided in Appendix A. The block diagram of the buoy's dynamic model is depicted in Fig. 1. The "intrinsic" model of the buoy G_i in continuous-time state space form is

$$\dot{x}_i = A_i x_i + B_i (F_e + F_a) \quad (2a)$$

$$y_i = C_i x_i + D_i (F_e + F_a) \quad (2b)$$

where the state vector is

$$x = \begin{bmatrix} z \\ \dot{z} \\ x_r \end{bmatrix}, \quad (3)$$

and

$$A_i = \begin{bmatrix} -\frac{B_f}{m+m_\infty} & -\frac{K}{m+m_\infty} & -\frac{C_r}{m+m_\infty} \\ 1 & 0 & \mathbf{0}_{1 \times n_r} \\ B_r & \mathbf{0}_{n_r \times 1} & A_r \end{bmatrix} \quad B_i = \begin{bmatrix} \frac{1}{m+m_\infty} \\ 0 \\ \mathbf{0}_{n_r \times 1} \end{bmatrix} \quad (4)$$

$$C_i = \begin{bmatrix} 0 & 1 & \mathbf{0}_{1 \times n_r} \\ -\frac{B_f}{m+m_\infty} & -\frac{K}{m+m_\infty} & -\frac{C_r}{m+m_\infty} \end{bmatrix} \quad D_i = \begin{bmatrix} 0 \\ \frac{1}{m+m_\infty} \end{bmatrix} \quad (5)$$

and where the matrices $A_r \in \mathbb{R}^{n_r \times n_r}$, $B_r \in \mathbb{R}^{n_r \times 1}$ and $C_r \in \mathbb{R}^{1 \times n_r}$ describe the radiation force F_r dynamics as

$$\dot{x}_r = A_r x_r + B_r \dot{z} \quad (6a)$$

$$F_r = C_r x_r. \quad (6b)$$

The mass of the buoy is denoted by m , the hydrostatic restoring coefficient by K , the friction coefficient by B_f and m_∞ is the asymptotic value of the added mass for $\omega \rightarrow \infty$. Two steps are now required to formulate system in (2) as required in (1)

1. Convert to discrete time
2. Derive matrices A, B, C, D, G and H

If the matrix A_i is not singular, then step 1 can be carried out by using the

$$A = e^{A_i T_c} \quad (7)$$

$$B = A_i^{-1} (A - I) B_i \quad (8)$$

where T_c is the sampling time. The matrices C, D, G and H are:

$$C = C_i \quad D = D_i \quad (9)$$

$$G = B \quad H = D. \quad (10)$$

The time-varying version of the algorithm given in 1 whereas the steady-state version (much faster computation) is given in 2.

Algorithm 1 Time-varying Unknown Input and State Estimator.

▷ Initialize:

- 1: $\hat{x}_{0|0} = \mathbb{E}[x_0]$
- 2: $\hat{d}_0 = H^\dagger (y_0 - C\hat{x}_{0|0} - Du_0)$
- 3: $P_{0|0}^x = \mathcal{P}_0^x$
- 4: $P_0^d = \mathcal{P}_0^d$
- 5: $P_0^{xd} = \mathcal{P}_0^{xd}$
- 6: $Q = \mathbb{E}[w w^T]$
- 7: $R = \mathbb{E}[v v^T]$

▷ Estimation loop for N time steps (Time step = T_c)

- 8: **for** $k = 1$ to N **do**
- ▷ One-Step prediction
- 9: $\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_{k-1} + G\hat{d}_{k-1}$
- 10: $P_{k|k-1}^x = AP_{k-1|k-1}^x A^T + GP_{k-1}^{xdT} A^T + AP_{k-1}^{xd} G^T + GP_{k-1}^d G^T + Q$
- 11: $\tilde{R}_k = CP_{k|k-1}^x C^T$
- ▷ Measurements update
- 12: $K_k = P_{k|k-1}^x C^T \tilde{R}_k^{-1}$
- 13: $L_k = K_k (I - H (H^T \tilde{R}_k^{-1} H)^{-1} H^T \tilde{R}_k^{-1})$
- 14: $\hat{x}_{k|k} = \hat{x}_{k|k-1} + L_k (y_k - C\hat{x}_{k|k-1} - Du_k)$
- 15: $P_{k|k}^x = (I - L_k C) P_{k|k-1}^x (I - L_k C)^T + L_k R L_k^T$
- ▷ Estimation of unknown input
- 16: $\tilde{R}_k^* = (I - CL_k) \tilde{R}_k (I - CL_k)^T$
- 17: $P_k^d = (H^T \tilde{R}_k^{*-1} H)^{-1}$
- 18: $M_k = P_k^d H^T \tilde{R}_k^{*-1}$
- 19: $\hat{d}_k = M_k (y_k - C\hat{x}_{k|k} - Du_k)$
- 20: $P_k^{xd} = -P_{k|k}^x C^T M_k^T + L_k R M_k^T$
- 21: **end for**

Algorithm 2 Steady-State Unknown Input and State Estimator.

▷ Initialize:

- 1: $\hat{x}_{0|0} = \mathbb{E}[x_0]$
- 2: $\hat{d}_0 = H^\dagger (y_0 - C\hat{x}_{0|0} - Du_0)$
- 3: $L_\infty = \lim_{k \rightarrow \infty} L_k$
- 4: $M_\infty = \lim_{k \rightarrow \infty} M_k$

▷ Estimation loop for N time steps (Time step = T_c)

- 5: **for** $k = 1$ to N **do**
- 6: $\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_{k-1} + G\hat{d}_{k-1}$ ▷ One-Step prediction
- 7: $\hat{x}_{k|k} = \hat{x}_{k|k-1} + L_\infty (y_k - C\hat{x}_{k|k-1} - Du_k)$ ▷ State estimation
- 8: $\hat{d}_k = M_\infty (y_k - C\hat{x}_{k|k} - Du_k)$ ▷ Unknown input estimation
- 9: **end for**

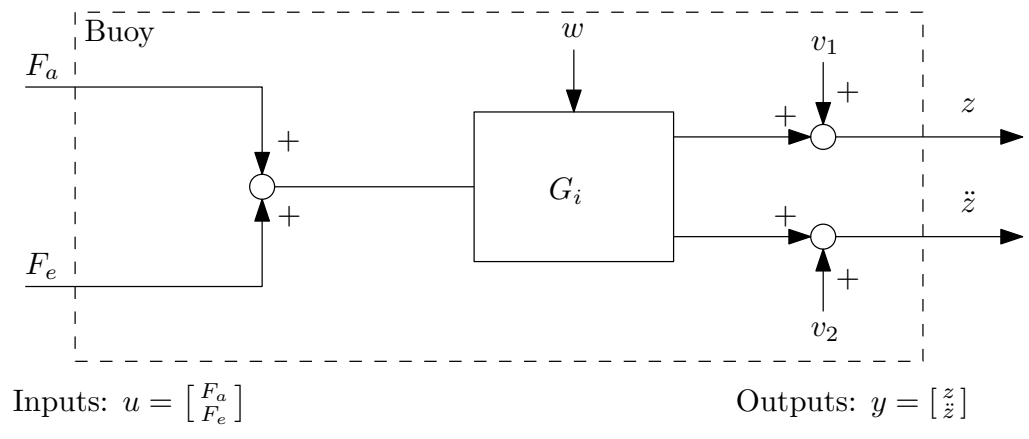


Figure 1. Block diagram of the buoy's dynamic model when position and acceleration are measured

3 Using pressure

This Section provides an example for using pressure and acceleration to predict the state of a WEC. Sample code for implementing this example is provided in Appendix B. The block diagram of the buoy's dynamic model is depicted in Fig. 1. The "intrinsic" model of the buoy G_i in continuous-time state space form is

$$\dot{x}_i = A_i x_i + B_i (F_e + F_a) \quad (11a)$$

$$y_i = C_i x_i + D_i (F_e + F_a) \quad (11b)$$

where the output vector y_i is

$$y_i = \begin{bmatrix} \ddot{z} \\ P_r \end{bmatrix}, \quad (12)$$

and the matrices composing the state space model in (11) have been identified from experimental data.

The state-space model of the excitation pressure in continuous-time (G_e) is:

$$\dot{x}_e = A_e x_e + B_e P_e \quad (13a)$$

$$F_e = C_e x_e + D_e P_e. \quad (13b)$$

According to the diagram in Fig. 2 the models in (11) and (13) can be combined to form the state-state space model

$$\dot{x} = A_c x + B_c \begin{bmatrix} F_a \\ P_e \end{bmatrix} \quad (14a)$$

$$y = C_c x + D_c \begin{bmatrix} F_a \\ P_e \end{bmatrix} \quad (14b)$$

where the state vector is

$$x = \begin{bmatrix} x_i \\ x_e \end{bmatrix} \quad (15)$$

and where the output vector is

$$y = \begin{bmatrix} \ddot{z} \\ P \end{bmatrix} = \begin{bmatrix} \ddot{z} \\ P_r + P_e \end{bmatrix} = \begin{bmatrix} \ddot{z} \\ P_r \end{bmatrix} + \begin{bmatrix} 0 \\ P_e \end{bmatrix} = y_i + \begin{bmatrix} 0 \\ P_e \end{bmatrix}. \quad (16)$$

The system matrices are

$$A_c = \begin{bmatrix} A_i & B_i C_e \\ \mathbf{0} & A_e \end{bmatrix} \quad B_c = \begin{bmatrix} B_i & B_u D_e \\ \mathbf{0} & B_e \end{bmatrix} \quad (17)$$

$$C_c = \begin{bmatrix} C_i & D_i C_e \end{bmatrix} \quad D_c = \begin{bmatrix} D_i & D_i D_e + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} \quad (18)$$

Two steps are now required to formulate system in (14) as required in (1):

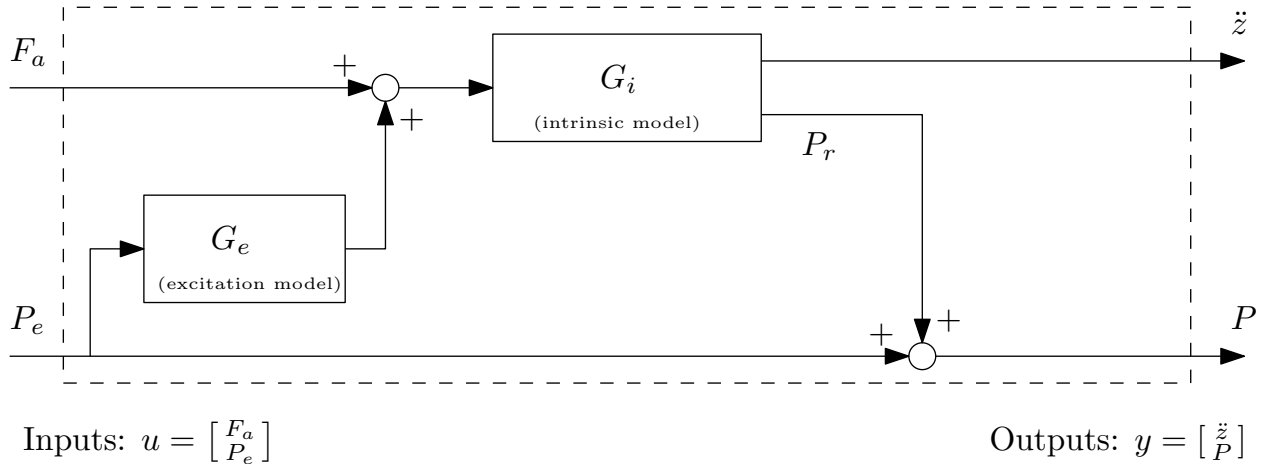


Figure 2. Block diagram of the buoy's dynamic model when pressure and acceleration are measured.

1. Convert to discrete time
2. Derive matrices A , B , C , D , G and H

If the matrix A_c is not singular, then step 1 can be carried out by using the

$$A = e^{A_c T_c} \quad (19)$$

$$\bar{B} = A_c^{-1} (A - I) B_c \quad (20)$$

where T_c is the sampling time. \bar{B} and D_c are $2 \times n$ matrices: B is the first column of \bar{B} , G is the second column of \bar{B} , D is the first column of D_c and H is the second column of D_c , that is:

$$\bar{B} = [B \quad G] \quad (21)$$

$$C = C_c \quad D_c = [D \quad H] \quad (22)$$

$$(23)$$

The time-varying version of the algorithm given in 3 whereas the steady-state version (much faster computation) is given in 4.

Algorithm 3 Time-varying Unknown Input and State Estimator.

▷ Initialize:

- 1: $\hat{x}_{0|0} = \mathbb{E}[x_0]$
- 2: $\hat{d}_0 = H^\dagger (y_0 - C\hat{x}_{0|0} - Du_0)$
- 3: $P_{0|0}^x = \mathcal{P}_0^x$
- 4: $P_0^d = \mathcal{P}_0^d$
- 5: $P_0^{xd} = \mathcal{P}_0^{xd}$
- 6: $Q = \mathbb{E}[w w^T]$
- 7: $R = \mathbb{E}[v v^T]$

▷ Estimation loop for N time steps (Time step = T_c)

- 8: **for** $k = 1$ to N **do**
- ▷ One-Step prediction
- 9: $\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_{k-1} + G\hat{d}_{k-1}$
- 10: $P_{k|k-1}^x = AP_{k-1|k-1}^x A^T + GP_{k-1}^{xd} A^T + AP_{k-1}^{xd} G^T + GP_{k-1}^d G^T + Q$
- 11: $\tilde{R}_k = CP_{k|k-1}^x C^T$
- ▷ Measurements update
- 12: $K_k = P_{k|k-1}^x C^T \tilde{R}_k^{-1}$
- 13: $L_k = K_k (I - H (H^T \tilde{R}_k^{-1} H)^{-1} H^T \tilde{R}_k^{-1})$
- 14: $\hat{x}_{k|k} = \hat{x}_{k|k-1} + L_k (y_k - C\hat{x}_{k|k-1} - Du_k)$
- 15: $P_{k|k}^x = (I - L_k C) P_{k|k-1}^x (I - L_k C)^T + L_k R L_k^T$
- ▷ Estimation of unknown input
- 16: $\tilde{R}_k^* = (I - CL_k) \tilde{R}_k (I - CL_k)^T$
- 17: $P_k^d = (H^T \tilde{R}_k^{*-1} H)^{-1}$
- 18: $M_k = P_k^d H^T \tilde{R}_k^{*-1}$
- 19: $\hat{d}_k = M_k (y_k - C\hat{x}_{k|k} - Du_k)$
- 20: $P_k^{xd} = -P_{k|k}^x C^T M_k^T + L_k R M_k^T$
- 21: **end for**

Algorithm 4 Steady-State Unknown Input and State Estimator.

▷ Initialize:

- 1: $\hat{x}_{0|0} = \mathbb{E}[x_0]$
- 2: $\hat{d}_0 = H^\dagger (y_0 - C\hat{x}_{0|0} - Du_0)$
- 3: $L_\infty = \lim_{k \rightarrow \infty} L_k$
- 4: $M_\infty = \lim_{k \rightarrow \infty} M_k$

▷ Estimation loop for N time steps (Time step = T_c)

- 5: **for** $k = 1$ to N **do**
- 6: $\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_{k-1} + G\hat{d}_{k-1}$ ▷ One-Step prediction
- 7: $\hat{x}_{k|k} = \hat{x}_{k|k-1} + L_\infty (y_k - C\hat{x}_{k|k-1} - Du_k)$ ▷ State estimation
- 8: $\hat{d}_k = M_\infty (y_k - C\hat{x}_{k|k} - Du_k)$ ▷ Unknown input estimation
- 9: **end for**

References

- [1] Sze Zheng Yong, Minghui Zhu, and Emilio Frazzoli. A unified filter for simultaneous input and state estimation of linear discrete-time stochastic systems. *Automatica*, 63:321 – 329, 2016.

A Sample code: position and acceleration measurements

This section contain sample MATLAB code for implementing a position/acceleration state estimator for a WEC.

```
1 % this scripts is to test the Unified Linear Input and State Estimator
2 % (ULISE algorithm) described in
3 %
4 % S. Z. Yong, M. Zhu, and E. Frazzoli, A unified filter for simultaneous
5 % input and state estimation of linear discrete-time stochastic systems,
6 % Automatica, vol. 63, pp. 321-329, 2016.
7 %
8 % Both Time-Varying and Steady-State versions are implemented.
9 % Position and acceleration measurements are used to estimate state and
10 % excitation force
11 %
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21
22
23 clc
24 clear
25
26 % load identified parametric WEC model model
27 WEC = load('WEC_param_model_1DOF.mat');
28
29 Tc = 1e-3; % sampling time
30 N = 5e4; % number of simulation steps
31
32 % load excitation force and interpolate
33 Fe = load('Exc_time_series.mat');
34 t = (1:N)*Tc;
35 d = interp1(Fe.t_trim, Fe.Fexc_td, t, 'pchip')*1e-3; %(excitation force in kN)
36
37 u = 0.5*sin(2*pi*0.75*t); % control input (PTO force in kN). Open loop, no control implemented
38
39 mass = 858.3987;
40 Ainf_hat = 822.3799;
41 K = 2.3981e+04;
42
43 Ar = WEC.rad_sys.a;
44 Br = WEC.rad_sys.b;
45 Cr = WEC.rad_sys.c;
46
47 Bf = WEC.B_eq_mat(9);
48
49 Ac = [-Bf/(mass + Ainf_hat)    -K/(mass + Ainf_hat)    -Cr/(mass + Ainf_hat);
50       1                        0                        zeros(1,length(Ar));
51       Br                        zeros(length(Ar),1)      Ar                ];
52
53 % input matrix (1e3 factor is used to account for the forces that expressed are in kN)
54 Bc = 1e3*[1/(mass + Ainf_hat);
55          0                        ;
56          zeros(length(Ar),1)];
57
58 % measurements: position and acceleration
```

```

59 C = [0 1 zeros(1,length(Ar));
      Ac(1,:)          ];
61
63 D = [0; Bc(1)];
65
67 n = size(C,2);
69 p = size(C,1);
71
73 % convert continuous time model to discrete time
75 A = expm(Ac*Tc);
77 B = Ac\ (A - eye(n))*Bc;
79
81 G = B;
83 H = D;
85
87 % process noise
89 w = .00001*(rand(N,4)-0.5);
91 % measurements noise
93 v = (rand(N,p)-0.5) * diag([.0001,0.05]);
95
97 Q = cov(w);
99 R = cov(v);
101
103 %% Time varying filter
105
107 % initialize variables
109 P_x_k_k = 0.001*eye(n);
111 P_xd_k = 0.001*ones(n,1);
113 P_d_k = 0.001;
115
117 x = zeros(n,1);
119 x_k_k = x;
121 d_k = 0;
123
125 In = eye(n);
127 Ip = eye(p);
129
131 % preallocation
133 d_k_vec = zeros(N,1);
135 x_k_vec = zeros(n,N);
137 x_vec = zeros(n,N);
139 y_vec = zeros(p,N);
141
143 tic
145
147 for ii = 1:N
149
151     x = A*x + B*u(ii) + G*d(ii) + w(ii,:);
153     y = C*x + D*u(ii) + H*d(ii) + v(ii,:);
155
157     x_vec(:,ii) = x;
159     y_vec(:,ii) = y;
161
163     x_k_k1 = A*x_k_k + B*u(ii) + G*d_k;
165     P_x_k_k1 = A*P_x_k_k*A' + G*P_xd_k'*A' + A*P_xd_k*G' + G*P_d_k*G' + Q;
167     R_t_k = C*P_x_k_k1*C' + R;
169     Kk = P_x_k_k1*C'/R_t_k;
171     Lk = Kk*(Ip - ((H'/(H'/R_t_k)*H))*H'/R_t_k);
173     x_k_k = x_k_k1 + Lk*(y - C*x_k_k1 - D*u(ii));
175     P_x_k_k = (In-Lk*C)*P_x_k_k1*(In-Lk*C)' + Lk*R*Lk';
177     R_ts_k = (Ip-C*Lk)*R_t_k*(Ip-C*Lk)';
179     P_d_k = inv((H'/R_ts_k)*H);
181     Mk = ((H'/R_ts_k)*H)\(H'/R_ts_k);
183     d_k = Mk*(y - C*x_k_k - D*u(ii));
185     P_xd_k = -P_x_k_k*C'*Mk' + Lk*R*Mk';
187
189     d_k_vec(ii) = d_k;

```

```

127     x_k_vec(:,ii) = x_k_k;
129 end
131 t_tv = toc;
132 disp('done TV')
133 %% steady state filter
135 L_inf = Lk;
137 M_inf = Mk;
139 % preallocation
140 d_k_vec_inf = zeros(N,1);
141 x_vec_inf = zeros(n,N);
142 y_vec_inf = zeros(p,N);
143 x_vec_est_inf = x_vec_inf;
145 % initialization
146 x = zeros(n,1);
147 x_k_k_inf = zeros(n,1);
148 d_kl_inf = 0;
149 tic
151 for ii = 1:N
153     x = A*x + B*u(ii) + G*d(ii) + w(ii,:);
154     y = C*x + D*u(ii) + H*d(ii) + v(ii,:);
155
156     x_vec_inf(:,ii) = x;
157     y_vec_inf(:,ii) = y;
159     x_k_kl_inf = A*x_k_k_inf + B*u(ii) + G*d_kl_inf;
160     x_k_k_inf = x_k_kl_inf + L_inf*(y - C*x_k_kl_inf - D*u(ii) );
161     d_kl_inf = M_inf*(y - C*x_k_k_inf - D*u(ii));
163     d_k_vec_inf(ii) = d_kl_inf;
164     x_vec_est_inf(:,ii) = x_k_k_inf;
165 end
167 t_ss = toc;
168 disp('done SS')
169
171 %% plotting
173 disp(['Time to compute Time-Varying filter: ' num2str(t_tv) 's'])
174 disp(['Time to compute Steady-State filter: ' num2str(t_ss) 's'])
175
176 figure(1)
177 plot(t, d_k_vec, t, d)
178 xlabel('time (s)')
179 ylabel('(kN)')
180 grid on
181 title('Time-Varying filter')
182 legend({'$\hat{F}e_{\infty}$', '$Fe$'}, 'Interpreter', 'latex')
183
184 figure(2)
185 plot(t, d_k_vec_inf, t, d)
186 xlabel('time (s)')
187 ylabel('(kN)')
188 grid on
189 title('Steady-State filter')
190 legend({'$\hat{F}e$', '$Fe$'}, 'Interpreter', 'latex')
191
192 figure(3)
193 plot(t, d_k_vec - d_k_vec_inf)
194 grid on

```

```

195 xlabel('Time (s)')
    title('Difference between Time-Varying and Steady-State filters')
197 legend('e_d')

199 figure(4)
    subplot 211
201 plot(t, y_vec(1,:))
    xlabel('Time (s)')
203 ylabel(' (m)')
    title('Measured (noisy) Outputs')
205 legend({'$z$'}, 'Interpreter', 'latex')
    grid on
207 subplot 212
    plot(t, y_vec(2,:))
209 xlabel('Time (s)')
    ylabel(' (m/s^2)')
211 legend({'$\ddot{z}$'}, 'Interpreter', 'latex')
    grid on
213

215 figure(5)
    subplot 211
217 plot(t, x_vec(1,:)', t, x_k_vec(1,:)', t, x_vec_est_inf(1,:))
    grid on
219 xlabel('time (s)')
    ylabel(' (m/s)')
221 legend({'$v$', '$\hat{v}$', '$\hat{v}_{\infty}$'}, 'Interpreter', 'latex')
    title('Estimated states')
223 grid on

225 subplot 212
    plot(t, x_vec(2,:)', t, x_k_vec(2,:)', t, x_vec_est_inf(2,:))
227 grid on
    xlabel('time (s)')
229 ylabel(' (m)')
    legend({'$z$', '$\hat{z}$', '$\hat{z}_{\infty}$'}, 'Interpreter', 'latex')
231 grid on

```

State_and_unknown_input_estimator_position_acceleration.m

B Sample code: pressure and acceleration measurements

This section contain sample MATLAB code for implementing a pressure/acceleration state estimator for a WEC.

```
1 % this scripts is to test the Unified Linear Input and State Estimator
2 % (ULISE algorithm) described in
3 %
4 % S. Z. Yong, M. Zhu, and E. Frazzoli, A unified filter for simultaneous
5 % input and state estimation of linear discrete-time stochastic systems,
6 % Automatica, vol. 63, pp. 321-329, 2016.
7 %
8 % Both Time-Varying and Steady-State versions are implemented. Pressure and
9 % acceleration measurements are used to estimate state and excitation force
10 %
11 % Sandia National Laboratories is a multi-mission laboratory managed and
12 % operated by National Technology and Engineering Solutions of Sandia,
13 % LLC., a wholly owned subsidiary of Honeywell International, Inc., for the
14 % U.S. Department of Energy's National Nuclear Security Administration
15 % under contract DE-NA0003525.
16 %
17 % G. Bacelli, R. Coe
18 % Sandia National Laboratories
19 % 2017
20
21
22
23
24
25 % load identified parametric WEC model model
26 WEC = load('WEC_param_model_1DOF.mat'); % radiatiojn impedance model
27 Gr = struct2array(load('Gr_model.mat')); % radiation pressure model
28 Ge = struct2array(load('Ge_model.mat')); % excitation pressure model
29
30 Tc = 1e-3;
31 N = 5e4;
32
33 % load excitation force
34 Fe = load('Exc_time_series.mat'); %(excitation force in kN)
35 t = (1:N)*Tc;
36 d = interp1(Fe.t_trim, Fe.Fexc_td, t, 'pchip')*1e-3; % control input (PTO force in kN). Open loop
37     , no control implemented
38
39 u = .5*sin(2*pi*0.75*t);
40
41 mass = 858.3987;
42 Ainf_hat = 822.3799;
43 K = 2.3981e+04;
44
45 Ar = WEC.rad_sys.a;
46 Br = WEC.rad_sys.b;
47 Cr = WEC.rad_sys.c;
48
49 Bf = WEC.B_eq_mat(9);
50
51 Am = [-Bf/(mass + Ainf_hat)    -K/(mass + Ainf_hat)    -Cr/(mass + Ainf_hat);
52       1                        0                      zeros(1,length(Ar));
53       Br                      zeros(length(Ar),1)    Ar];
54
55 % input matrix (1e3 factor is used to acccount for the forces that expressed are in kN)
56 Bm = 1e3*[1/(mass + Ainf_hat);
57          0                        ;
58          zeros(length(Ar),1)    ];
```

```

59 Cm = Am(1,:);
61 Dm = Bm(1);
63 Ai = blkdiag(Am, Gr.A);
Bi = [Bm; Gr.B];
65 Ci = blkdiag(Cm, Gr.C);
Di = [Dm; Gr.D];
67
ni = size(Ai,1);
69 ne = size(Ge.A,1);
Ac = [Ai, Bi*Ge.C;
71      zeros(ne, ni) Ge.A];
73 Bc = [Bi, Bi*Ge.D; zeros(ne,1), Ge.B];
75 C = [Ci, Di*Ge.C];
D = [Di, Di*Ge.D + [0;1] ];
77
sys_c = ss(Ac, Bc, C, D);
79 sys_d = c2d(sys_c, Tc);
81 Ge_d = c2d(Ge,Tc);
83 Ce = Ge_d.C;
De = Ge_d.D;
85
n = size(C,2);
87 p = size(C,1);
89 % convert continuous time model to discrete time
A = expm(Ac*Tc);
91 B = Ac\ (A - eye(n))*Bc;
93 G = B(:,1);
B = B(:,1);
95
H = D(:,2);
97 D = D(:,1);
99 % process noise
w = .00001*(rand(N,ni+ne)-0.5);
101 % measurements noise
v = (rand(N,p)-0.5) * diag([.05, 0.05]);
103
Q = cov(w);
105 R = cov(v);
107 %% Time Varying filter
109 % initialize variables
P_x_k_k = 0.001*eye(n);
111 P_xd_k = 0.001*ones(n,1);
P_d_k = 0.001;
113
x = zeros(n,1);
115 x_k_k = x;
d_k = 0;
117
In = eye(n);
119 Ip = eye(p);
121 % preallocation
x_vec = zeros(n,N);
123 y_vec = zeros(p,N);
d_k_vec = zeros(N,1);
125 x_k_vec = zeros(n,N);

```

```

Fe_est = zeros(N,1);
127 Fe = Fe_est;

129 tic

131 for ii = 1:N

133     x = A*x + B*u(ii) + G*d(ii) + w(ii,:);
134     y = C*x + D*u(ii) + H*d(ii) + v(ii,:);
135
136     Fe(ii) = Ce*x(ni+1:end) + De*d(ii);
137
138     x_vec(:,ii) = x;
139     y_vec(:,ii) = y;

141     x_k_k1 = A*x_k_k + B*u(ii) + G*d_k;
142     P_x_k_k1 = A*P_x_k_k*A' + G*P_xd_k'*A' + A*P_xd_k*G' + G*P_d_k*G' + Q;
143     R_t_k = C*P_x_k_k1*C' + R;
144     Kk = P_x_k_k1*C'/R_t_k;
145     Lk = Kk*(Ip-((H'/(H'/R_t_k)*H))*H'/R_t_k);
146     x_k_k = x_k_k1 + Lk*(y - C*x_k_k1 - D*u(ii));
147     P_x_k_k = (In-Lk*C)*P_x_k_k1*(In-Lk*C)' + Lk*R*Lk';
148     R_ts_k = (Ip-C*Lk)*R_t_k*(Ip-C*Lk)';
149     P_d_k = inv((H'/R_ts_k)*H);
150     Mk = ((H'/R_ts_k)*H)\(H'/R_ts_k);
151     d_k = Mk*(y - C*x_k_k - D*u(ii));
152     P_xd_k = -P_x_k_k*C'*Mk' + Lk*R*Mk';

153
154     d_k_vec(ii) = d_k;
155     x_k_vec(:,ii) = x_k_k;
156     Fe_est(ii) = Ce*x_k_k(ni+1:end) + De*d_k;

157 end

159 t_tv = toc;
161 disp('done TV')

163 %% Steady State filter

165 L_inf = Lk;
166 M_inf = Mk;
167

168 % preallocation
169 d_k_vec_inf = zeros(N,1);
170 x_vec_inf = zeros(n,N);
171 y_vec_inf = zeros(p,N);
172 x_vec_est_inf = x_vec_inf;
173 Fe_est_inf = zeros(N,1);

175 % initialization
176 x = zeros(n,1);
177 x_k_k_inf = zeros(n,1);
178 d_k1_inf = 0;

179 tic
181 for ii = 1:N

183     x = A*x + B*u(ii) + G*d(ii) + w(ii,:);
184     y = C*x + D*u(ii) + H*d(ii) + v(ii,:);
185
186     x_vec_inf(:,ii) = x;
187     y_vec_inf(:,ii) = y;

189     x_k_k1_inf = A*x_k_k_inf + B*u(ii) + G*d_k1_inf;
190     x_k_k_inf = x_k_k1_inf + L_inf*(y - C*x_k_k1_inf - D*u(ii));
191     d_k1_inf = M_inf*(y - C*x_k_k1_inf - D*u(ii));

193     d_k_vec_inf(ii) = d_k1_inf;

```

```

195     x_vec_est_inf(:,ii) = x_k_k_inf;
196     Fe_est_inf(ii) = Ce*x_k_k_inf(ni+1:end) + De*d_k1_inf;
197 end
198
199 t_ss = toc;
200 disp('done SS')
201 %% plotting
202
203 disp(['Time to compute Time-Varying filter: ' num2str(t_tv) 's'])
204 disp(['Time to compute Steady-State filter: ' num2str(t_ss) 's'])
205
206
207 figure(1)
208 subplot 211
209 plot(t, d_k_vec', t, d)
210 xlabel('time (s)')
211 ylabel(' (kPa)')
212 grid on
213 legend({'$\hat{P}e$', '$Pe$'}, 'Interpreter', 'latex')
214 title('Time-Varying filter: Unknown input (Excitation pressure Pe)')
215
216 subplot 212
217 plot(t, Fe_est, t, Fe)
218 grid on
219 ylabel(' (kN)')
220 xlabel('time (s)')
221 title('Excitation force')
222 legend({'$\hat{F}e$', '$Fe$'}, 'Interpreter', 'latex')
223
224 figure(2)
225 subplot 211
226 plot(t, d_k_vec_inf', t, d)
227 xlabel('time (s)')
228 ylabel(' (kPa)')
229 grid on
230 legend({'$\hat{P}e$', '$Pe$'}, 'Interpreter', 'latex')
231 title('Steady-State filter: Unknown input (Excitation pressure Pe)')
232
233 subplot 212
234 plot(t, Fe_est_inf, t, Fe)
235 grid on
236 ylabel(' (kN)')
237 xlabel('time (s)')
238 title('Excitation force')
239 legend({'$\hat{F}e$', '$Fe$'}, 'Interpreter', 'latex')
240
241 figure(3)
242 plot(t, d_k_vec - d_k_vec_inf)
243 grid on
244 xlabel('Time (s)')
245 title('Difference between Time-Varying and Steady-State filters')
246 legend('e_d')
247
248 figure(4)
249 subplot 211
250 plot(t, y_vec(1,:))
251 xlabel('Time (s)')
252 ylabel(' (m/s^2)')
253 title('Measured (noisy) Outputs')
254 legend({'$\ddot{z}$'}, 'Interpreter', 'latex')
255 grid on
256 subplot 212
257 plot(t, y_vec(2,:))
258 xlabel('Time (s)')
259 ylabel(' (kPa)', 'Interpreter', 'latex')
260 grid on
261 legend({'$P$'}, 'Interpreter', 'latex')

```



```

263 figure(5)
    subplot 211
265 plot(t, x_vec(1,:), t, x_k_vec(1,:), t, x_vec_est_inf(1,:))
    grid on
267 xlabel('time (s)')
    ylabel(' (m/s)')
269 legend({'$v$', '$\hat{v}$', '$\hat{v}_{\infty}$'}, 'Interpreter', 'latex')
    title('Estimated states')
271 grid on

273 subplot 212
    plot(t, x_vec(2,:), t, x_k_vec(2,:), t, x_vec_est_inf(2,:))
275 grid on
    xlabel('time (s)')
277 ylabel(' (m)')
    legend({'$z$', '$\hat{z}$', '$\hat{z}_{\infty}$'}, 'Interpreter', 'latex')
279 grid on

```

State_and_unknown_input_estimator_pressure_acceleration.m

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