

# Numerical modelling of two-scale turbine flow dynamics

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**Summary:** Two-scale actuator disc theory is shown to predict qualitatively the performance of a lateral array of tidal turbines in a depth-averaged channel model. The agreement between the numerical and analytical models is improved by extrapolating the numerical results to zero background roughness. Increasing background roughness is shown to reduce the optimal turbine spacing and increase the peak array power coefficient.

## Introduction

To maximise power output, a tidal turbine array should be designed to sweep the largest permissible fraction of the flow cross-section [1]. Since it would be impractical to fill a cross-section with turbines, developers must consider the variation of power potential with array width (array blockage,  $B_A$ ) and the size and spacing of individual turbines (local blockage,  $B_L$ ). The two-scale ADT [2] provides a useful description of the basic effects of  $B_A$  and  $B_L$  on the performance of a lateral array of turbines. Specifically, it predicts an optimal lateral spacing (optimal  $B_L$ ) to maximise the maximum power coefficient ( $C_{PG(max)}$ ) of the array. Subsequent studies employing three-dimensional RANS simulations [3] and physical experiments [4] have shown this theory to predict qualitatively the power and thrust behaviour of lateral rows of porous discs. The present work shows the theory to predict qualitatively the power performance of a lateral turbine array in a depth-averaged channel model.

## Methods

The discontinuous Galerkin version of ADCIRC is used to simulate flow through an idealised tidal channel. Following [5] and [6], turbines are modelled as line discontinuities based on the finite Froude number ADT [7]. This method accounts for local-scale mixing in a sub-grid scale model whilst array-scale mixing is simulated directly by the shallow water equations (fig. 1). For a single row of identical turbine edges,  $B_A$  is varied and  $B_L$  adjusted to maintain a global blockage ( $B_G$ ) of 0.1. A constant  $B_G$  ensures that, even though the number of edges varies with  $B_A$ , this is analogous to varying the lateral spacing between a fixed number of turbines. For each  $B_L$ , the turbine edges are tuned to produce  $C_{PG(max)}$ . In this way, the effect of lateral spacing on  $C_{PG(max)}$  is calculated numerically (NM) and compared with the predictions of the theory (AM). To represent the AM more accurately, the numerical results are then extrapolated to zero background roughness ( $C_F$ ).

## Results and Conclusions

In the simple case of fixed  $B_L$  and constant inflow, the power extracted by a tidal turbine initially increases with reducing throughflow velocity before decreasing as the applied thrust increasingly diverts flow around the turbine. The optimal thrust may be defined as that produced by the optimal tuning to maximise the useful power output. In the two-scale ADT,  $C_{PG(max)}$  also depends on the balance between local-scale slowing and array-scale choking. As the lateral spacing between turbines is reduced ( $B_L$  is increased)  $C_{PG(max)}$  initially increases as the decreasing potential for flow diversion around each turbine permits higher optimal thrusts. As  $B_L$  is further increased  $C_{PG(max)}$  reduces as the potential for diversion around the entire array increases, necessitating lower optimal thrusts. The value of  $B_L$  that maximises  $C_{PG(max)}$  is the one for which the optimal thrust is greatest.

Fig. 2a shows qualitative agreement between the models, which is improved by extrapolating the numerical results to zero  $C_F$ . Contrary to the findings of [3], the results of the NM are lower than those of the AM for low  $B_L$  and higher for high  $B_L$ . The remaining differences between the models may explain this disparity. Free surface deformation is likely to have increased the numerical  $C_{PG(max)}$  values [8], whereas two-dimensional flow is likely to have decreased them. The net result of these competing effects is unclear. Further, due to the low  $C_F$  selected, the array mixing length-scale in the NM is not short compared to the channel length (fig. 1). The significance of this will be considered in future work.

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As reported in [9], for a given  $B_L$ ,  $C_{PG(max)}$  increases with increasing  $C_F$  (fig. 2b). There are two reasons for this: firstly, increasing the resistance of the bypass flow relative to the core flows improves the performance of the turbines, even for a fixed tuning, and secondly, this change in relative resistance increases the optimal thrust, allowing the turbines to be retuned to extract even more power. Further, the amount by which these effects increase  $C_{PG(max)}$  values increases with  $B_L$  because, for optimally tuned turbines, the difference between core and bypass velocities increases with  $B_L$ , which also means that, after increasing  $C_F$ , the additional thrust that can be applied by retuning increases with  $B_L$ . The effect of increasing  $C_F$  on  $C_{PG(max)}$  increases with  $B_L$ , which explains why, for the two-scale ADT, increasing  $C_F$  increases both the peak  $C_{PG(max)}$  and the optimal  $B_L$  (fig. 2b).

It is important to note, however, that measurement of performance in terms of power coefficient neglects the effect of power extraction on the channel flow rate [1]. In this case, the assumption of constant inflow also conceals the fact that increasing  $C_F$  increases frictional dissipation, reducing the amount of power available for extraction.

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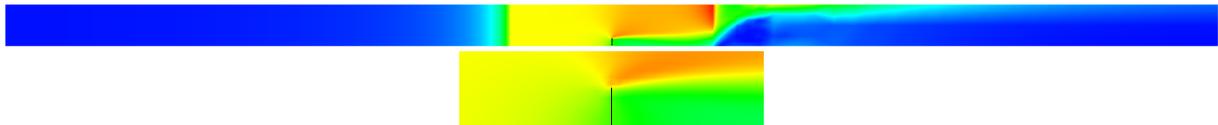


Fig. 1: Plan view of flow around the turbine array (black line) within the channel. A close-up of the flow around the array is shown below. Contours are of depth-averaged velocity and of arbitrary scale.

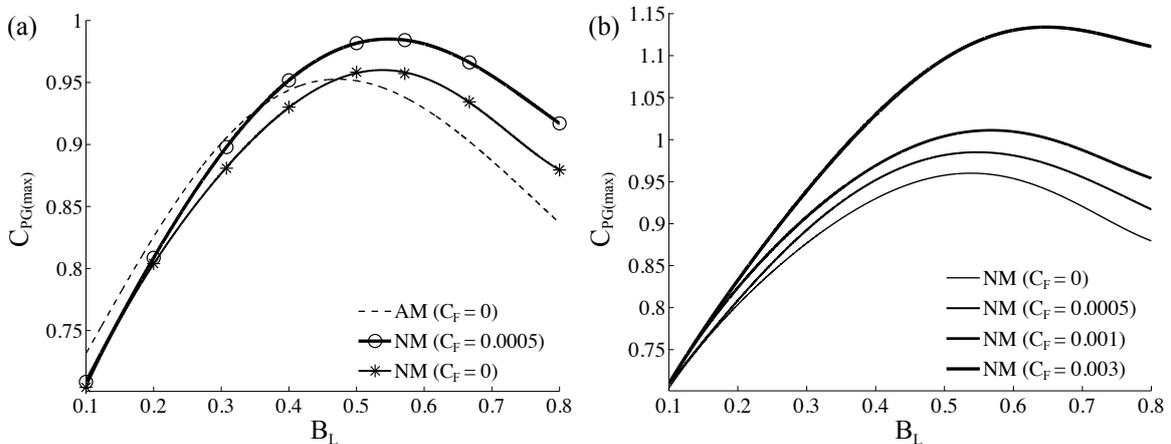


Fig. 2: The effect of lateral spacing (a) and background roughness (b) on  $C_{PG(max)}$ .