Panel Method for the Study of Wave-Structure Interaction in a 2D Circular Domain

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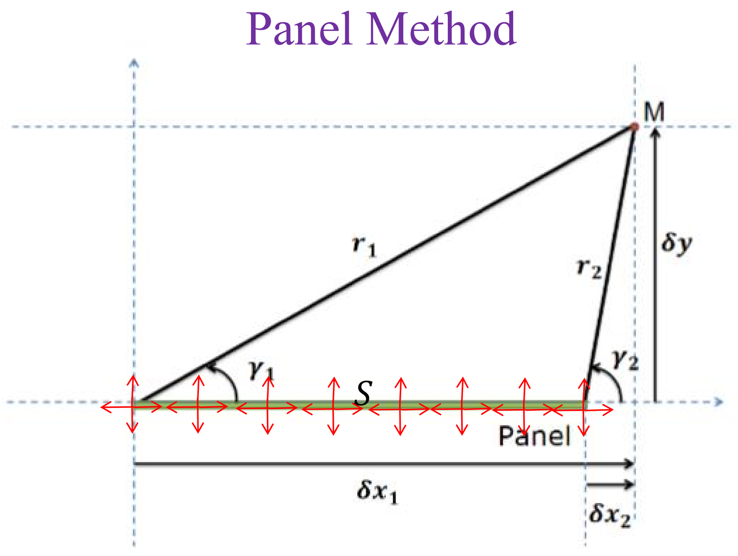
※Analytial Method for 2D Circular Fluid Domain

※Panel Method Applied with Images

※Proof of the Convergence of the Algorithm

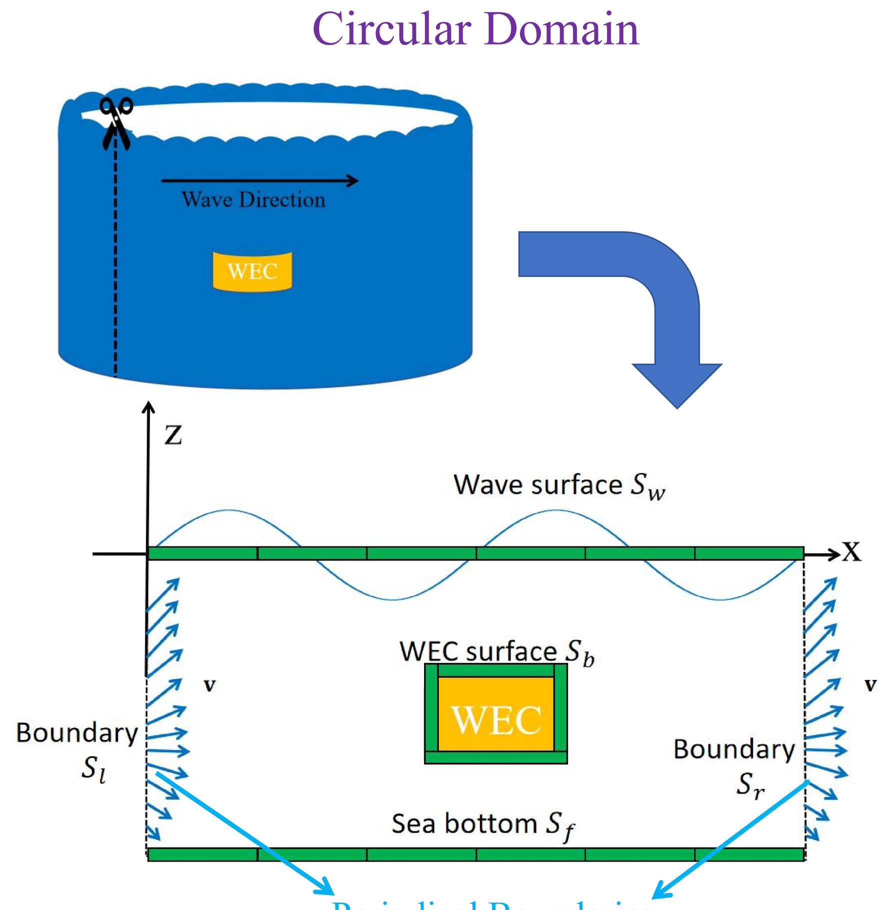
Introduction

Panel Method



- Flow singularities (sources, doublets, vortices, etc.) are placed on panels.
- Panel method works well in unsteady flow field and could show the coupling effect between each boundary.
- Panel method is able to capture the bilateral fluid-structure interaction.

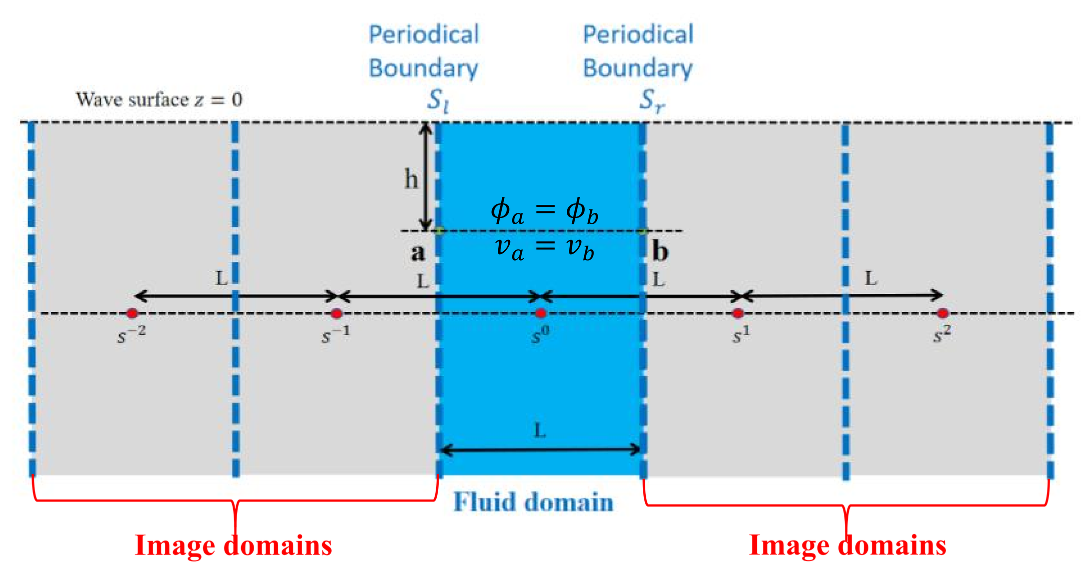
Circular Domain



- Only finite number of panels is required.
- No need for arbitrary boundary conditions.
  - Boundaries conditions are periodic.
- Panels are placed on wave surface, sea bottom, and WEC surface.

※Reasearch Question:

How to apply panel method in circular domain



Algorithm

B.c on Wave Surface

Wave height $\frac{\partial}{\partial t}\eta(t,x) = \frac{\partial \phi_w}{\partial z}\Big|_{(t,(x,\bar{z}))}$

Velocity potential $\frac{\partial}{\partial t}\phi_w(t,x) = -g\,\eta(t,x)$

Method of Images

strength density of panel

$$\phi(t,q) = \int_S G(q,s)\cdot c(t,s)\cdot ds$$

Contribution of unity strength panel

$$G(q,s) = \begin{cases} \frac{1}{2\pi}\ln|q-s| & \text{sources} \\ -\frac{1}{2\pi}\frac{(q-s)\cdot\hat{n}(s)}{|q-s|^2} & \text{doublets} \\ -\frac{1}{2\pi}\tan^{-1}\left(\frac{z_q-z_s}{x_q-x_s}\right) & \text{vortices} \end{cases}$$

※Consider M images from periodical B.c.

$$G'(q,s) = \sum_{m=-M}^{M} G(q,(x+mL,z))$$

Update the States of Wave

Contribution matrices; b: rigid body; w: wave surface

$$\underbrace{\begin{bmatrix} A_{v,b2b} & A_{v,w2b} \\ A_{\phi,b2w} & A_{\phi,w2w} \end{bmatrix}}_{A_1} \begin{bmatrix} c_b \\ c_w \end{bmatrix} = \begin{bmatrix} \bar{V}_{bn} \\ \bar{\phi}_w \end{bmatrix}$$

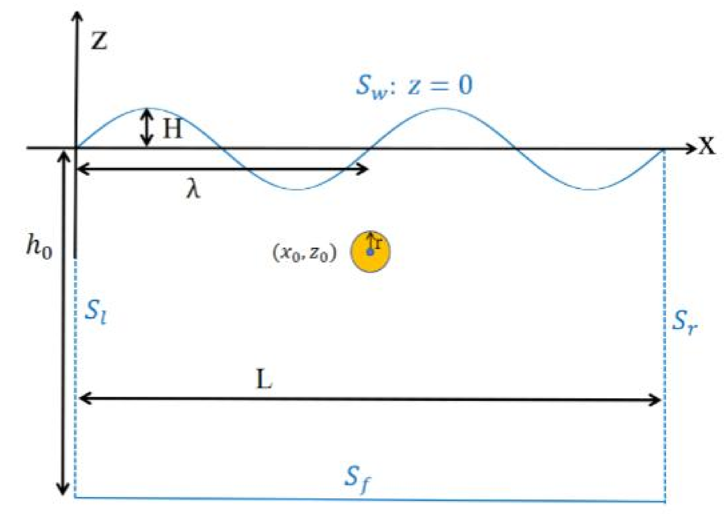
$$\frac{\partial \bar{\phi}_w}{\partial z} = \underbrace{\left[\ \frac{\partial}{\partial z}A_{\phi,b2w} \ \middle|\ \frac{\partial}{\partial z}A_{\phi,w2w}\ \right]}_{A_2} \begin{bmatrix} c_b \\ c_w \end{bmatrix} = A_2 A_1^{-1} \begin{bmatrix} \bar{V}_{bn} \\ \bar{\phi}_w \end{bmatrix}$$

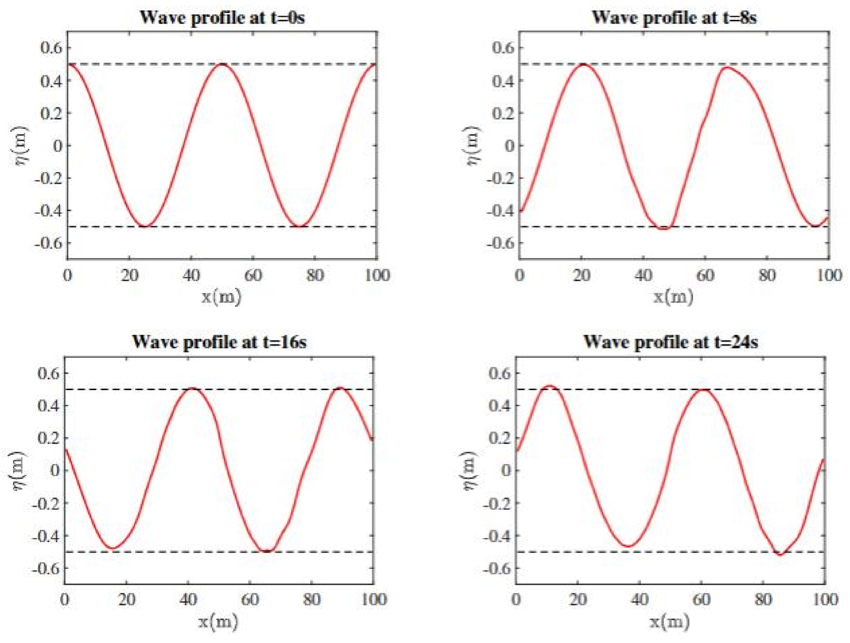
$$A_2 A_1^{-1} = \left[\ A_{b2w} \ \middle|\ A_{w2w}\ \right]$$

$$\frac{d}{dt}\begin{bmatrix} \bar{\eta} \\ \bar{\phi}_w \end{bmatrix} = \begin{bmatrix} 0 & A_{w2w} \\ -gI & 0 \end{bmatrix} \begin{bmatrix} \bar{\eta} \\ \bar{\phi}_w \end{bmatrix} + \begin{bmatrix} A_{b2w} \\ 0 \end{bmatrix} \bar{V}_{bn}(t)$$

States — States — Input

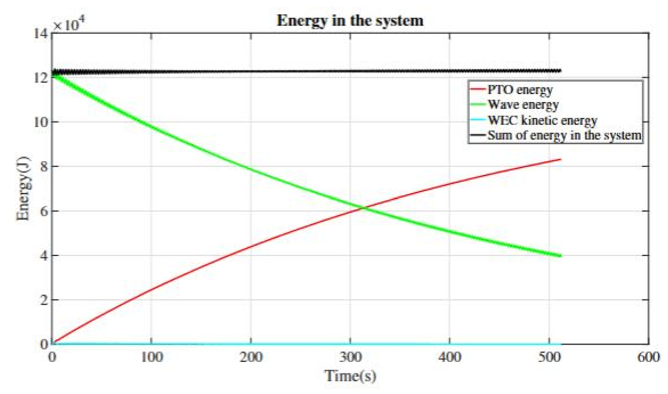
Simulation





Case 1:

- WEC is assume to be fixed.
- In the conventional analytical method, the wave profile is not influenced by the WEC.
- With the panel method approach, the wave profile changes based on the how strong the bilateral fluid structure interaction is.

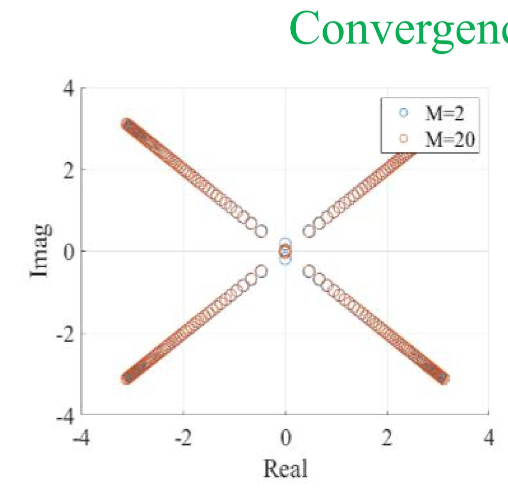


Case 2:

- A damping controller (P controller) is applied on the WEC.
- The WEC will extract energy from the wave, the extracted energy will equal to the energy decrease in the fluid domain.

Stability

Convergence of the Algorithm: Check Eigenvalues



Vortices should not be used as the singularities since the introduction of images to enforce the periodic boundary condition leads to unstable dynamics. Moreover, this instability cannot be improved by including more images.

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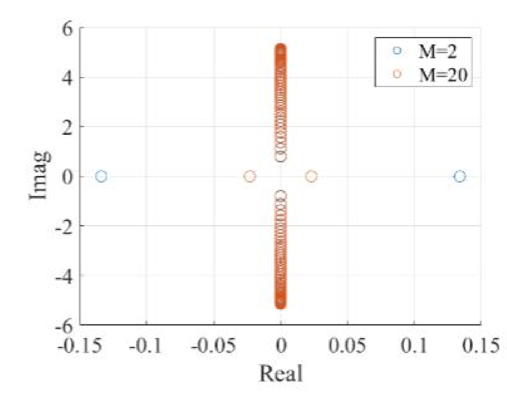

Sources can be used especially when the number of images to be included or the width of the domain are large enough.

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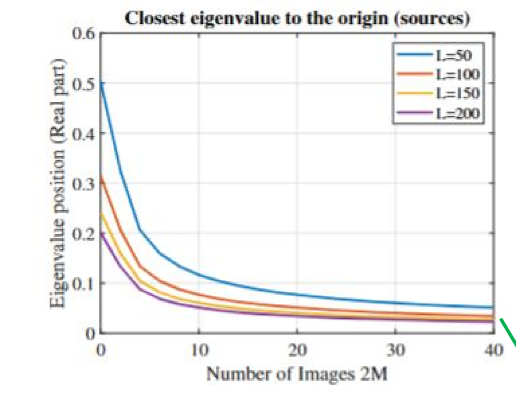
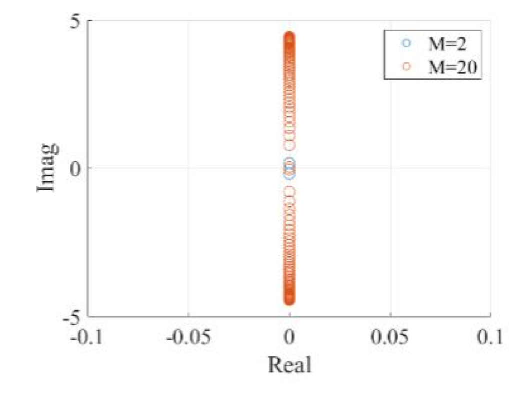


Converge to the fundamental frequency of the circular domain



Doublets are the best singularities to use to panel method on a 2D circular domain as the system dynamics remain marginally stable as expected of physical system.

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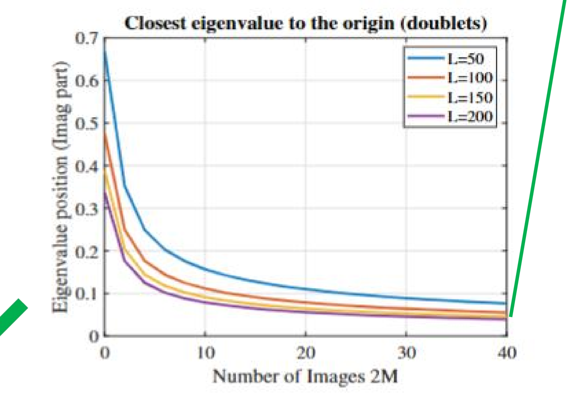

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