PAPER • **OPEN ACCESS**

Motion analysis and power output optimization design of wave energy device

To cite this article: Lijie Chen *et al* 2023 *J. Phys.: Conf. Ser.* **2565** 012002

Motion analysis and power output optimization design of wave energy device

Lijie Chen, Xiao Liu, Weimin Yan *, Guoyong Shi, Zhibing Gao, Zexian Huang

Wuhan Donghu University, Wuhan, Hubei, China

*Correspond author: 1319858663@qq.com

Abstract. Wave energy is an essential marine renewable energy. Improving the energy conversion efficiency of wave energy devices is the key aspect of wave energy utilization research. This paper deals with the movement status of the wave energy device and its power optimization. Depending on Newton's second law, the second-order ordinary differential equation model is established for the float and oscillator respectively. According to whether the damping coefficient is constant, the pending method coefficient is utilized to solve the general second-order solution ordinary differential equation model, and the drooping displacement and velocity graphs under different damping coefficients are obtained. The analysis shows that the oscillating wave energy device improved by the second-order ordinary differential equation model could become stable within a very short time. The motion displacement and velocity of the wave energy device at five random moments were taken to check the accuracy of the model, and the results show that the model has an excellent promoting effect on the motion state of the wave energy device at sea and reduces the depletion of the device in terms of accuracy of motion displacement and velocity.

1. Introduction

In the current society, the economy and society are developing continuously, however, social progress also brings the double problems of humans about energy demand and environmental pollution. Therefore, the research and development of the renewable energy industry have become a hot spot in world research. China has established a goal of achieving carbon neutrality by 2060, so supporting green energy and developing renewable energy is one of its key tasks. When waves roll into the shore and back again, they give rise to a lot of energy. The ocean covers 70% of the earth's surface, with extensive distribution and abundant reserves, and wave energy has considerable application prospects [2]. Its role in China's future energy structure cannot be underestimated. The development of wave energy is just an inevitable requirement of the national energy development strategy [1].

2. Research status

The wave load calculation theory mainly consists of potential flow theory and equation-based viscous fluid method. When the wave absorber is a simple three-dimensional geometry, such as a cylinder, sphere, hemisphere, square body, it can be quickly calculated by analytical method. Newman computed the exciting force and moment by applying the Haskind relation [5]. Yeung established an analytical method to obtain additional mass and damping coefficients for offshore projects [6]. Hulmea deduced the hydrodynamic coefficients of floating hemispheres undergoing periodically forced vibration [7]. In the past two decades, many studies have adopted analytical methods to analyze the energy export of single-point energy-absorbing devices [8,9]. The corresponding hydrodynamic coefficients are obtained from the matching conditions of the inner domain and the outer domain for the radiation problems of swing, heave, and pitch motion [10].

3. Establishment of second-order nonhomogeneous ordinary differential equation model

The oscillating wave energy device moves at six degrees of freedom on the sea surface. In this paper, only heave motions are considered. To better describe its motion, the earth and float are selected as the reference system respectively. Figure 1 shows the analysis of the forces on the vibrator and the float in turn. When the device only moves in a vertical motion, the float is subject to the excitation force and damping force of seawater wave, the float's gravity, the static water restoring force, the spring's resistance, and the linear damper's resistance. Because the internal vibrator of the floating body exerts different forces on the spring and the linear damper (the force of PTO [3] is greater than or equal to the gravity of the vibrator), the mass action of the vibrator is ignored. The forces acting on the oscillator are inertial force, its gravity, linear damping force and spring tension. According to the known information, wave exciting force F_L satisfies:

$$F_L = f\cos\omega t \tag{1}$$

wave damping force F_X satisfies:

$$F_X = k_X v_Y = k_X \frac{dx}{dt}. \tag{2}$$

The linear damping force is proportional to the relative velocity, which refers to the relative velocity of the float and the oscillator:

$$F_Z = k_Z v = k_Z \frac{dx}{dt}. \tag{3}$$

As the device is in the process of heave motion and the buoyancy of the floating body changes, it will produce still-water restoring force [4], so only considering the heave motion, the still-water restoring force is satisfied as follows:

$$\begin{cases} F_j = -CX \\ C = \rho g A_w = \rho g S_j \end{cases} \tag{4}$$

where X is the height change of the draft depth of the floating body, and A_w is the waterplane area.

Spring force F_T meets:

$$F_T = k_T x_T \tag{5}$$

Since the force of the PTO system [3] includes linear spring elastic force and linear damping force, it can be expressed as:

$$F_{PTO} = -F_T - F_Z. \tag{6}$$

Based on Newton's second law, the second-order non-homogeneous differential equation model of float and oscillator with constant coefficients is established:

$$\begin{cases} (m_F + \Delta m)\dfrac{dx^2}{d^2t} + k_X \dfrac{dx}{dt} + Cx = f\cos\omega t + F_{PTO} \\ F_G + F_T + m_Z g + F_z = m_Z a \end{cases} \Rightarrow \begin{cases} \dfrac{dx^2}{d^2t} - (\dfrac{k_X + k_Z}{m_F + \Delta m})\dfrac{dx}{dt} + \dfrac{c + k_T}{m_F + \Delta m}x = \dfrac{f\cos\omega t}{m_F + \Delta m} \\ m\dfrac{d^2x}{dt^2} - k_T x - k_Z v = m_Z(a_F - g) \end{cases} \tag{7}$$

where $m_F, \Delta m, m_Z$ is the mass of the float and the additional mass generated by the additional inertial force, and the mass of the oscillator respectively, f indicates the amplitude of the wave excitation force, and F_G indicates the inertial force.

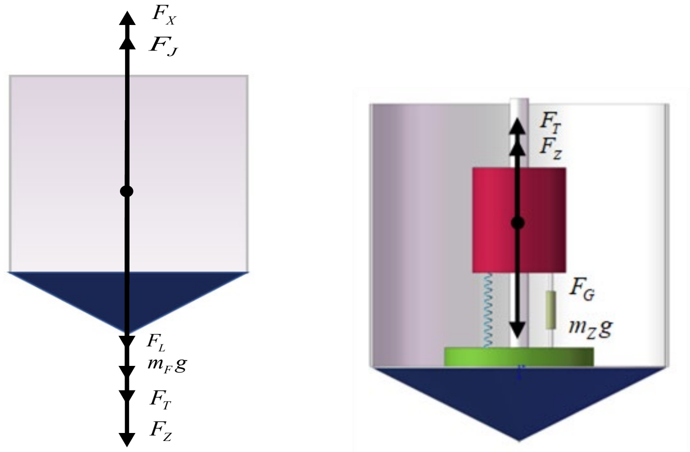


Figure 1. Force analysis of float and oscillator.

4. The solution of the second-order constant coefficient non-simultaneous differential equation model

To study the variation of its lifting and sinking motion more intuitively, the model can be solved by using the general solution of the second-order constant coefficient non-simultaneous differential equation to obtain the corresponding lifting and sinking displacements and velocities at different moments.

The specific solution of the model is as follows: since the general solution of the second-order constant-coefficient non-associative differential system can be regarded as consisting of its corresponding second-order constant-coefficient linear differential equation plus one of its special solutions, this paper first finds the corresponding general solution of the model equation and its special solution to obtain the final solution of the differential equation, transform the second-order non-simultaneous differential equations of floats and oscillators into the second-order linear differential equations of the same order.

$$\begin{cases} \frac{d^2x}{dt^2} + \left(\frac{k_X + k_Z}{m_F + \Delta m}\right)\frac{dx}{dt} + \left(\frac{c+k}{m+\Delta m}\right)x = 0 \\ \frac{d^2x}{dt^2} - \frac{k_Z}{m}\frac{dx}{dt} - \frac{k_T}{m}x = 0 \end{cases} \tag{8}$$

The general solution is obtained: $x = C_1 e^{\lambda_1 t} + C_2^{\lambda_2 t}$

Among them, the float: $\lambda_1 = \frac{-\frac{k_X + k_Z}{m_F + \Delta m} + \sqrt{\left(\frac{k_X + k_Z}{m_F + \Delta m}\right)^2 - 4\left(\frac{c+k}{m+\Delta m}\right)}}{2}$ $\lambda_2 = \frac{-\frac{k_X + k_Z}{m_F + \Delta m} - \sqrt{\left(\frac{k_X + k_Z}{m_F + \Delta m}\right)^2 - 4\left(\frac{c+k}{m+\Delta m}\right)}}{2}$

Oscillator: $\lambda_1^{`} = \frac{-\frac{k_Z}{m} + \sqrt{\left(\frac{k_Z}{m}\right)^2 - 4\frac{k}{m}}}{2}$, $\lambda_2^{`} = \frac{-\frac{k_Z}{m} - \sqrt{\left(\frac{k_Z}{m}\right)^2 - 4\frac{k}{m}}}{2}$.

Since $\pm\omega$ is not an eigen root, the particular solution is substituted into the inhomogeneous ordinary differential equation and simplified to obtain:

$$\begin{cases} f\cos\omega t = (-\omega^2 A\cos\omega t - \omega^2 B\sin\omega t) + \left(\frac{k_Z + k_X}{m+\Delta m}\right)(-\omega A\sin\omega t + \omega B\cos\omega t) + \frac{c+k}{m+\Delta m}(A\cos\omega t + B\sin\omega t) \\ f^{`}\cos\omega t = (-\omega^2 A\cos\omega t - \omega^2 B\sin\omega t) + \frac{k_Z}{m}(-\omega A\sin\omega t + \omega B\cos\omega t) + \frac{k}{m}(A\cos\omega t + B\sin\omega t) \end{cases} \tag{9}$$

In solving, when $\omega t = 0$ or $\omega t = 90$, we can get:

$$\text{Float: } \begin{cases} -\omega^2 B + \dfrac{k_X + k_Z}{m_F + \Delta m}(-\omega A) + \dfrac{(c+k)B}{m_F + \Delta m} = 0 \\ -\omega^2 A + \dfrac{k_X + k_Z}{m_F + \Delta m}\omega B + \dfrac{(c+k)A}{m_F + \Delta m} = f \end{cases} \quad \text{oscillator: } \begin{cases} -\omega^2 A + \dfrac{k_Z}{m}\omega B + \dfrac{kA}{m} = f^{`} \\ -\omega^2 B + \dfrac{k_Z}{m}(-\omega A) + \dfrac{kB}{m} = 0 \end{cases} \tag{10}$$

Therefore, to sum up, the general solution of float and oscillator is:

$$\begin{cases} x = C_1 e^{\lambda_1 t} + C_2^{\lambda_2 t} - A\cos\omega t - B\sin\omega t \\ x = C_1 e^{\lambda_1 t} + C_2^{\lambda_2 t} + A\cos\omega t + B\sin\omega t \end{cases} \tag{11}$$

Since there are two cases of k_Z [11], according to the specific values of k_Z in different cases combined with the above formula, the undetermined coefficient method is adopted to obtain the special solution equation of float and oscillator.

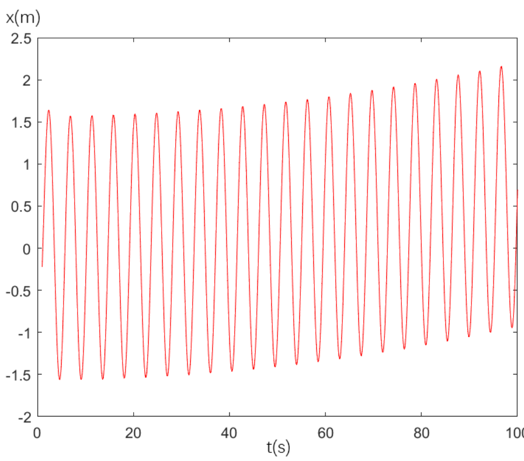
In case 1, $k_Z = 10000$, the float and the oscillator's motion and velocity equations are obtained respectively according to the known information:

$$\begin{cases} x = 1.023e^{-1.309t} + 0.002^{0.354t} + 0.142\cos 1.4005t - 0.312\sin 1.4005t \\ v = -1.34e^{-1.309t} + \ln^{5.693} 0.354 * 0.02^{-1.182t} + 0.199\sin 1.4005t - 0.437\cos 1.4005t \\ x = -0.94e^{-5.8t} + 0.39^{5.7t} - 0.062\cos 1.4005t \\ v = 5.452e^{-5.8t} + \ln^{0.39} 5.7 * 0.39^{5.7t} + 0.087\sin 1.4005t - 0.459\cos\omega t \end{cases}. \tag{12}$$

In case 2, the motion and velocity equations of the float and the motion and velocity equations of the oscillator are obtained respectively according to the known information:

$$\begin{cases} x = 1.023e^{-1.309t} + 0.002^{0.354t} + 0.142\cos 1.4005t - 0.312\sin 1.4005t \\ v = -1.34e^{-1.309t} + \ln^{5.693} 0.354 * 0.02^{-1.182t} + 0.199\sin 1.4005t - 0.437\cos 1.4005t \\ x = 1.59e^{-1.12t} + 0.345^{0.529t} - 0.2\cos 1.4005t + 0.2\sin 1.4005t \\ v = 1.78e^{-1.12t} + \ln^{0.35} 0.529 * 0.35^{0.529t} + 0.28\sin 1.4005t + 0.28\cos 1.4005t \end{cases}. \tag{13}$$

To observe the state of their vertical motion, this paper shows the changes in vertical displacement and velocity when the linear damping coefficient is fixed for the first 40 cycles, as shown in Figure 2 and Figure 3, and the changes in vertical displacement and velocity when the linear damping coefficient is changed for the first 40 cycles, as shown in Figures 4 and 5. After observation, it can be found that the oscillator and float tend to be stable within a very short time.



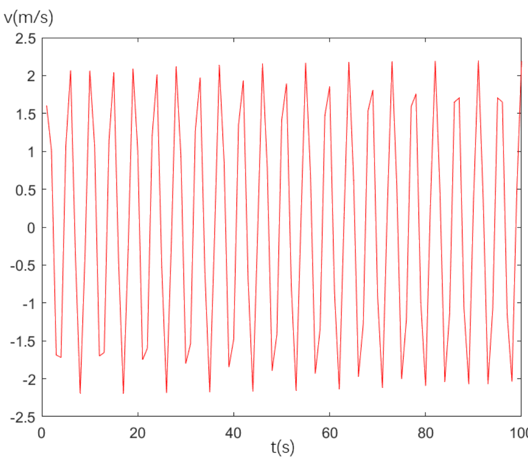


Figure 2. Variation of heave displacement and velocity when float linear damping is fixed.

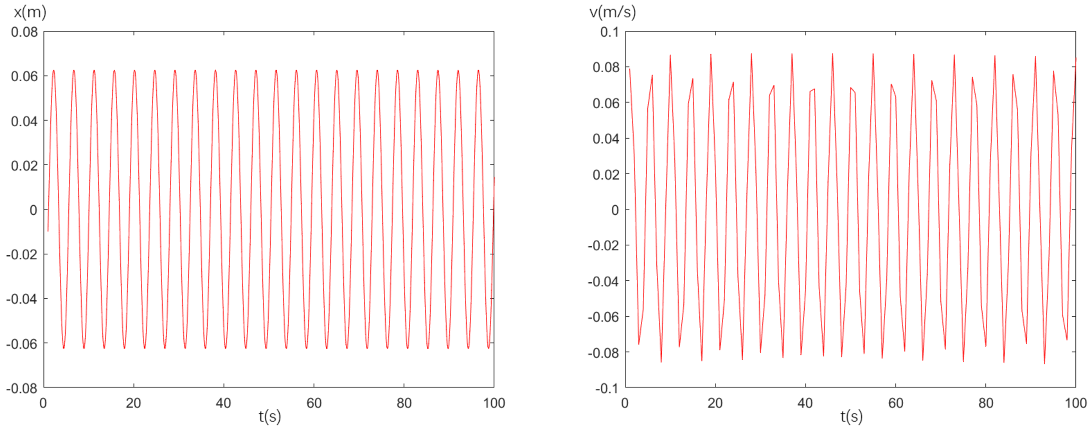


Figure 3. Variation of heave displacement and velocity when linear damping of the oscillator is fixed.

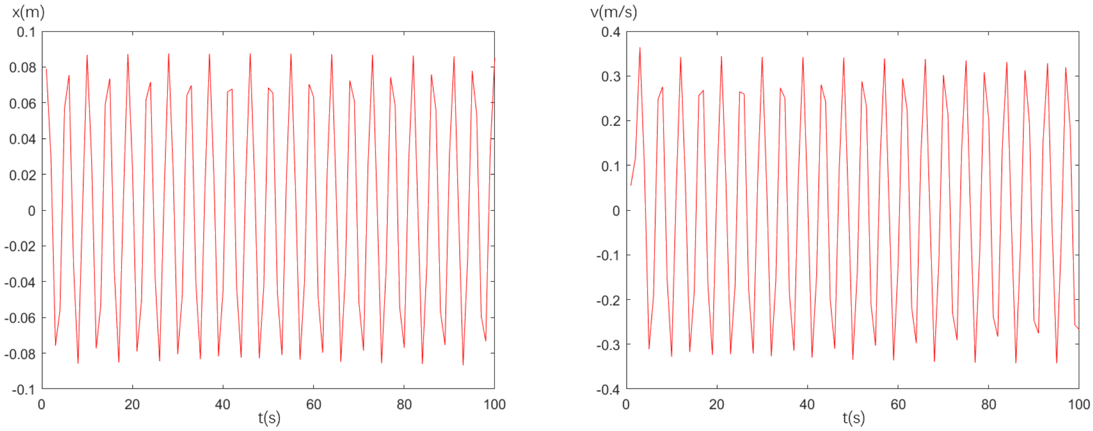


Figure 4. Variation of heave displacement and velocity when floating linear damping changes.

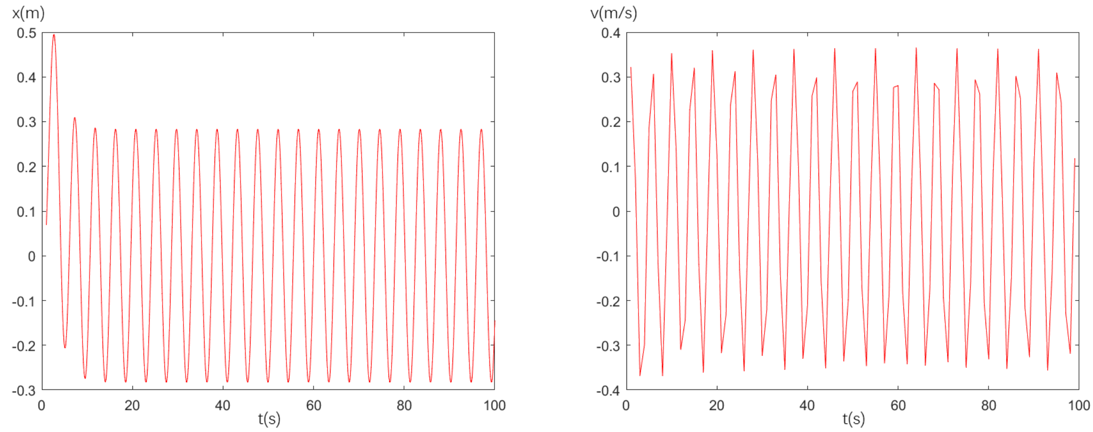


Figure 5. The displacement and velocity of the float and the vibrator are calculated five times (t = 10 s, 20 s, 40 s, 60 s, 80 s and 100 s) to study the change of the heave displacement and velocity when the linear damping of the vibrator changes.

Table 1 Displacement and velocity of float and oscillator at each time

Time (s)	Float		Oscillator	
	Displacement (m)	Speed (m/s)	Displacement (m)	Speed (m/s)
10	-0.59	-0.64	-0.31	-0.69
20	-0.94	-0.84	-0.76	-0.76
40	0.28	-0.31	0.29	0.33
60	-0.02	-0.04	-0.05	-0.03
100	-0.05	-0.01	-0.02	-0.06

5. Conclusions

In this paper, a second-order inhomogeneous differential equation model with constant coefficients is provided and built for an oscillating wave energy device in the ocean. Firstly, the reference frame of the float and vibrator was established, and according to the relationship between force and acceleration in Newton's second law of motion, the model is solved by the undetermined coefficient method, and finally, the motion and velocity equations of the float and oscillator with different linear damping coefficients are obtained. The oscillating wave energy device improved by this model can become stable in a very short time and can facilitate the sea surface wave motion of the oscillating wave energy device. In this study, the displacement and time of the float and vibrator at five moments are randomly selected as examples to verify the accuracy of the model. The results show that the model variant reduces the losses caused by errors in the oscillating wave energy device and optimizes its power output, which is practical and instructive for the development of wave energy.

References

[1] Liu Y J, Wu S, Wang D S, Wang R H. Research progress of ocean wave energy generation device [J]. Journal of Shandong University (Engineering Science), 2021, 51 (05): 63 - 75.
[2] Zhang Y Q, Sheng S W, You Y G, Wang K L, Wang Z P. Current situation and direction of wave energy generation technology application [J]. Advances in New Energy, 2019, 7 (04): 374 - 378. (in Chinese)
[3] Zheng X B, Zhang L, Ma Y. Hydrodynamic Calculation and Energy Conversion Characteristic Analysis of Double Floating Body Wave Energy Device [J]. Science and Technology Review, 2014, 32 (19): 26 - 30.
[4] Zeng Dong, Ye Hengkui, Sun Jianglong. Motion Analysis of a Box Floating Body in Waves [J]. China Ship Research, 2006 (Z1): 5 - 8.
[5] Chatjigeorgiou I K. Hydrodynamic exciting forces on a submerged oblate spheroid in regular waves [J]. Computers & Fluids, 2012, 57 (none): 151 - 162.
[6] Conca C, Osses A, Planchard J. Added mass and damping in fluid-structure interaction [J]. Computer Methods in Applied Mechanics & Engineering, 1997, 146 (3 - 4): 387 - 405.
[7] Zhang B L, Ma L, Han Q L. Sliding mode H∞ control for offshore steel jacket platforms subject to nonlinear self-excited wave force and external disturbance [J]. Nonlinear Analysis Real World Applications, 2013, 14 (1).
[8] Eriksson M, Isberg J, Leijon M. Hydrodynamic modeling of a direct drive wave energy converter [J]. International Journal of Engineering Science, 2005, 43 (17 - 18): 1377 - 1387.
[9] Rosati M, Kelly T, Violini D G, et al. Data-based hydrodynamic modeling of a fixed OWC wave energy converter [J]. 2021.
[10] Drobyshevski Y. Hydrodynamic coefficients of a floating, truncated vertical cylinder in shallow water[J]. Ocean Engineering, 2004, 31 (3 - 4): 269 - 304.
[11] Study on Nonlinear Factors Based on hydroelastic method and Calculation of Ship Design Load [C]//. Proceedings of Marine Steel Structure Branch of China Steel Structure Association, 2015: 49 - 54.