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Research paper

Theoretical and experimental transverse vibration analysis of a non-uniform composite helical tidal turbine foil

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ABSTRACT

Recently, tidal energy has gained attention as an attractive renewable source of power generation. In this context, tidal turbine foils must operate in harsh conditions and extremely variable dynamic loads. Considering the expensive dynamic tests required for these large structures, developing validated dynamic models could be valuable for failure prediction and design purposes. In this paper, the free and forced vibration behaviour of the cantilevered part of a non-uniform helical tidal turbine foil made from the composite material is investigated theoretically and experimentally. In this regard, governing vibration equations of the foil with related boundary conditions are presented. The free vibration analysis of the foil is performed by solving a boundary-value problem, where a closed-form solution, based on the mode summation method, for the forced vibration of the foil is employed. Finally, the obtained numerical results (natural frequency and dynamic displacements) are compared with the corresponding results from experiments to validate the vibration equations and the employed solution. Acceptable agreement between numerical and experimental data confirms that the vibration model presented in this study can be employed to predict the dynamic response, parametric study, optimization, and design of the tidal foil without performing costly experiments.

1. Introduction

Due to its potential to provide renewable and predictable power generation, tidal energy has attracted much commercial and research attention recently. Converting tidal stream energy to electrical energy uses tidal energy converters which typically consist of a rotating turbine connected to a generator. Underwater tidal turbines developed for this purpose are required to perform under harsh operating conditions and challenging mechanical loads, with minimum servicing, for a design life of circa 20 years. Therefore, tidal turbine foils should meet high technical criteria in terms of strength, natural frequency, endurance, and corrosion resistance.

In this regard, various research has been presented in the context of strength analysis, endurance and lifetime prediction, health monitoring, and fluid-solid interaction in tidal turbine blades. (Nachtane et al., 2020) presented a review of technology developments, hydrodynamics

and mechanical design, numerical models, and optimization methods employed for tidal current turbines. (Walker and Thies, 2021) published a review of component and system reliability in 58 tidal stream energy deployments between 2003 and August 2020. (Vennell et al., 2015) set out an overview of the issues and compromises in macro- and micro-designing the layout of the large tidal turbine arrays required to realize this potential. Moreover, A review paper with a focus on the state of the art in the structural testing of the axial flow composite tidal turbine foils has been presented by (Munaweera Thanthirige et al., 2023). Furthermore, (Finnegan et al., 2022) presented the manufacture of a full-scale 1 MW tidal turbine foil made from glass fibre-reinforced powder epoxy composite material. They presented a numerical model of the foil, manufactured, and performed some static, fatigue and dynamic tests on this large structure.

Fluid-solid interaction (FSI) phenomena in tidal turbines should be addressed for the purpose of load calculations, static, and fatigue

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Fig. 1. Test setup for fatigue and dynamic testing.

analysis. For example, (Badshah et al., 2018) presented a coupled modular FSI approach for the performance evaluation and structural load characterization of a tidal current turbine under uniform and profiled flow. Furthermore, (Young et al., 2010) developed and validated a coupled boundary element method-finite element method to simulate the transient fluid-structure interaction response of tidal turbines subject to spatially varying inflow.

Tidal turbines are designed to work under high dynamic loads. In this regard, various research has been presented to evaluate the durability and fatigue life prediction of tidal turbines. For instance, (Nachtane et al., 2018) evaluated the durability of composite materials of a ducted tidal turbine under critical loads (hydrodynamic and hydrostatic pressures) with the implementation of a failure criterion using the finite element analysis (FEA). Moreover, (Gonabadi et al., 2022) presented a fatigue life prediction methodology for composite tidal turbine blades based on combined hydrodynamic and finite element structural models with the view to characterize the fatigue properties and failure modes of candidate composite material manufactured by vacuum-assisted resin transfer moulding process.

Most researchers have focused on the contexts of technology, strength analysis, endurance prediction, and fluid-solid interaction modelling in tidal turbines. However, investigation on modelling, testing and analysis of vibration behaviour in tidal turbines for the purpose of resonance detection or condition monitoring are limited and are reviewed in the following.

(De Cal, 2019) employed vibration monitoring to the detection of early misalignment and rub failures in a tidal turbine. (Galloway et al., 2016) investigated the use of deep learning approaches for fault detection within a tidal turbine's generator from vibration data. Moreover, (Galloway et al., 2017) presented an approach for condition monitoring of a tidal turbine's gearbox from vibration measurements with low sample rates based on the weighted least squares regression. (Murray et al., 2018) investigated the added-mass effects on natural frequency, thrust loads, and dynamic response of a horizontal-axis tidal turbine.

Two-dimensional fluid-structure interaction analysis of a vertical axis tidal turbine foil was studied by (Arini et al., 2018). Also, they predicted a resonant condition, or a lock-in frequency induced by wake generation at the tidal turbine foil during the turbine operation. Furthermore, (Arini et al., 2019) investigated numerically the influence of unsteady tidal flow on the flow-induced vibration of a vertical-axis tidal turbine foil. (Ullah et al., 2019) investigated the modal and fatigue performance of a horizontal axis tidal turbine using fluid-structure interaction.

(Xu et al., 2022) studied the effect of vibration and oscillation of an instrumented testing apparatus on the energy generation of a horizontal-axis tidal turbine. They concluded that the enhanced

vibration reduction method is effective and necessary for tidal turbine apparatus and its instrumentation.

According to the presented literature review and to the best of the authors' knowledge, investigations on the vibration behaviour of tidal turbine foils in terms of modelling, experiments, and analysis are limited. In this paper, the transverse vibration of a non-uniform composite tidal turbine foil is studied theoretically and experimentally. In this regard, a mathematical model for vibration behaviour of the nonuniform composite tidal turbine foil is presented and validated by performing experimental tests. This validated model can predict the dynamic response of the foil under any desired excitation without performing real-world expensive tests. Moreover, this model can be used for the purpose of parametric study, optimization, and design.

In Section 2, the experimental setup for vibration tests of the tidal turbine foil is explained. In Section 3, the governing vibration equation of the tidal turbine foil is derived and the related free and forced vibration analyses are presented. In Section 4, numerical results for free and forced vibration of the foil are presented and compared with corresponding experimental results. Finally, conclusions are presented in Section 5.

2. Materials and methods

2.1. Aim and objectives

The aim of this study is to investigate theoretically and experimentally the free and forced vibration behaviour of the cantilevered part of a non-uniform helical tidal turbine foil made from composite material.

However, in order to achieve this aim, the following objectives must be completed:

- Development of an experimental setup to perform dynamic tests
- Derivation of the governing vibration equations
- Identification of the geometric and material properties of the foil including Young's modulus, density, cross-sectional area and second moment, damping ratio, etc.
- Solving of the numerical governing differential equations
- Validation of the vibration model with experimental tests

2.2. Methodology

In this paper, a vibration model is presented for the cantilevered part of a non-uniform helical tidal turbine foil made from composite material. This model is validated by experimental tests and can be used for dynamic response prediction, parametric study, optimization, and design.



Fig. 2. Unbalanced Rotating Mass (URM) system employed for exciting the foil.

In this regard, an experimental setup is developed to perform dynamic testing of the foil. To apply forced excitation, an Unbalanced Rotating Mass (URM) system is employed which can apply harmonic force with adjustable frequency. A laser displacement sensor is used for measurement purposes. All data are gathered using a data acquisition system for post-processing. Then, vibration equations governing the non-uniform composite helical tidal turbine foil are derived based on Euler-Bernoulli assumptions and considering the viscous damping. The solutions for free and forced vibration of the foil are presented.

Afterwards, the geometric and material parameters of the foil are identified and numerical results for free and forced vibration of the foil are presented using MATLAB. Finally, the obtained numerical results are compared with the corresponding experimental results to verify the presented vibration mode. Agreement between numerical and



(a)



(b)

Fig. 3. (a) The investigated part of the foil including URM system with clamps supported by strut (b) The related schematic model as a cantilevered non-uniform composite beam with tip mass and the considered coordinate system.



Fig. 4. The half-circular unbalanced mass in the URM system (a) and its schematic (b).



Fig. 5. Representation of one of the foil sections and related coordinate system.

experimental results confirms the validity of the vibration model for prediction, parametric study, optimization, and design purposes.

2.3. Tidal foil and related experimental setup

In line with a project on developing a helical tidal turbine foil manufactured using carbon fibre prepreg composite material, various structural tests are conducted at Large Structures Laboratory. These tests include:

- Dynamic testing to establish the foil natural frequencies and modes.
- Static testing to determine the structural integrity of the foil.
- Fatigue testing to gain an insight into the performance of the foil in operation.
- Residual strength tests, including a destructive static test.

In this section the related experimental setup is explained. Fig. 1 indicates an overview of the experimental setup for fatigue and dynamic testing of the tidal turbine foil made from composite material. As shown in this figure, the foil and struts are mounted on a steel support frame, which is connected to the strong floor via several connection points (with 500 KN capacity per connection point).

An Unbalanced Rotating Mass (URM) system, which is powered by a motor of a controllable frequency, is installed on the tip of the foil to excite the foil with desirable frequency. Fig. 2 shows the URM system mounted with two clamps at the tip of the foil. Moreover, a laser displacement sensor is employed to measure the tip displacement of the foil.

To perform the free vibration test, while the URM system is off, an initial displacement is applied on the tip of the foil and released to make it vibrate freely. The tip displacement is measured by the laser displacement sensor. The obtained results are presented in Section 4 to calculate the damping ratio of the foil. Also, by applying the Fast Fourier Transform (FFT), the natural frequencies of the foil will be obtained.

Forced vibration tests are performed using the URM system which is adjustable for various frequency excitations. Tip displacement of the foil is measured using the laser displacement sensor and presented in Section 4.

3. The governing vibration equations

In this study, cantilevered part of the investigated tidal turbine foil

including the URM system and related clamps (shown in Fig. 3(a)) is modelled as a non-uniform composite cantilevered beam with a concentrated tip mass and an inertia force applied to its free end due to the unbalanced mass in the URM system. The related schematic model depicting the considered coordinate system is presented in Fig. 3(b).

The cross-section of the foil is depicted in Fig. 5. The bending stiffnesses of the foil in the xz and xy planes are directly related to the crosssectional second moment of area around the y-axis (I_{yy}) and the z-axis (I_{zz}), respectively. Given that the cross-sectional distribution of the foil favours the y-axis over the z-axis, it follows that I_{zz} is notably greater than I_{yy} . Consequently, the bending stiffness of the foil in the xy plane is higher than that in the xz plane. This substantial difference in bending stiffness effectively decouples the vibration of the foil in the xz plane from its vibration in the xy plane. In other words, vibration analyses in these mentioned planes can be conducted independently. Therefore, the governing equation of the foil in the xy plane remains identical to that in the xz plane, with the consideration of $I_{zz}(x)$ instead of $I_{yy}(x)$.

The URM system applies a rotating centrifugal force to the foil, thereby exciting the foil with force magnitudes that are equal in both the xy and xz planes. However, due to the lower stiffness of the foil in the xz plane compared to the xy plane, monitoring the vibration of the foil in the xz plane becomes more crucial, particularly near the resonance frequency. Consequently, in this study, the displacement of the foil in the z-direction was measured, and a vibration model of the foil in the xz plane was developed and validated through experiments.

To derive the vibration equation of the foil, the following assumptions are considered:

- The URM system, clamps and the part of foil surrounded by the clamps are modelled as a concentrated mass at the tip.
- Euler-Bernoulli assumptions are considered for the beam.
- The damping of the beam is modelled as viscous.
- The flexibility of strut-foil joint is considered negligible and is modelled as a fixed boundary condition.

In this regard, the concentrated tip mass including the masses of the URM system, clamps and part of the foil surrounded by the clamps is denoted by *M*. Moreover, the length, mass density and damping coefficient of the beam are indicated by *L*, ρ and *c* respectively. Also, the distances from the excitation and measurement points to the cantilevered end of the beam are shown by L_e and L_r respectively. In addition, E(x), A(x) and $I_{yy}(x)$ are used to indicate the Young's modulus, crosssection area and second moment of area of the beam around the y-axis, respectively, that are varied in the longitudinal direction of the foil (x).

The governing partial differential equation for the transverse vibration of a non-uniform beam can be obtained using Hamilton's Principle or Newton-Euler equations as below (Rao, 2019):

$$\frac{\partial^2}{\partial x^2} \left(E(x) I_{yy}(x) \frac{\partial^2 w(x,t)}{\partial x^2} \right) + c \frac{\partial w(x,t)}{\partial t} + \rho A(x) \frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t)$$
(1)

where w(x, t) is the transverse vibration of the beam that is a function of the location x and time t. Also, f(x, t) is the excitation force applied to the beam. In this study, the excitation is provided by the URM system which includes an electric motor and an unbalanced mass. The unbalanced mass has a half-circular shape as shown in Fig. 4. The excitation force will be the vertical component of the inertia force of the unbalanced mass as below:

$$f(\mathbf{x},t) = \int r\omega_e^2 \sin(\omega_e t) d\mathbf{m} = \frac{2}{3} \, \mathbf{m} R \omega_e^2 \sin(\omega_e t) \tag{2}$$

where, *m* and *R* are the total mass and radius of the half-circular unbalanced mass. Also, ω_e is the excitation frequency (URM frequency).

The boundary conditions for solving the partial differential Eq. (1) are as follows:

- At the clamped end of the foil, x = 0, the transverse deflection of the foil and its slope is zero, i.e.,

$$w(0,t) = 0 \quad \frac{\partial w(0,t)}{\partial x} = 0 \tag{3}$$

- At the free end of the foil, x = L, the bending moment is zero and the shear force is equal to the inertia force due to the attached mass *M*, i. e.,

$$EI\frac{\partial^2 w(L,t)}{\partial x^2} = 0 \quad \frac{\partial}{\partial x} \left(E(x)I_{yy}(x)\frac{\partial^2 w(L,t)}{\partial x^2} \right) = M\frac{\partial^2 w(L,t)}{\partial t^2}$$
(4)

3.1. Free vibration analysis

For undamped free vibration analysis, the excitation force (f(x, t)) and damping coefficient (*c*) are considered to be zero. Therefore, Eq. (1) is written as below:

$$\frac{\partial^2}{\partial x^2} \left(E(x) I_{yy}(x) \frac{\partial^2 w(x,t)}{\partial x^2} \right) + \rho A(x) \frac{\partial^2 w(x,t)}{\partial t^2} = 0$$
(5)

The solution of Eq. (5) can be found by the method of separation variables as:

$$w(x,t) = \varphi(x)\eta(t) \tag{6}$$

By Substituting Eq. (6) into Eq. (5) and rearranging, the following is obtained:

$$\frac{1}{\rho A(x)\varphi(x)} \frac{d^2}{dx^2} \left(E(x)I_{yy}(x) \frac{d^2\varphi(x)}{dx^2} \right) = - \frac{1}{\eta(t)} \frac{d^2\eta(t)}{dt^2} = a = \omega^2$$
(7)

To achieve a physical solution for $\eta(t)$, i.e., harmonic solution, $a = \omega^2$ should be a positive constant. It is noted that $\varphi(x)$ and ω are the mode shape function and natural frequency of the foil, respectively.

The Eq. (7) can be written as the following two equations:

$$\frac{d^2}{dx^2} \left(E(x) I_{yy}(x) \frac{d^2 \varphi(x)}{dx^2} \right) - \rho A(x) \varphi(x) \omega^2 = 0$$
(8)

$$\frac{d^2\eta(t)}{dt^2} + \omega^2 \eta(t) = 0 \tag{9}$$

Solution of the Eq. (9) is as below:

 $\eta(t) = A\sin(\omega t) + B\cos(\omega t) \tag{10}$

where A and B are constants that can be found using the initial conditions.

By substituting Eq. (6) into Eqs. (3) and (4), the boundary conditions can be presented as below:

$$\varphi(0) = 0 \quad \frac{d\varphi(0)}{dx} = 0 \tag{11}$$

$$\frac{d^2\varphi(L)}{dx^2} = 0 \quad E(L)I_{yy}(L)\frac{d^3\varphi(L)}{dx^3} = -M\omega^2\varphi(L)$$
(12)

The differential equation (8) with boundary conditions (11) and (12) constitute a boundary-value problem that can be solved to achieve the natural frequencies and mode shape functions of the foil.

3.2. Forced vibration analysis

According to the mode summation method, the solution of the forced vibration Eq. (1) can be assumed to be a linear combination of the mode shape functions of the foil as below:

$$w(\mathbf{x},t) = \sum_{i=1}^{\infty} \varphi_i(\mathbf{x})\eta_i(t)$$
(13)

where, $\varphi_i(x)$ is the *i* th mode shape function and $\eta_i(t)$ is the *i* th mode participation coefficient. By substituting Eq. (13) into Eq. (1):

$$\sum_{i=1}^{\infty} \frac{d^2}{dx^2} \left(E(x)I_{yy}(x)\frac{d^2\varphi_i(x)}{dx^2} \right) \eta_i(t) + c$$

$$\times \sum_{i=1}^{\infty} \varphi_i(x)\frac{d\eta_i(t)}{dt} + \rho A(x) \sum_{i=1}^{\infty} \varphi_i(x)\frac{d^2\eta_i(t)}{dt^2} = f(x,t)$$
(14)

By substituting Eq. (8) for the mode shape functions into Eq. (14), the following is obtained:

$$\rho A(x) \sum_{i=1}^{\infty} \varphi_i(x) \omega_i^2 \eta_i(t) + c \sum_{i=1}^{\infty} \varphi_i(x) \frac{d\eta_i(t)}{dt} + \rho A(x) \sum_{i=1}^{\infty} \varphi_i(x) \frac{d^2 \eta_i(t)}{dt^2} = f(x,t)$$
(15)

where ω_i is the natural frequency of the *i* th mode shape. Multiplying both sides of Eq. (15) by $\varphi_i(x)$ and integrating from 0 to *L* results in:

$$\sum_{i=1}^{\infty} \eta_i(t) \omega_i^2 \int_0^L \rho A(\mathbf{x}) \varphi_i(\mathbf{x}) \varphi_j(\mathbf{x}) d\mathbf{x} + c \sum_{i=1}^{\infty} \frac{d\eta_i(t)}{dt} \int_0^L \varphi_i(\mathbf{x}) \varphi_j(\mathbf{x}) d\mathbf{x}$$
$$+ \sum_{i=1}^{\infty} \frac{d^2 \eta_i(t)}{dt^2} \int_0^L \rho A(\mathbf{x}) \varphi_i(\mathbf{x}) \varphi_j(\mathbf{x}) d\mathbf{x}$$
$$= \int_0^L \varphi_j(\mathbf{x}) f(\mathbf{x}, t) d\mathbf{x}$$
(16)

On the other hand, according to the orthogonality of normal modes:

$$\int_{0}^{L} \rho A(x)\varphi_{i}(x)\varphi_{j}(x)dx = \begin{cases} 0 & i \neq j\\ M_{i} & i = j \end{cases}$$
(17)

By substituting Eq. (17) into Eq. (16), all terms in each of the summations vanish except for the one term for which i = j, leaving:

$$M_i \frac{d^2 \eta_i(t)}{dt^2} + C_i \frac{d\eta_i(t)}{dt} + K_i \eta_i(t) = Q_i(t)$$
(18)

where, M_i , C_i , K_i and $Q_i(t)$ are the generalized mass, damping, stiffness and force corresponding to the *i* th mode as follows:

$$C_{i} = c \int_{0}^{L} \varphi_{i}^{2}(x) dx$$

$$K_{i} = M_{i} \omega_{i}^{2}$$

$$Q_{i}(t) = \int_{0}^{L} \varphi_{i}(x) f(x, t) dx$$
(19)

L

By dividing both side of Eq. (18) by M_i and considering that $\frac{C_i}{M_i} = 2\zeta_i \omega_i$ and $\frac{K_i}{M_i} = \omega_i^2$, the following equation is obtained:

$$\frac{d^2\eta_i(t)}{dt^2} + 2\zeta_i\omega_i\frac{d\eta_i(t)}{dt} + \omega_i^2\eta_i(t) = \frac{Q_i(t)}{M_i}$$
(20)

where ζ_i is the damping ratio corresponding to the *i* th mode shape. By considering $\varphi_i(x)$ such that $M_i = 1$ (mass normalisation), Eq. (20) can be presented in a simpler form as below:

$$\frac{d^2\eta_i(t)}{dt^2} + 2\zeta_i\omega_i\frac{d\eta_i(t)}{dt} + \omega_i^2\eta_i(t) = Q_i(t)$$
(21)

The complete solution of Eq. (21) depends on the damping ratio ζ_i as below:

- No damped case ($\zeta_i = 0$):

$$\eta_i(t) = A_i \sin(\omega_i t) + B_i \cos(\omega_i t) + \frac{1}{\omega_i} \int_0^t \sin(\omega_i \tau) Q_i(t-\tau) d\tau$$
(22)

- Underdamped case (0 < ζ_i < 1):

 $\eta_i(t) = e^{-\zeta_i \omega_i t} \left(A_i \sin(\omega_{di} t) + B_i \cos(\omega_{di} t) \right)$

$$+\frac{1}{\omega_{di}}\int_{0}^{\cdot}e^{-\zeta_{i}\omega_{i}\tau}\sin(\omega_{di}\tau)Q_{i}(t-\tau)d\tau$$
(23)

- Critically damped case ($\zeta_i = 1$):

$$\eta_i(t) = e^{-\omega_i t} (A_i + B_i t) + \int_0^t \tau e^{-\omega_i \tau} Q_i(t-\tau) d\tau$$
(24)

- Overdamped case ($\zeta_i > 1$):

$$\eta_{i}(t) = A_{i}e^{-\alpha_{i}t} + B_{i} e^{-\beta_{i}t} - \frac{1}{2\omega_{i}\sqrt{\zeta_{i}^{2}-1}} \int_{0}^{t} \left(e^{-\alpha_{i}\tau} - e^{-\beta_{i}\tau}\right)Q_{i}(t-\tau)d\tau$$

$$\alpha_{i} = \omega_{i} \left(\zeta_{i} + \sqrt{\zeta_{i}^{2}-1}\right)$$

$$\beta_{i} = \omega_{i} \left(\zeta_{i} - \sqrt{\zeta_{i}^{2}-1}\right)$$
(25)

where, A_i and B_i are constants which can be evaluated using initial conditions of the foil. Also, $\omega_{di} = \omega_i \sqrt{1 - \zeta_i^2}$ is the damped natural frequency of the *i*-th mode shape.

In brief, $\varphi_i(x)$ can be found by solving the boundary-value problem (Eq. (8), (11) and (12)) and then, it becomes mass-normalized by setting $M_i = 1$ in Eq. (17). Moreover, $\eta_i(t)$ is obtained by one of Eqs. (22)–(25) and finally, the forced vibration response will be obtained from Eq. (13).

4. Numerical and experimental results

In this section, after determining the foil parameters, the numerical and experimental results for free and forced vibration of the foil are presented and compared.

It is worth mentioning that in this study, the initial condition and the applied force on the foil excites its first mode shape more than the other



Fig. 6. Variation of the foil specifications in its longitudinal direction: cross-sectional area (A) and product of Young's modulus and second moment of area (EI_{yy}) .

modes. In other words, the participation coefficient for the first mode shape is significantly greater than other participation coefficients, i.e.,

$$\eta_1(t) \gg \eta_2(t), \eta_3(t), \dots$$
 (26)

Therefore, Eq. (13) can be considered as below:

$$w(\mathbf{x},t) \approx \varphi_1(\mathbf{x})\eta_1(t) \tag{27}$$

As a result, the first mode shape and natural frequency of the foil need to be investigated.

4.1. Foil case study specifications

The turbine foil is fabricated from carbon fibre prepreg material that is laid down on a mould and consolidated in an autoclave with a vacuum bag. The laminate schedule is composed of diverse carbon fibre plies such as unidirectional, biaxial, and woven to provide stiffness, strength, and good resistance to impacts. The laminate schedule varies along the span of the foil and can be decomposed into some sections. Fig. 5 shows one of the sections of the foil. Each section thickness is estimated by using the cured ply thickness provided by the material supplier. Moreover, the area and second moment of area for each section are calculated using CAD software (Solidworks). Furthermore, an ABD matrix method (Lindell, 2023) was used to calculate the equivalent Young's modulus in each section of the foil. This approach is based on the constitutive equation of each ply within the laminate, taking into account ply material properties, material strengths, ply fibre orientation and stacking sequence. By utilizing this method, it becomes feasible to analyse laminated composite plates according to classical lamination theory (CLT) and to determine apparent laminate material, ply stiffness and compliance matrices.

As the cantilevered part of the composite tidal foil is non-uniform, its cross-sectional area (A(x)), Young's modulus (E(x)), and second moment of area around the *y*-axis ($I_{yy}(x)$) are varied in longitudinal direction of the foil (x). The diagrams of A(x) and $E(x)I_{yy}(x)$ are presented in Fig. 6, and the related fitted functions are as follows:

$$A(x) = -0.0016 x + 0.003 \tag{28}$$

 $E(x)I_{yy}(x) = 3974.3 x^3 - 205900 x^2 - 13758 x + 270152$ ⁽²⁹⁾

It is noted that as shown in Fig. 3, x = 0 is related to the position of the strut (support in the cantilevered beam model).

In this study, the viscous damping is considered for the foil. By considering that the time response of viscous-damped systems under initial conditions is decayed exponentially with the rate of $-\zeta \omega_n$, the damping ratio of the foil can be calculated. In this regard, an initial displacement is applied at the tip of the foil and released to make it



Fig. 7. Experimental results for tip displacement of the foil in free vibration (a) time response and related envelop curve (b) envelop curve and its fitted curve.



Fig. 8. Tip displacement of the foil in the frequency domain for free vibration (from experiment).

Table 1

Parameters of the cantilevered part of the composite tidal foil and the URM system.

Parameter	Definition	Value
L L _e	Total length of the cantilevered foil Distance from the excitation point to the cantilevered support	0.98 m 0.5 m for Case 1 0.855 m for Case 2
L _r	Distance from the measuring point to the cantilevered support	0.95 m
<i>A(x)</i>	Cross-sectional area	Eq. (28) and Fig. 6
$E(\mathbf{x})I_{\mathbf{y}\mathbf{y}}(\mathbf{x})$	Product of Effective Young's modulus of the laminate in the longitudinal direction and second moment of area	Eq. (29) and Fig. 6
Μ	Concentrated tip mass (Summation of the URM system mass (48 Kg), mass of two clamps (6.45 Kg and 5.98 Kg) and mass of the foil surrounded by the clamps (0.54 Kg))	60.97 Kg
ρ	Mass density	1550 kg/m ³
ζ	Damping ratio	0.0015
т	Total mass of the half-circular unbalanced mass in the URM system	3.77 Kg
R	Radius of the half-circular unbalanced mass in the URM system	95 mm

vibrate freely and tip displacement of the foil is measured. Fig. 7(a) indicates the measured time response and related envelop curve. Also, Fig. 7(b) shows the envelop curve and its fitted exponential curve with the rate of -0.05466 (which theoretically equals to $-\zeta \omega_n$).

On the other hand, by applying the Fast Fourier Transform (FFT) on the free vibration time response presented in Fig. 7(a), the frequency response is obtained which is presented in Fig. 8. This figure indicates a peak at 5.92 Hz which is the first natural frequency of the foil with the URM attached. By considering that the exponential rate of decay in Fig. 7(b) is -0.05466, the damping ratio of the foil is calculated as $\zeta = 0.0015$.

Other parameters of the cantilevered part of the composite tidal foil are presented in Table 1.

4.2. Free vibration results

The boundary value problem (Eq. (8) and boundary conditions (11) and (12)) by considering the parameters in Table 1 is solved numerically

Table 2

Natural frequency of the foil, both with and without the URM system, obtained from experiment and numerical solution.

	Numerical	Experiment	Error
without URM	38.34 Hz	37.50 Hz	2.24 %
with URM	6.73 Hz	5.92 Hz	13.68 %



Fig. 9. The mass-normalized first mode shape function of the non-uniform composite tidal foil (numerical results and fitted curve).

using MATLAB to achieve the first natural frequency and mode shape function of the cantilevered part of the foil. It is worth mentioning that this numerical solution needs initial guesses for the natural frequency and mode shape function of the foil. In this study, the natural frequency and mode shape function of an equivalent uniform foil (with average values for A(x) and $E(x)I_{yy}(x)$) is considered as the initial guesses to achieve more accurate results for the non-uniform foil.

The obtained results for the first natural frequency of the nonuniform composite tidal foil, both with and without the URM system, from experiments and numerical solutions, are presented in Table 2.

Referring to Table 2, the natural frequency of the foil with the URM



Fig. 10. Tip displacement of the foil by applying an initial displacement (numerical and experimental results).



Fig. 11. Uncovered URM system showing the location of the unbalanced rotating mass in Case 1.

system is greater than that of the foil without the URM system. This discrepancy occurs because the URM system adds excessive mass to the foil, thereby decreasing its natural frequency. Furthermore, as indicated by Table 2, the percentage error in calculating the natural frequency of the foil with the URM system is greater than that of the foil without the URM system. This is due to the fact that in the mathematical modelling, the URM system has been treated as a concentrated mass, whereas in reality, it has a distributed mass. This matter introduces errors in calculating the natural frequency of the foil with the URM system. However, the developed mathematical model is more accurate in predicting the natural frequency of the foil without the URM system. The other source of error between numerical and experimental values can be related to the modelling of the strut-foil joint. This joint has some flexibility in reality; however, it has been considered as a fixed boundary condition in the mathematical model.

Moreover, the obtained result for the mass-normalized first mode shape function of the non-uniform composite tidal foil from numerical calculations is presented in Fig. 9. In addition, the following polynomial function is fitted to the numerical result which is presented in Fig. 9.

$$\varphi_1(\mathbf{x}) = -3.58 \, \mathbf{x}^5 + 7.5 \, \mathbf{x}^4 - 3.52 \, \mathbf{x}^3 - 1.91 \, \mathbf{x}^2 + 0.2 \, \mathbf{x} \tag{30}$$

To validate the free vibration response of the foil, an initial displacement is applied at the tip of the foil in the test setup and released to make free vibration. In this way, the first mode shape of the foil is excited remarkably in comparison with other mode shapes. Then, the related tip displacement of the foil is measured using the laser displacement sensor. On the other hand, according to the calculated damping ratio in the previous section, the considered foil is underdamped (i.e. $\zeta < 1$) and therefore, Eq. (23) can be employed to calculate the participation coefficient related to the first mode shape of the foil $(\eta_1(t))$ by considering zero excitation force. Then, using the first mode shape function (Eq. (30)) and employing Eq. (27), the time response of the tip displacement of the foil in free vibration will be calculated.

Fig. 10 indicates the numerical and experimental results for tip displacement of the foil in free vibration. Referring to this figure, although there is an acceptable agreement between numerical and experimental data in terms of the displacement values, there is a phase difference between these data. This matter is related to the difference between the calculated and experimental values of the natural frequency presented in Table 2. In other words, the numerical solution predicts a greater value for the natural frequency (or lower value for the natural period) of the foil that is observable in Fig. 10.

4.3. Forced vibration results

As calculated in Subsection 4.1, the damping ratio of the foil is lower than unity (underdamped). Therefore, Eq. (23) is considered an analytical solution for the *i* th mode participation coefficient ($\eta_i(t)$) of

the foil. As discussed in the beginning of Section 4, $\eta_1(t)$ should be calculated. Then, using the first mode shape function, $\varphi_1(x)$, computed in the previous section (Eq. (30)) and Eq. (27), the foil displacement under the applied force will be obtained.

In this study, as explained in Section 2, a URM system is employed at the tip of the foil to apply excitation force. In this system, unbalanced rotating masses (shown in Fig. 4(a)) can be adjusted at one or both sides to achieve desirable centrifugal forces. The parameters of the URM system are presented in Table 1. In this study, experiments are performed with two different adjustments of an unbalanced rotating mass explained as follows:

4.3.1. URM adjustment case 1

In this case, the unbalanced rotating mass is adjusted on the side of the URM system near the cantilevered side of the foil. The uncovered URM system showing the location of unbalanced rotating mass is depicted in Fig. 11. The excitation frequency is set to be $\omega_e = 5.635$ Hz. Fig. 12(a) indicates time response of the tip displacement of the foil obtained from the experiment. Fig. 12(b) compares numerical results with experimental data in a small period of 2 s to be more observable.

4.3.2. URM adjustment case 2

In this case, the unbalanced rotating mass is adjusted on the side of the URM system near the tip side of the foil. Fig. 13 indicates the uncovered URM system showing the location of unbalanced rotating mass. The excitation frequency is set to be $\omega_e = 6.326$ Hz. Fig. 14(a) indicates time response of the tip displacement of the foil obtained from the experiment. In Fig. 14(b), numerical and experimental data are compared in a small period of 2 s.

Referring to Fig. 12(b) and 14(b), it is obvious that there is a little phase difference between numerical and experimental results. This can be related to the approximation in calculating the damping ratio. Furthermore, the difference between the vibration amplitudes of the numerical and experimental results may be attributed to certain assumptions made in the vibration model. Specifically, these include the consideration of viscous-type damping, the treatment of the URM system as a concentrated mass at the tip of the foil, and the neglect of the flexibility of the strut-foil joint.

Although the developed mathematical model in this study predicts a higher natural frequency for the foil (indicating either higher equivalent stiffness and/or lower equivalent mass), it also predicts higher vibration amplitudes in the forced vibration results presented in Fig. 12(b) and 14 (b). This matter arises because, in these forced vibration results, the excitation frequencies (5.635 Hz and 6.326 Hz) are very close to the natural frequency of the foil (5.92 Hz). Under such conditions, even a small variation in the damping ratio can lead to a remarkable variation in the vibration amplitude. Therefore, the prediction of higher vibration amplitude by the mathematical model in Fig. 12(b) and 14(b) suggests



Fig. 12. Tip displacement of the foil by applying a harmonic force - Case 1 a) experimental data b) comparison of numerical and experimental data.

that the calculated damping ratio in this study is underestimated. This underestimation results in the prediction of higher vibration amplitudes near the resonance frequency.

5. Conclusions

In this study, a validated vibration model is presented for a non-

uniform composite helical tidal turbine foil. In this regard, vibration equations governing the non-uniform composite tidal turbine foil were derived and the related numerical results were presented and compared with experiments. The obtained results for free and forced vibration using the numerical solution and experimental tests showed that there is a good agreement between numerical and experimental data. The observed differences can be related to some assumptions of the vibration



Fig. 13. Uncovered URM system showing the location of the unbalanced rotating mass in Case 2.

model, especially consideration of the viscous-type damping, the assumption of the URM system as a concentrated mass at the tip of the foil and neglecting the flexibility of strut-foil joint. In this study, the governing vibration equation of the foil was derived by incorporating viscous damping, while the equivalent Young's modulus in each section of the foil was determined using an ABD Matrix approach, considering the mechanical properties of each ply. A future study may explore the derivation of the vibration equation of the foil by incorporating viscoelastic material assumptions.

The developed model in this study can be used to predict the dynamic response of the tidal foil with acceptable accuracy under any desirable dynamic load without the need to perform dynamic tests on the full-scale structure. Moreover, this model is useful for parametric study, optimization, and design of the tidal foil, which will accelerate the design process for future iterations of the foil. The methodology presented may also be applied to other cantilever composite structures and, therefore, has applications in wind energy, aerospace, drones and construction.



Fig. 14. Tip displacement of the foil by applying a harmonic force - Case 2 (a) experimental data (b) comparison of numerical and experimental data.

CRediT authorship contribution statement

Vahid Fakhari: Writing – original draft. Tenis Ranjan Munaweera Thanthirige: Methodology, Investigation, Formal analysis, Data curation. Michael Flanagan: Writing – review & editing. Ciaran Kennedy: Methodology, Investigation, Data curation, Conceptualization. Yadong Jiang: Investigation. Micheal O'Conghaile: Project administration. Tomas Flanagan: Supervision, Project administration, Funding acquisition. Clement Courade: Investigation, Data curation. Patrick Cronin: Project administration, Funding acquisition. Conor Dillon: Data curation. Jamie Goggins: Project administration, Funding acquisition. William Finnegan: Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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