Passive control for wave energy systems: An overview

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Motivation: Why passive control?
Reactive vs. Passive control strategies

Passive control refers to **WEC control strategies** that enforce **unidirectional power flow** through the **PTO**.

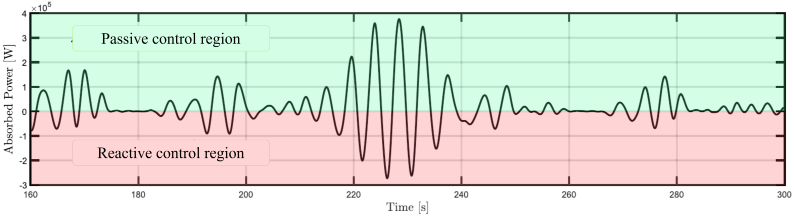
Why Passive Control Matters for WECs

Passive control refers to **WEC control strategies** that enforce **unidirectional power flow** through the **PTO**.

Change of control objective paradigm:

For many years, WEC control objective focused on maximising absorbed energy, leading to **reactive** control strategies, often positioning passive control as a lower-performance approach.

- Reactive control can potentially achieve high energy absorption relying on:
  - bidirectional power flow,
  - usually large motion and force operational spaces,



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More recently, control objective has shifted towards a techno-economic perspective, where **energy production** is assessed jointly with **CapEx** and **OpEx** to reduce the levelised cost of energy **(LCoE)**.

$$\min. \quad LCoE = \frac{CapEx + OpEx}{E}$$

Within this evolving **techno-economic** context:

Limitations of reactive control:

Why passive control matters:

Within this evolving **techno-economic** context:

Limitations of reactive control:

- Increased device motion and PTO forces, may accelerate structural fatigue increasing OpEx.

Why passive control matters:

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Passive Control under a Techno-Economic Perspective

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- Requirement for bidirectional PTO architectures, increasing system complexity and potentially the CapEx

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However, while passive control may appear conceptually simpler and beneficial in the broader techno-economical context, the nonlinear passivity (power) constraint poses a significant challenge for both control design and real-time implementation.

Brief review: Passive control approaches

Passive control: Preliminaries

The energy-maximising **passive control problem** is typically formulated as follows:

Unstructured PTO force f_{pto}:

Structured PTO force $f_{pto} = -B_{pto}(t)x_2(t)$:

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- Passivity: mixed control-state nonlinear (NL) constraint, nonconvex in the general case.
- Potentially quadratic objective function (OF).

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Passivity constraint: Main consequences

Enforcing passivity leads to nonconvex OCP and potentially NL dynamics, thus **challenging** the design of computationally efficient control schemes.

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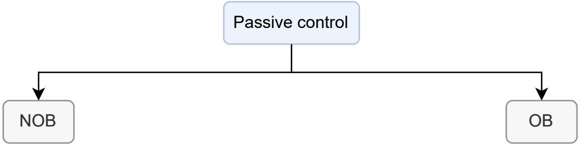
Passive control main challenge:

Design a control strategy that **balances** the **trade-off** between: **power absorption performance** & **computational cost**, while respecting the WEC physical **constraints**.

Passive control approaches

To address the main challenges, different passive control approaches have been proposed in wave energy literature, broadly **categorised** as:

- **Non-optimisation-based (NOB) control:** (based on fundamental understanding of the problem hydrodynamics to design the control)
- **Optimisation-based (OB) control:** (based on optimisation theory and associated numerical algorithms)



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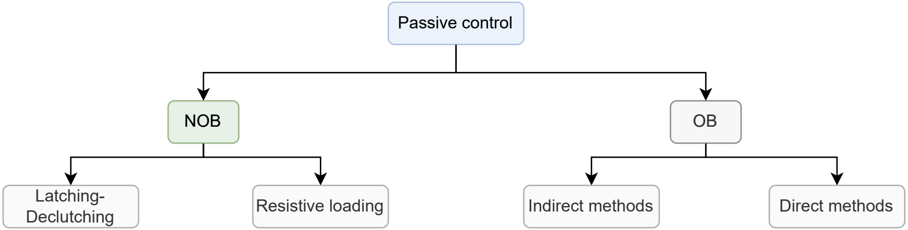
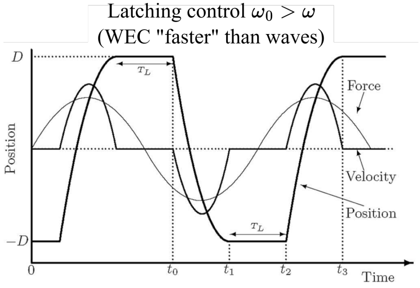


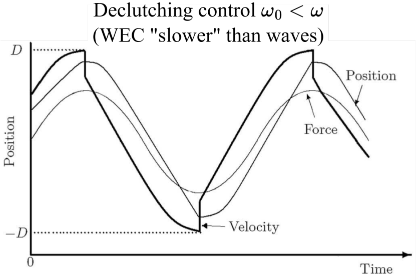
Brief review: Passive control approaches

Non-optimisation-based (NOB)

Latching and **declutching** constitute **early approaches** of **passive** control strategies.

- Both exploit classical wave energy theory: WEC response is shaped so that the **velocity** of the device is **in phase** with the **wave excitation force** (well-known impedance matching result) [1, 2].





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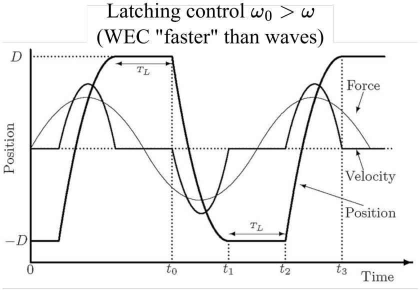
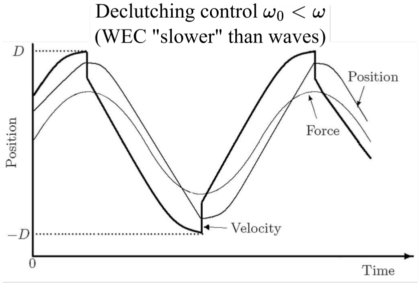
- Control problem usually reduces to compute the optimal latching-declutching times.





NOB Resistive Loading

Resistive loading implements a **structured** PTO force $f_{pto}(t) = -B_{pto}(t)\, x_2(t)$, where the **damping** coefficient is defined by exploiting the impedance-matching result:

$$B_{pto}(t) = |Z_w(j\omega)| = \sqrt{B_r^2(\omega) + \big(\omega(m_w + A_r(\omega) - S_h/\omega)\big)^2}, \quad \omega = \begin{cases} \{\omega_e, \omega_{pk}\} \rightarrow \textbf{constant}, \\ \hat{\omega}(t) \rightarrow \textbf{time-varying}. \end{cases}$$

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NOB controllers:

Benefits

Simple concept & real-time implementation.

Drawbacks

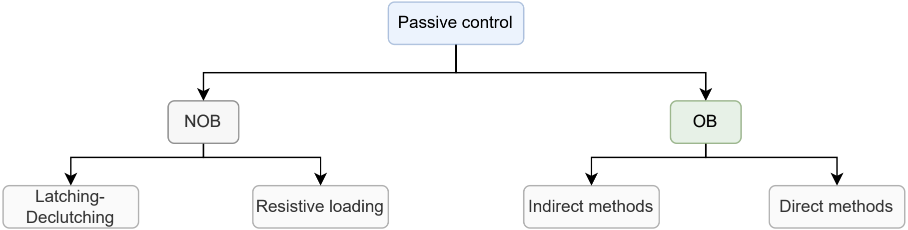
(i) Inherently **suboptimal**. **(ii)** Do not contemplate WEC **constraints** in the control definition. **(iii)** Latch-Decl strategy extension to **polychromatic** sea states is not straightforward.

Brief review: Passive control approaches

Optimisation-based (OB)

OB Indirect methods
(First optimise, then discretise)

OB Direct methods
(First discretise, then optimise)



(OB) Indirect methods
(First optimise, then discretise)

- **Formulation:** Optimality conditions derived via PMP, then discretised to solve the BVP.

(OB) Direct methods
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Although both indirect and direct methods are viable for wave energy systems, **indirect** approach present various **implementation difficulties** compared to **direct** methods [3, 4].

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- **Numerial robustness:** BVP presents high sensitivity to costate (non-physical vars.) initialisation, possible ill-conditioned or unstable costates dynamics.

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- **Numerial robustness:** improves since do **not deal** with **costates dynamics** and **BVP**.

Brief review: Passive control approaches

OB Direct methods

OB direct methods: OCP to NLP

OB Direct methods approach: First discretise, ...

Unstructured OCP:

$$\max.\ \mathcal{J}(f_{pto}) = -\int_{t_1}^{t_2} f_{pto}\, x_2 \,\mathrm{d}t$$

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- **Reactive control:** NLP is usually a **QP** and **KKT** left-hand-side **matrix** **gradients** are **constant**: do not depend on current iteration of $\tilde{w}$.

OB direct methods: OCP to NLP

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Passive control: **nonconvex** NLP & **gradients** in the **KKT** matrix explicitly **depend** on **current iteration** $\tilde{w}$, yielding **much more computationally demanding** NLP problems to solve.

OB Direct methods: Computational cost

How can we estimate the **computational cost** of **solving NLP** problems**?** Ii is mainly determined by **two components** [3, 5]:

$$T_{\text{iter}} \approx T_{\text{LA}} + T_G$$

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(i) **Linear algebra cost**: solving the KKT system at each iteration:

(ii) **Gradients evaluation cost**: computation of KKT sys. **elements**.

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(i) **Linear algebra cost**: solving the KKT system at each iteration:

- **Dimension** of the KKT system, (↑)
- **Sparsity** pattern and nnz elements **ordering** in KKT matrix, (↓)

(ii) **Gradients evaluation cost**: computation of KKT sys. **elements**.

OB Direct methods: Computational cost

How can we estimate the **computational cost** of **solving NLP** problems? Ii is mainly determined by **two components** [3, 5]:

$$T_{\text{iter}} \approx T_{\text{LA}} + T_G$$

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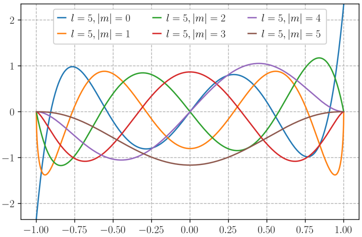
KKT sys. **Dimension** & **Sparsity** (**T_{LA}**) depend on: **(i) discretisation method**, and **(ii) NLP formulation**.

(i) How the **discretisation** method influences the **computational cost**:

Global support, pseudospectral (PS):

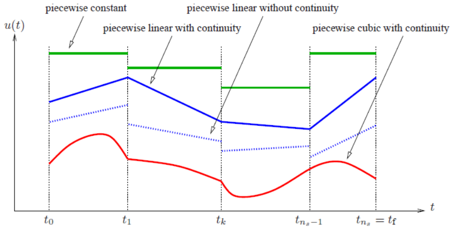
(Fourier - HRCF - Legendre)



- **Global domain**: overall horizon.
- **Strong coupling**: Each decision var. influences the solution everywhere.

Compact support:

(ZOH - FOH)



- **Local domain**: interval-wise.
- **Local coupling**: Each decision var. influences only neighbours.

(i) How the **discretisation** method influences the **computational cost**:

Global support, pseudospectral (PS):
(Fourier - HRCF - Legendre)

- **Dense** representation (↑)

Compact support:
(ZOH - FOH)

- **Sparse** representation. (↓)

Generic **NLP:**

$$\max.\ \tilde{\mathcal{J}}(\tilde{w}) = A_1\tilde{w} + B_1 q_1(\tilde{w})$$

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- **Convergence**: spectral for smooth signals ⇒ less nodes ⇒ dimension drops (↓)

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(ii) How the **NLP formulation** influences the **computational cost**. There are two main formulations:

Simultaneous:

- **Full discretisation**: **states & inputs** (control-wave excitation)

Sequential:

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Simultaneous:

- **Full discretisation**: **states & inputs** (control-wave excitation)
- **Particular case**: for linear dyn., **states** can be **cancelled** analytically ⇒ **condensed** NLP in terms of **control vars. only**.

Sequential:

- **Only inputs** are **discretised** (control-wave excitation)
- **Particular case**: for linear dyn., closed-form solution can be used to **cancel** the **states** (exact discretisation) ⇒ **condensed** NLP in terms of **control vars. only**.

While condensing significantly reduces the dimension of the NLP (↓), and is the standard approach for **reactive control**, the implications for **passive** control are **different...**

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- KKT matrix needs **re-computation** and **re-factorisation** at every iteration (↑).

Computational cost & NLP formulation

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Condensing in general is less attractive for WEC problems with nonlinear passivity constraints (compared to the reactive control case).

Several direct method approaches have been developed in literature:

- SIM(n-cond & cond)-**GH**PSC(Fourier,Legendre)-uns.NLC [6, 7, 8, 9, 10].
- SIM(cond)-**RH**PSC(Fourier + windowing) (implemented for reactive control) [11, 12].
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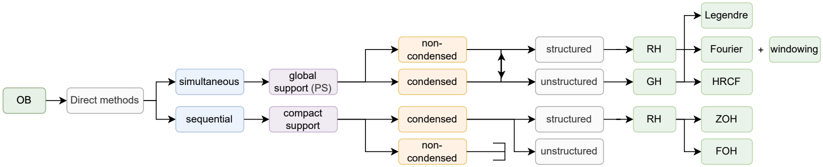
Despite the great **variety** of passive control approaches in literature, these studies are often **focused** on benchmarking a **single class of methods**, therefore providing **limited guidance** for **method selection** based on the **passive problem structure**.



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Despite the great **variety** of passive control approaches in literature, these studies are often **focused** on benchmarking a **single class of methods**, therefore providing **limited guidance** for **method selection** based on the **passive problem structure**.

Open-question: Which is the **most suitable direct** optimal control scheme—comprising OCP discretisation and NLP formulation—for **passive** control of wave energy systems, balancing the trade-off between power absorption, computational cost, constraint satisfaction, and modelling fidelity?

Wavepiston: Passive control with real industrial relevance

Passive control strategies are not only a interesting control exercise, but it began to have **real industrial applications**.

Wavepiston: From passive OCP to a real case scenario.

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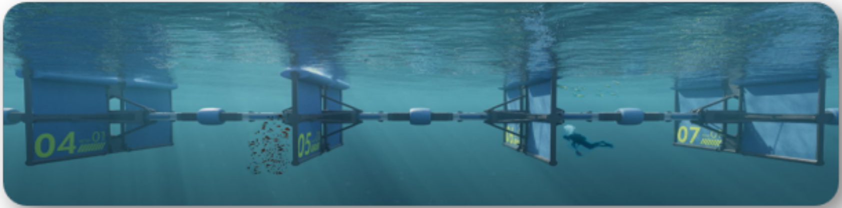
Wavepiston vision and the role of passive control:

- **Wavepiston technology development**: is driven by a strong focus on minimising LCoE seeking for simplicity, reliability, and scalability.



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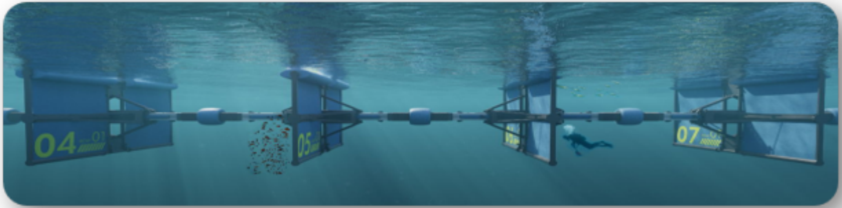
- **Wavepiston technology development**: is driven by a strong focus on minimising LCoE seeking for simplicity, reliability, and scalability.
- The use of a passive PTO is a deliberate design choice.
- Therefore, passive control strategies constitute a central element of Wavepiston philosophy.



Open research challenge: Wavepiston passive control (WAPPAC) competition

The **WavePiston Passive Control (WAPPAC)** competition is an open research challenge offered to the wave energy and control communities:

- **Participants**: invited to design passive control strategies for the Wavepiston device.
- **Objective**: minimise LCoE-surrogate, while respecting the PTO passivity constraint.

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Competition purpose: foster collaboration across the wave energy and control communities to advance the state-of-the-art in passive control through a realistic, industry-relevant case study.

Thank you!

Centre for Ocean Energy Research

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Research Ireland

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