

# Multi-Parameter Optimization of Hybrid Arrays of Point Absorber Wave Energy Converters

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**Abstract**—In this paper the influence of having different sizes of wave energy converters in the same park is evaluated by the extension of a customizable tool based on a genetic algorithm. Two different cost function are used in the optimization scheme and their results compared. The method has been applied to simultaneous optimization of some important design variables of a wave energy farm, such as the geometry of the buoys and the damping coefficient of the electric generator. Spatial layout optimization has also been performed with hybrids parks, i.e. arrays consisting of point absorbers of different dimensions. It is shown that the choice of the cost function has a great impact on the results of the optimization and that a slightly larger power production can be achieved by deploying devices of different sizes in a hybrid park.

**Index Terms**—Wave energy arrays, genetic algorithm, optimization, different WECs, wave parks, hybrid parks.

## I. INTRODUCTION

Point-absorber wave energy converters (WECs) of relatively small rated power will in general be deployed in large arrays of many devices to achieve a large enough power to cover for costs of installation and maintenance. The devices in the park will interact both hydrodynamically and electrically, and the performance of the full wave energy park is influenced by many parameters such as park layout, distance between devices, power take-off and dimensions of the individual devices. An optimal park will produce maximal annual energy with low power fluctuations and low costs for installation and maintenance. For this reason, the optimization of WEC array design parameters is one of the most challenging and urgent goals within the wave energy research community.

Most of the previous array modelling and experimental studies published in the field have been based on arrays of identical devices. Few previous works have shown insight on possible benefits resulting by deployment of parks with different sizes of the devices (hybrid parks) ([1], [2]), but no systematic optimization approach has yet been pursued.

In this paper two different applications using a customizable tool based on a genetic algorithm are presented; the model was firstly introduced in [3] and [4] and here extended to perform optimization of hybrid parks by the hydrodynamic model developed in [2].

The optimized parks consist of up to 12 point absorber wave energy converters of the kind developed at Uppsala University, whose working principle consists of a linear direct-driven

permanent magnet generator located on the seabed driven by a floating buoy at the sea surface [5].

Genetic algorithm optimization methods applied to wave energy arrays have been initiated in the works of [6]–[9]: the object of these works included the optimization of the geometric layout of identical point-absorber WECs [6]–[9], or the optimization of the power take-off (PTO) parameters given a fixed layout and floaters geometries [8].

In [3] and [4], the genetic algorithm used in this paper was introduced for optimization of the radius, draft and damping coefficient of a single WEC and validated against parameter sweep (PS) of the same variables. The agreement was excellent (less than 0.2 % difference in the final average power output calculated by the two methods was obtained in 20 simulations) and computational cost was shown to be low (GA converged to solution in around 11 % of the time required to perform PS). In this paper, the method is further developed and used to study arrays of non-homogeneous WECs size and PTO constants which is of highest relevance to find optimal wave power parks.

## II. METHOD

### A. Hydrodynamics and WEC model

Consider a park of  $N_b$  wave energy converters (WECs) labelled by  $i = 1, \dots, N_b$ . Each WEC consists of a float connected to a direct-driven generator at the seabed. The floats are truncated cylinders with individual radius  $R^i$  and draft  $d^i$ , and the generator is characterized by individual power take-off constants  $\Gamma^i$ , giving an instantaneous power of each WEC as  $P^i(t) = \Gamma^i [\dot{z}^i(t)]^2$ , where  $z^i(t)$  is the position of the translator in the linear generator. The line connecting the float with the translator is assumed to be stiff, implying that the coupled equation of motion between the surface buoy and the translator can be written as one equation as

$$m\ddot{z}^i(t) = F_{\text{exc}}^i(t) + F_{\text{rad}}^i(t) + F_{\text{PTO}}^i(t) + F_{\text{stat}}^i(t) \quad (1)$$

where  $m$  is the total mass of the float and the translator,  $F_{\text{exc}}^i$  and  $F_{\text{rad}}^i$  are the hydrodynamic excitation and radiation forces, respectively,  $F_{\text{PTO}}^i(t) = -\Gamma^i \dot{z}^i(t)$  the power take-off force and  $F_{\text{stat}}^i(t) = \rho g \pi R^{i2} (d^i - z^i(t))$  the hydrostatic restoring force. Only heave motion is taken into account, an approximation which has been shown to agree well with a full-scale system in offshore conditions [10]. In addition, unlimited stroke length

is assumed, which is a reasonable assumption in operational waves, but doesn't hold in more energetic sea states. By Fourier transformation, the problem can be considered in the frequency domain, in which the equation of motion (1) takes the form

$$\left[ -\omega^2(m^i + m_{\text{add}}^i) - i\omega(B^i + \Gamma^i) - \rho g \pi R^{i2} \right] z^i = f_{\text{exc}}^i \eta^i \quad (2)$$

where the radiation force has been divided into added mass and radiation damping as  $F_{\text{rad}}^i(\omega) = [\omega^2 m_{\text{add}}^i(\omega) + i\omega B^i(\omega)] z^i$  and  $\eta^i(\omega)$  is the frequency domain amplitude of the incident waves. The hydrodynamic forces are computed as surface integrals over the wetted surfaces of the buoys,  $\bar{F} = i\omega \rho \iint_S \phi d\bar{S}$ .

To compute the fluid velocity potential, potential flow theory is assumed, i.e. the fluid is assumed to be non-viscous, irrotational and incompressible. In addition, the waves are assumed to be non-steep, implying that the boundary conditions at the free surface, sea bed and any rigid body can be linearized and the first order approximation taken. The fluid velocity potential is a superposition of incident, scattered and radiated waves,  $\phi^i = \phi_I^i + \phi_S^i + \phi_R^i$ , and satisfies the Laplace equation in the fluid domain.

The fluid domain has a constant depth  $h = 25$  m, which corresponds to the wave energy test site at Lysekil, Sweden, and water density  $\rho = 1025$  kg/m<sup>3</sup>. Local coordinate systems  $(r, \theta, z)$  are chosen with the origin in each cylinder buoy. The vertical coordinate is chosen such that  $z = 0$  at the still water surface, and  $z = -h$  at the seabed.

Based on the multiple scattering method, first presented in [11] and further developed in [12], [13], an analytical method has been developed, allowing for fast computation of the hydrodynamical coupling with scattered and radiated waves in the park [14]. The method has been further extended in [2] to allow for floats of different dimensions and topologies. In this paper, different buoy radius  $R^i$  and draft  $d^i$  are considered, but we restrict to cylinder buoys.

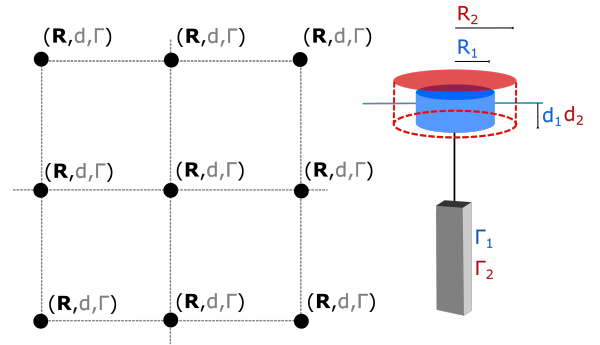
The input to the model consists of 20 min time series of irregular waves measured off-shore at the Lysekil research test site at the west coast of Sweden. The 20 min time series, characterized by a specific significant wave height  $H_s = 1.53$  m and energy period  $T_e = 5.01$  s, is repeated twice to have 60 minutes of input waves into the model. In all simulations waves propagate in the positive x direction.

The dynamics of the WECs is determined in the time domain by inverse Fourier transform of the solution to the equation of motion (2) and the instantaneous power absorption of each WEC is computed. The power output of the full park will be the sum of all  $N_b$  WECs,

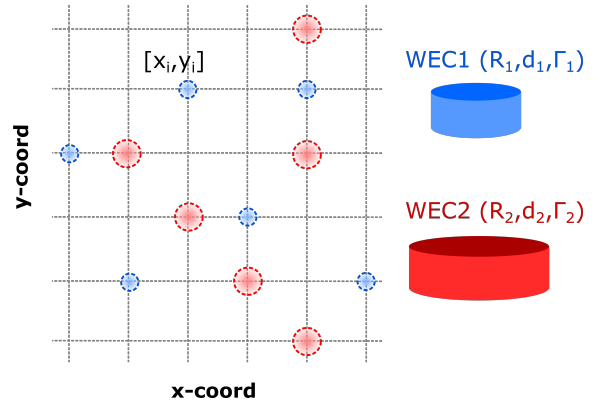
$$P_{\text{tot}}(t) = \sum_{i=1}^{N_b} P^i(t). \quad (3)$$

### B. Genetic algorithm

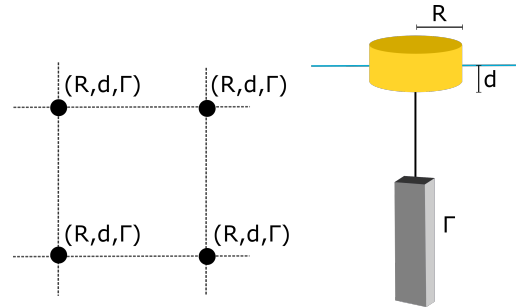
The genetic algorithm (GA) is a metaheuristic optimization method based on the theory of evolution which was developed in [15]. The procedure can be applied to many different problems and subjects and is based on a "genetic evolution" over



(a) Sketch of the WEC concept and parameters of the optimization in a fixed layout. Optimization only on the radii values of the WECs. Values are shown in Table I.



(b) Sketch of the layout optimization of WECs of fixed different topologies. Values are shown in Table I.



(c) Sketch of the multiple parameter optimization in a fixed layout.

Fig. 1. Case study 1 (a), 2 (b) and 3 (c).

a set of solutions, until sufficiently good results are obtained. In this case and in general for wave energy converters arrays optimization problems, many different parameters affect the performance of the park. In other words, a large parameter space is involved; therefore parameter sweep is infeasible. GA is a well suited method for these kind of problems, considering that the shape of the cost function is not known and probably multi-peaked [6].

This paper shows results from three different applications of the tool to hybrid parks. In the first study the optimal geometry is sought for a fixed gridded regular layout of 9 WECs (Fig. 1(a)). The method allows simultaneous optimization of

TABLE I  
WECs BUOYS GEOMETRY AND PTO CONSTANTS.

Geometry	Radius	Draft	Mass	PTO $\Gamma$
#1	2.0 m	0.5 m	6440 kg	70 kNs/m
#2	3.5 m	0.6 m	23668 kg	200 kNs/m

radii, drafts and PTO constants of all the devices in the park, but it has here been implemented only on two different radii values of the WECs. A value of draft and  $\Gamma$  is assigned according to Table I respectively. The second application shows a layout optimization of 6 WECs of geometry #1 and 6 WECs of geometry #2 (Fig. 1(b)). The third application is a multiple parameter optimization of an array of 4 WECs (Fig. 1(c)). The radius, draft and PTO coefficient of each WEC are free to take values within the ranges  $R \in [2 : 0.5 : 3.5]$  m,  $d \in [0.3 : 0.05 : 0.6]$  m and  $\Gamma \in [15 : 1 : 250]$  kN. The alternative cost function (5) is used for comparison with the non-dimensional cost function in (4).

The first *population* is a set of a fixed number of *chromosomes* created randomly which contains a certain number of *genes*, depending on the variables involved in the optimization. For the case study 1 shown in Fig. 1(a), each chromosome contains 9 different genes, i.e. one value of  $R^i$  for each of the 9 WECs in the array. Case study 2 shown in Fig. 1(b) contains 12 genes in each chromosome, so that every device is represented by a couple of coordinates  $[x_i, y_i]$  (where  $i$  is the  $i$ -th device in the park). The possible positions allowed are placed on a 6 x 6 grid with a separating distance of 15 m. Case study 3 shown in Fig. 1(c) has 12 genes as well, but this time containing one value of  $R^i$ ,  $d^i$  and  $\Gamma^i$  for each device.

As described in the introduction, an optimal wave energy park should produce maximal power with minimized costs (and possible other constraints, such as minimized power fluctuations and used ocean area). Around 70 % of the capital cost for a wave energy device can be attributed to costs for structure and mechanical systems [16]. Hence, a crude estimation for the installation cost of a wave device is its mass. In the GA, a fitness or objective function is used to evaluate the solutions of the optimization, and is here defined in two ways. In the first case, where the individual buoy dimensions of WECs in a park with fixed layout are to be optimized, the fitness function is the non-dimensionalized ratio between the total produced power of the park and the total mass of the devices,

$$f_{\text{cost}} = - \frac{(P_{\text{tot}} - P_{\text{small}})/(P_{\text{big}} - P_{\text{small}})}{(m_{\text{tot}} - m_{\text{small}})/(m_{\text{big}} - m_{\text{small}})}. \quad (4)$$

Here,  $P_{\text{tot}}$  is the total mean power of the considered hybrid array,  $P_{\text{big}}$  is the total power of the park when all WECs are of the largest allowed dimension, whereas  $P_{\text{small}}$  is the total power when all WECs are of the smallest dimension. Analogously,  $m_{\text{tot}}$  is the total mass of the WECs in the considered array,  $m_{\text{big}}$  is the total mass of the park when all WECs are of the largest allowed dimension and  $m_{\text{small}}$  of the smallest. Given

than the drafts are fixed, the masses are calculated according to Archimedes' principle. Values are reported in Table I. Both the numerator and denominator of the fitness function in (4) range from 0 to 1. In the second case, where the layout is optimized of a park of 6 small and 6 big devices, the total mass of the park will be equal for all cases, and the the fitness function is taken simply as the negative value of the power production of the park, as in equation (3).

The cost function in (4) is the range of two non-dimensionalized values both running from 0 to 1, thus the relative change in power output and total mass have an equal impact on the cost function. However, this might not be the obvious choice to optimize wave energy arrays. To study this in more detail, an alternative dimensional cost function has been used in case 3, defined as

$$f'_{\text{cost}} = - \frac{P_{\text{tot}}}{m_{\text{tot}}}. \quad (5)$$

The alternative cost function is simply the ratio of the total output power and the total mass of the devices in the array. Since the two values have different ranges, the impact of the relative changes in the output power or the total mass on the cost function may be different, which possibly reflects an optimization process in wave energy development in a more realistic way.

To clarify; in Case study 1, the buoy radius is optimized for 9 WECs, and the cost function being optimized is the non-dimensionalised power-to-mass ratio. In Case study 2, the layout is optimized for an array consisting of 6 large and 6 smaller WECs, and since the mass will be equal for all cases, the cost function is simply the total power. In Case study 3, the layout is fix for 4 WECs, and the radius, draft and PTO coefficient are free. A dimensionalised cost function in equation (5) is optimized in this case.

The WECs are considered fully hydrodynamically coupled and an analytical fast multiple scattering method is used to compute the hydrodynamic parameters, as discussed in the section II-A.

The convergence criteria implemented in the method, i.e. the criteria needed to stop the search when an acceptable solution is reached are:

- I. a maximum number of iterations is reached;
- II. all the chromosomes in the actual population are the same;
- III. the solution ceases to improve after a certain number of iterations.

If one of this conditions is fulfilled, the algorithm stops and the first chromosome of the ranked population is taken as final optimal solution.

If convergence is not reached, reproduction is performed and genetic material between two parents chromosomes is exchanged so that potentially positive distinctive genes from both individuals will be inherited by every child (*offsprings*). This procedure is performed with a single crossover point for case 1 and 2, whether for case 3 we have a crossover point

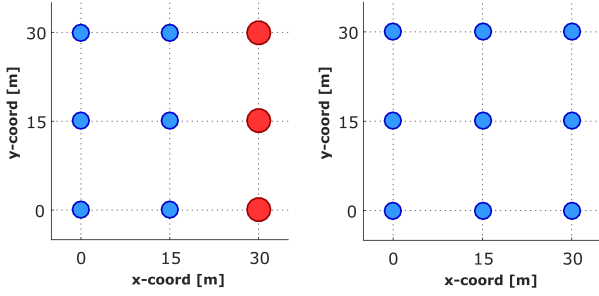


Fig. 2. Case study 1: best (left) and worst (right) solution.

for each variable. The *parents* chromosomes are the first upper 50 % of the population.

To ensure that the algorithm does not get stuck in local minima, 20 % of the variables (i.e. genes) in the population are randomly changed by *mutation*. This ensures that other regions in the solution space will be explored. However, the best solution is preserved unaltered in the following generation by *elitism* operator, so that it is not affected by potentially negative mutations. The discussed parameters used in the GA optimization have been chosen by trial-and-error to give fast solution convergence.

Consequentially, a new population is built by the combination of the *parents* chromosomes and the newly generated *offsprings*; the evaluation and reproduction processes are then iterated for a certain number of *generations*.

For more details about the structure of the algorithm please refer to [4].

### III. RESULTS

#### A. Case study 1

Fig. 2 shows the results obtained in the first case study in terms of best and worst layout, according to the value of  $f_{cost}$ . The optimal solution has the first two rows facing the incoming waves of small buoys (i.e.  $x = 0$  m and  $x = 15$  m), while the line at  $x = 30$  m consists of bigger buoys.

Fig. 3 shows the values of the non-dimensionalized power ratio (numerator of equation (4)) as a function of the non-dimensionalized mass ratio (denominator of equation (4)). Given a fixed value of the mass (i.e. costs), is it possible to get different power production, according to the internal location of big and small buoys. The highest ratio between the relative produced power and the relative mass (with respect to a park with all buoys of the smallest and largest geometry) is obtained with an hybrid park of 6 small and 3 big devices.

#### B. Case study 2

Results of case study 2 have shown that the GA optimization was able to find a layout that produce 7 % more than the average configuration (Fig. 4 left), while the worst layout (Fig. 4 right) produce around 12 % less than the average configuration. The average configuration has here been taken as the median value of all configurations ordered from worst

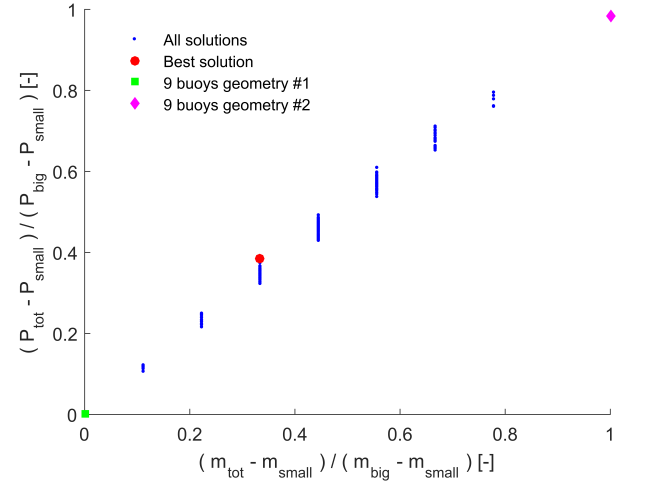


Fig. 3. Value of the non-dimensionalized power ratio (numerator of equation (4)) as a function of the non-dimensionalized mass ratio (denominator of equation (4)).

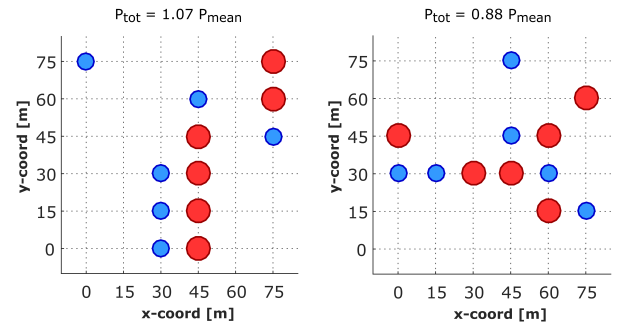


Fig. 4. Case study 2: best (left) and worst (right) solution.

to best. The hybrid park best solution was obtained with 121 number of iterations, but a qualitative guess would suggest that a higher number of iterations would result in a geometrical layout represented in Fig. 5 instead. To verify this hypothesis, the power of this park has been computed separately and has been found just 0.6 % higher than the solution in Fig. 4. Hence, the found optimal park configuration is not truly optimal, but the difference is negligible.

As a comparison to the optimal configuration, we can consider the deployment of two distinctive parks of big and small WECs separately (located on two adjacent line facing the wave front) (Fig. 6). The resulting output power would be around 164.5 kW and 74.5 kW respectively. The sum of the power production would then be 239 kW. The deployment of two of hybrid parks would give a total power output of about 245.5 kW, which is around 2.7 % higher than having two distinguished homogeneous parks of big and small devices. In other words, for a given number of small and large devices, a slightly larger power production is obtained if the devices are deployed in hybrid arrays of mixed sizes.

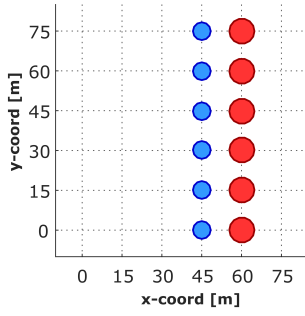


Fig. 5. Qualitative guess solution for case study 2.

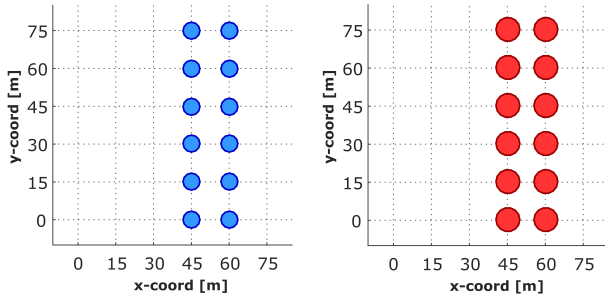


Fig. 6. Representation of homogeneous park of 6 small devices (left) or 6 big devices (right).

### C. Case study 3

In the third studied application, the parameters radius, draft and PTO coefficients have been simultaneously optimized in an array of 4 WECs. The coordinates of the WECs are fixed on a grid with separation distance 15 m, see Fig. 7. To compare different cost functions used for the optimization, the alternative cost function (5) has been used here.

Unlike the results obtained in case study 1, here the optimal solution after 5000 iterations is the one shown in Fig. 7, with only the smallest WECs and drafts. The GA has been able to match smallest possible radius with smallest draft and corresponding optimal PTO coefficients.

## IV. DISCUSSION AND CONCLUSION

The present paper describes a method to perform multiple parameter optimization of wave energy parks, and compared different cost functions for the optimization scheme.

From the comparisons of case studies 1 and 3, it can be concluded that using the dimensional cost function (5), the relative changes in the mass have a larger impact on the cost function than the relative changes in the output power, since the optimal configuration is the one with only smallest WECs and small draft and corresponding PTO coefficients. In other words, the improvement in output power when including larger WECs is too small to make a difference, as compared to the larger mass, or increased costs.

Hence, we see that the choice of cost function determines the outcome of the optimization; with the non-dimensionalized

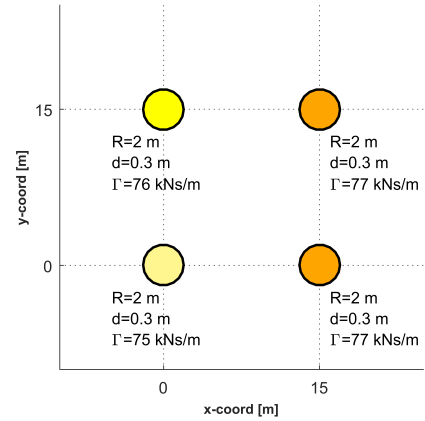


Fig. 7. Optimal solution obtained in case study 3.

cost function defined in equation (4), the *worst* solution is similar to the *best* solution obtained with the dimensional cost function in equation (5). Both these configurations consists of small WECs, and in case study 3, the relative changes in the mass – which is a crude estimate of the construction costs – have a larger impact than the changes in total power, thus resulting in an optimal solution.

An objective, non-biased comparison would require the knowledge of an economical cost function defined as the ratio between the total income of the produced electricity and the capital (CAPEX) and operational (OPEX) costs. If the cost per produced kWh is given by  $g(P_{out})$  and the CAPEX and OPEX could be written as a function of the total device masses as  $h(m_{tot})$ , such a cost function could be defined as

$$f_{cost} = -\frac{g(P_{out})}{h(m_{tot})}. \quad (6)$$

The dimensional cost function in (5) can be seen as a first step towards such a more refined cost function based on economical assessment of the park.

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