Frequency array and wave phase realizations for wave energy converter control optimization

Jeff T. Grasberger, Ryan G. Coe, Daniel T. Gaebele, Michael C. Devin, Carlos A. Michelen Ströfer, Giorgio Bacelli

Abstract—**Vital to the progression of the wave energy industry is wave energy converter optimization which often relies on frequency-domain evaluations, utilized for efficiency as compared to time-domain. Despite being an integral factor in the resultant solution, the frequency array is rarely described in such studies. This study shows the impacts of the frequency array components and illustrates a general process by which to select a proper frequency array. The main factors to consider are the range and number of frequencies. Furthermore, this study introduces irregular wave phase realizations and suggests the importance of optimizing the system for multiple random sets of wave component phase. Ultimately, the importance of proper selection of both the frequency array and wave phase realizations to the optimization solution is demonstrated for the Pioneer WEC using WecOptTool.**

Keywords — **Control co-design, Optimization, Wave energy conversion**

I. Introduction

WAVE energy conversion is a unique renewable energy technology with the potential to play a significant role in the transition to renewable energy yet remains vastly untapped. Progression in the field of wave energy harvesting requires co-design practices which holistically consider multiple facets of WEC design. These facets of WEC design include geometry, mechanical components, electrical components, and controls. A common method for co-design is to model WEC dynamics in the frequency domain, allowing for effective numerical models with low computational expense. Modeling WEC dynamics in the frequency domain requires selection of a discrete frequency array at which to calculate the hydrodynamics and apply the equations of motion.

Furthermore, when assessing WEC design in irregular wave conditions, a wave phase must be applied at each frequency in the array, creating a phase realization. The phase realization is traditionally randomized but can impact the expected design performance. Thus, best practice (although not always followed) is to take the average result from modeling multiple random phase realizations, but the impact of the number of realizations is not well understood. The importance of the wave phase realization is intertwined with the frequency array and requires careful consideration to ensure precise results.

WEC frequency domain optimization studies have been carried out extensively across a wide variety of WEC archetypes, yet guidance and impact of the frequency array and wave phase realizations is very limited. [1] documents and reviews many frequency domain optimization studies, yet never specifies the importance of the frequency array. Some studies do not specify the frequency array used at all such as [2] [3], and [4], while others describe the array used but do not provide reasoning or state a consideration of the wave phase realization such as [5], [6], [7], and [8].

This paper explores frequency-domain models of WECs for co-design and optimization purposes with a focus on frequency array selection and wave phase realizations in order to demonstrate the importance of a rigorous frequency array and realization analysis. Specifically, this paper uses the pseudo-spectral method for optimization of WECs, which solves the dynamics in the frequency domain and evaluates constraints in the time domain, adding an extra layer of complexity.

II. Wave Energy Converter Modeling and Optimization

A. Dynamic Model

The equation of motion of a wave energy converter in the pitch degree of freedom is defined as.

$$r(t) = M\ddot{z} - f_r(t) - f_h(t) - f_f(t) - f_e(t) - f_a(t) \quad (1)$$

Here, M is a mass/inertia matrix, $\ddot{z}$ is the WEC acceleration vector, and the different generalized force/torque vectors are the radiation force f_r due to wave generation, the hydrostatic force f_h, the hydrodynamic frictional force f_f,

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[1] *Capytaine*: https://ancell.in/capytaine/latest/

the wave excitation force f_e, and any additional forces f_a such as PTO and mooring forces. The hydrodynamic forces listed above are obtained from solutions to the radiation and diffraction problems using the boundary element method code *Capytaine*[1] [9].

The wave energy converter dynamics can also be understood through the calculation of the device's intrinsic impedance. The impedance is a ratio of the device response velocity to the input force from the waves, defined by (2). The input forces from the wave can be easily related to the WEC response with the impedance, a valuable tool in WEC design.

$$Z_i(\omega) = j\omega\big(M + A(\omega)\big) + B(\omega) + B_f + \frac{K_{hs}}{j\omega} \qquad (2)$$

With the WEC impedance defined based on the boundary element method results, and the wave excitation defined based on the input wave conditions, the basic WEC dynamics can be calculated in the frequency domain.

The Pioneer WEC is used as a case study here. The Pioneer WEC is a pitch resonator concept developed in [10]. The concept consists of a flywheel connected to an existing buoy by a magnetic torsional spring and in parallel with a generator. The WEC concept and power take-off (PTO) is described in more detail in [11]. The Pioneer WEC is of particular interest for this study because of its narrow-banded response. Even though the Pioneer WEC is used in this paper for demonstration, the design concepts presented are intended to be applicable to all frequency-domain optimization of WECs.

B. *Pseudo-Spectral Optimization*

Pseudo-spectral optimization is a relatively broad method of optimization which describes solving the system in the frequency domain while also including time-domain evaluations [12] – [14]. The time-domain evaluations may include constraints and the objective function. For wave energy converters, pseudo-spectral optimization can be particularly useful for optimizing the dynamics in a specific wave condition because waves are generally defined as a spectrum and WECs are designed to be oscillatory systems. This lends itself to efficient dynamic calculations in the frequency domain. Then, the objective function (power) and constraints (displacement, torque, etc.) can be quickly converted to a time-domain realization for evaluation.

WecOptTool[2] is a WEC optimization tool that utilizes a pseudo-spectral method for optimizing the WEC control trajectory and other design variables. This study presents WecOptTool's Pioneer tutorial as an example for demonstrating the importance of the frequency array and realization selection.

A Fourier decomposition of the WEC position is completed for a discrete frequency array $\omega = [\omega_0 \;\; 2\omega_0 \;\; \dots \;\; N_\omega\omega_0]$ of length N_ω, where ω_0 is the fundamental frequency. An unstructured optimal controller, which can apply an arbitrary PTO force at each time step, is used in this study. Thus, the control coefficients are formatted as an array of equal length to the WEC position array. The Fourier and control coefficients are stored together in a single state variable (x). The optimization problem in (3) can then include solving for the optimal control state to maximize the electrical power while solving the WEC dynamics.

$$\begin{gathered} \min_x J(x) \\ s.t. \\ r(x) = 0 \\ c_{ineq}(x) \geq 0 \\ c_{eq} = 0, \end{gathered} \qquad (3)$$

Here, $J(x)$ is the objective function (e.g., average electrical power), $r(x)$ captures the WEC dynamics in residual form (Section II-A), and c_{eq} and c_{ineq} are arbitrary equality and inequality constraints. In this study, the objective function is the average electrical power, and an inequality constraint is applied for some specified cases to limit the maximum PTO force as noted in each case. WecOptTool uses a sequential least squares programming (SLSQP) optimization algorithm to solve the problem.

III. Frequency Array Components and Selection

Any solution involving the frequency domain requires a discrete frequency array. The frequency array used in pseudo-spectral methods is defined by a number of frequencies at which the excitation is defined and the dynamics evaluated. This study will be focused solely on the pseudo-spectral method using equally spaced frequency arrays with the spacing equal to the fundamental frequency (a current requirement of WecOptTool). The three main concerns relating to the frequency array are the range of frequencies, number of frequencies, and phase realization.

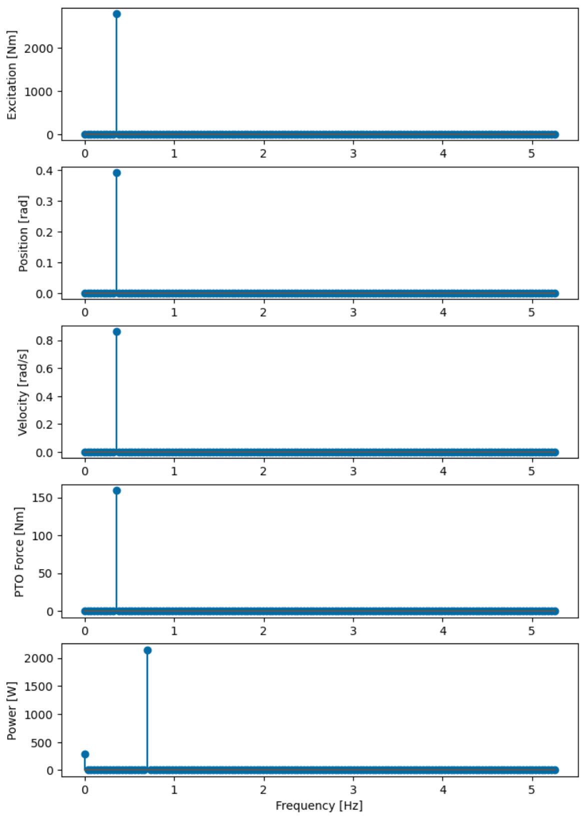

Fig. 1. Pioneer response spectrum for linear (unconstrained) case

[2] *WecOptTool*: https://github.com/sandialabs/WecOptTool

A. *Frequency Range*

The frequency range should be selected to satisfy two main requirements: capture the wave spectrum accurately and include the entire WEC response (including nonlinearities).

First, for simplicity, the WEC system response to a regular wave can be analyzed. For the linear case, the WEC responds only at the wave frequency (Fig. 1). When nonlinearities are incorporated (in this case, a constraint on the maximum PTO force), the WEC system also experiences response components at the odd harmonics of the wave frequency as shown in Fig. 2. The power components are a result of the PTO force and are present at 0 Hz and the even harmonics. In order to capture these nonlinear responses, it is recommended to complete a convergence study to determine the upper extent of the frequency range needed to ensure the desired accuracy. This concept applies to both regular and irregular waves.

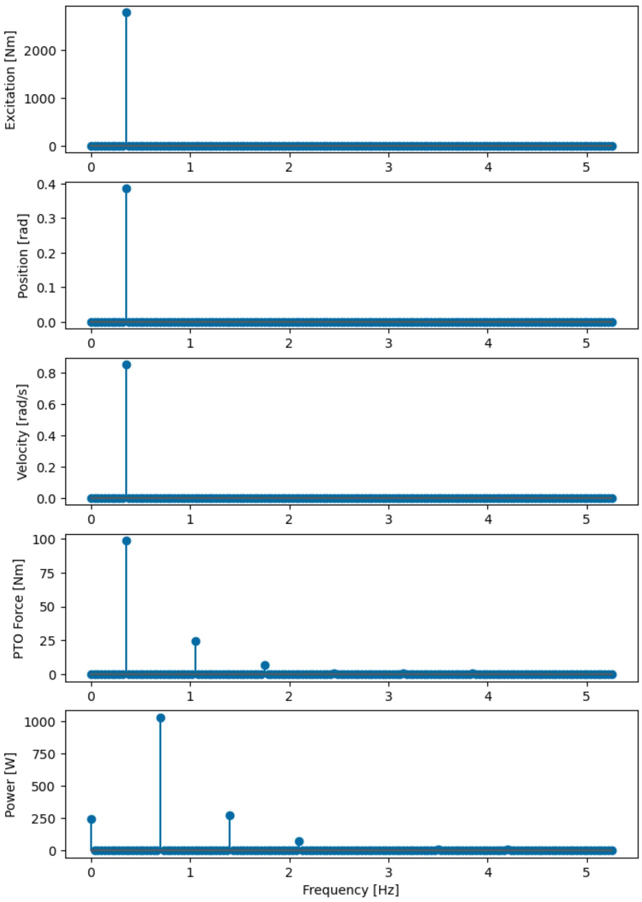

Fig. 2. Pioneer response spectrum for nonlinear (constrained PTO force) case

Irregular waves also require careful consideration to create an accurate representation of the wave spectrum. An irregular wave can be defined in terms of its wave elevation spectrum, which is a complex value representing the wave elevation at each frequency. The magnitude of the complex value corresponds to the magnitude of wave elevation. In an irregular wave, the excitation is spread out across the range of frequency values. When using the fundamental frequency (f1) as the frequency spacing, this means that the fundamental frequency needs to be small enough to create a smooth discretization of the wave spectrum. If the frequency spacing is not fine enough, the values of the wave elevation will not be a smooth representation of the wave spectrum, as shown in Fig. 3. With f1 equal to 0.1 Hz, the wave spectrum is very jagged and not well represented between the frequencies in the Array, but with f1 equal to 0.001 Hz, the resultant wave spectrum is much smoother and very well represented.

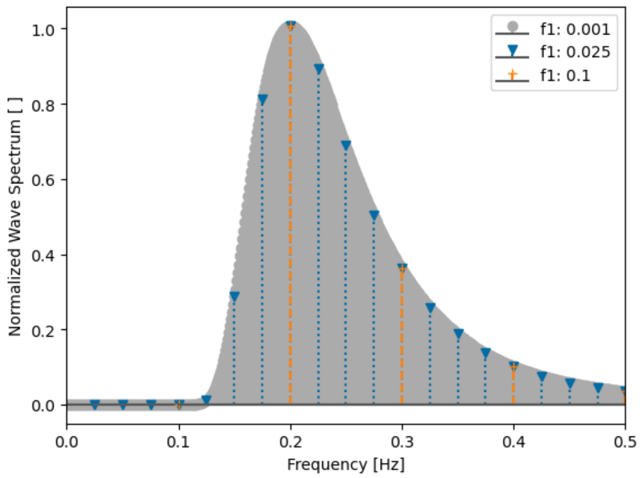

Fig. 3. JONSWAP spectrum with various fundamental frequencies

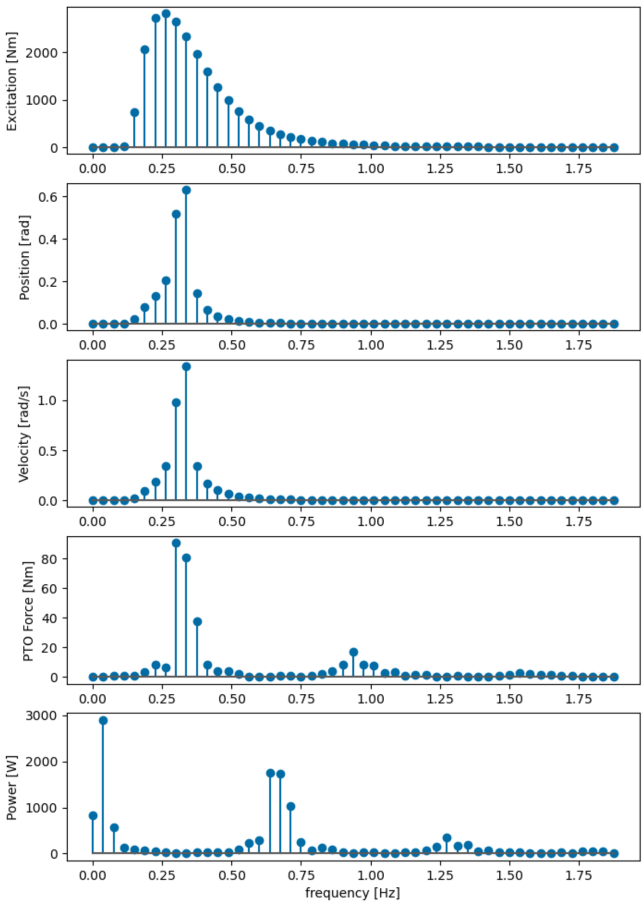

Fig. 4. Pioneer response spectrum for nonlinear (constrained PTO force) case in irregular waves

Generally, it is found that a fundamental frequency of about 1/10th of the peak wave frequency is able to produce a smooth, representative result, but exact values necessary may depend on the application. One way to check this is to compute the numerical integral of the wave elevation components and compare to the analytical integral of the wave spectrum equation to ensure the desired accuracy is achieved.

On the larger end of the frequency array, the first requirement is again that the wave spectrum is fully included. This is usually accomplished by including frequencies up to 3 times the wave frequency. In the case suggested above, this would require at least 30 frequencies. If the system dynamics are fully linear, ensuring the wave spectrum is included in the frequency array also leads to the WEC response being fully realized. On the other hand, for nonlinear cases the harmonic responses also need to be included. This is shown by the spectrum plot of the WEC response dynamics in Fig. 4 with the PTO force components lining up with both the wave spectrum (~0.325 Hz) and the odd harmonics (~0.975 and 1.625 Hz). It is also worth noting that, similar to for the regular wave, the power components occur at the even harmonics (~0, 0.65, and 1.3 Hz).

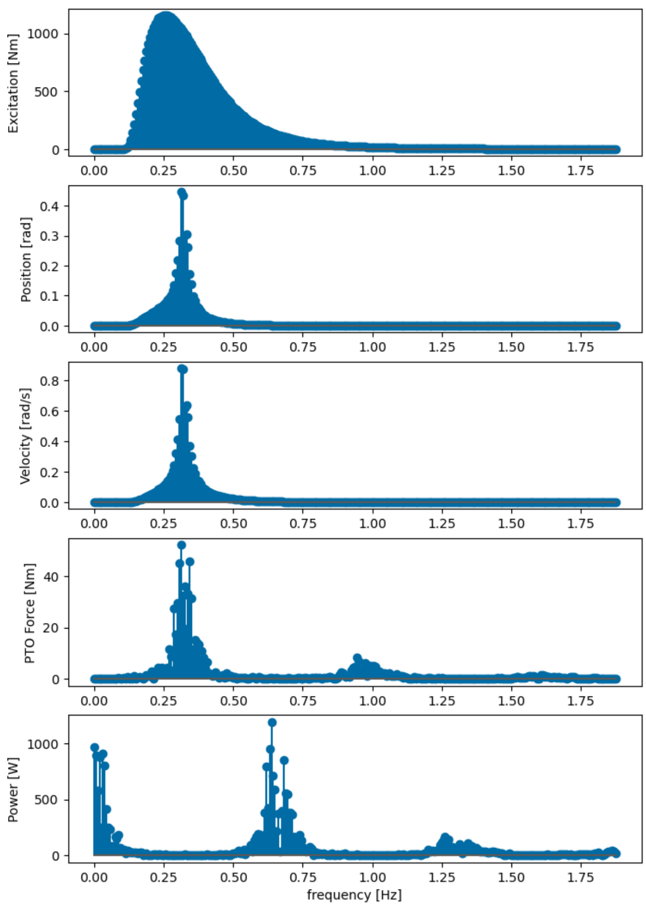

Fig. 5. Pioneer response spectrum for nonlinear (constrained PTO force) case in irregular waves with a refined frequency array

B. *Number of Frequencies*

The number of frequencies in the array impacts the accuracy of the result. Not only is it important to include enough frequencies to represent the wave spectrum, but it's also important to ensure the WEC response is accurately represented. This is particularly important for narrow-banded responses. The Pioneer WEC is a good example of a device with a narrow-banded response. With a low number of frequencies (Fig. 4), the WEC response is only represented at a few frequencies and is not fully represented by the calculated hydrodynamics. On the other hand, a larger number of frequencies (Fig. 5) allows for a smoother, finely discretized representation of the dynamics with a well-captured steep peak response.

Refining the frequency array makes a very significant difference in the resultant response and power output. The overall result of increasing the number of frequencies is shown in Fig. 6. The figure demonstrates a remarkable (about 20%) improvement in the resultant average power when increasing from an array of 50 to 300 frequencies. As the number of frequencies in the array increases, a more accurate representation of the system response is captured, but the computation time also increases significantly, as shown by Fig. 7. A proper evaluation balances the desired accuracy with computation time to select a reasonable number of frequencies. For the purposes of this study, 150 frequencies met a goal of about 2% error and still just took a few minutes to optimize.

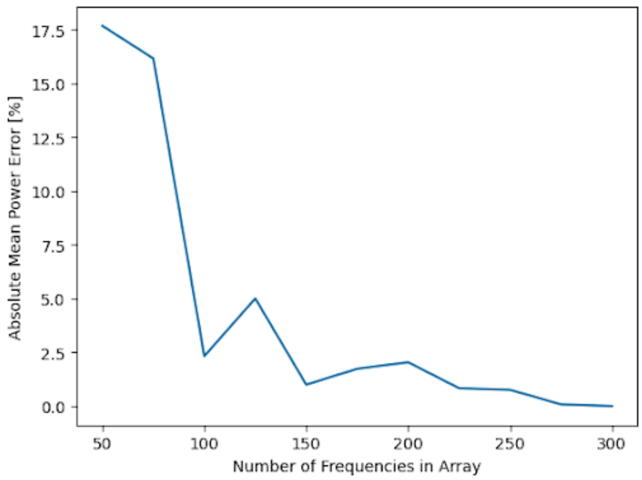

Fig. 6. Effect of increasing the number of frequencies on the error of the resultant average power

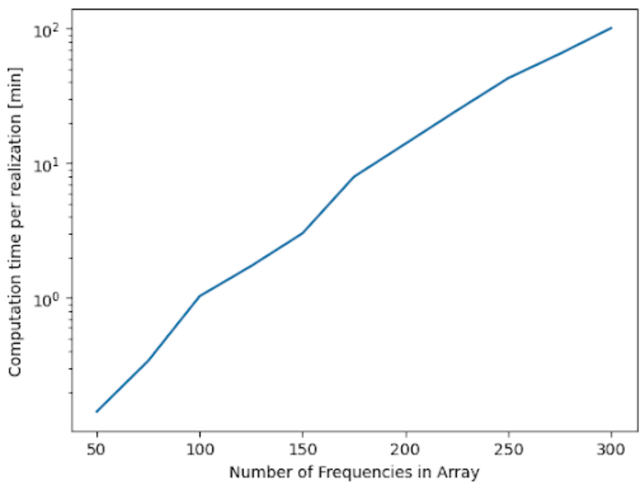

Fig. 7. Effect of increasing the number of frequencies on the optimization computation time

[2]*WecOptTool*: https://github.com/sandialabs/WecOptTool

C. Phase Realizations

For an irregular wave, a unique phase is applied to the excitation at each frequency in the array. Generally, random phase realizations are utilized, leading to varying resultant time-domain wave elevations as shown in Fig. 8. When applying a constraint, the unique amplitudes of each realization mean a different required control force and, thus, a unique impact of the constraint. In a time-domain simulation, running the solution for a longer period of time would resolve any discrepancies between phase realizations, but this is not possible with the pseudo-spectral method. Instead, multiple random phase realizations should be run, and the results averaged in order to get a precise result.

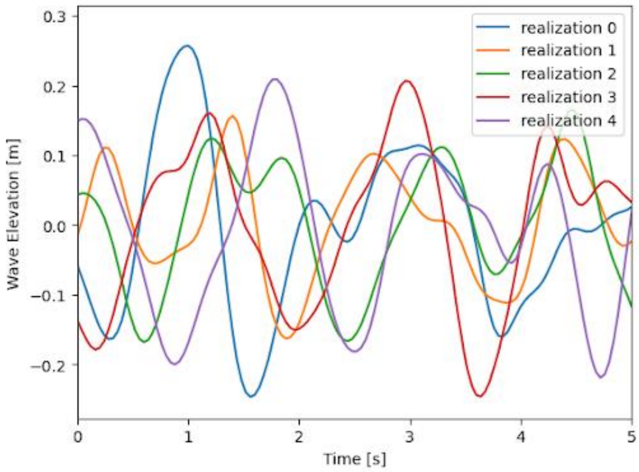

Fig. 8. Comparison of the time-domain wave elevation for five different phase realizations

The number of phase realizations effectively controls the precision of the result. As shown in Fig. 9, as the number of realizations increases, the resultant power gradually converges to a steady state value. Increasing the number `for a longer period of time in that it effectively increases the total simulation time (top axis of Fig. 9) For this example, just using one realization could lead to an error in the average electrical power of up to 4%, but this can depend on multiple variables. The number of realizations required to converge to a stable solution depends on a number of factors including the system dynamics, frequency array, and desired precision.

Some other factors being studied are the distribution of the realization results and the nuanced relationship with the frequency array, simulation time, and computation time. By increasing the number of frequencies in the frequency array, it is expected that the result will converge with less realizations, but the exact correlation is still being investigated.

IV. Results

To understand the impact of the frequency array in terms of a design optimization study, the torsional spring stiffness was varied from 780 to 850 Nm/rad. First, this was completed with a frequency array containing 50 frequencies (same array as Fig. 4) with the electrical power results shown in Fig. 10. In this example, the optimal spring stiffness is 810 Nm/rad which leads to an electrical power of almost 1.7 kW. On the other hand, when using a frequency array containing 150 frequencies (Fig. 11), the optimal spring stiffness is also 810 Nm/rad but leads to an electrical power of about 1.1 kW.

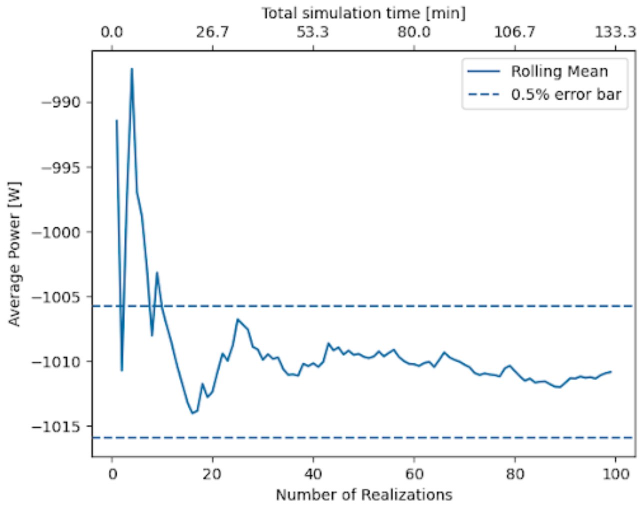

Fig. 9. Rolling average power vs. the number of realizations and corresponding simulation time

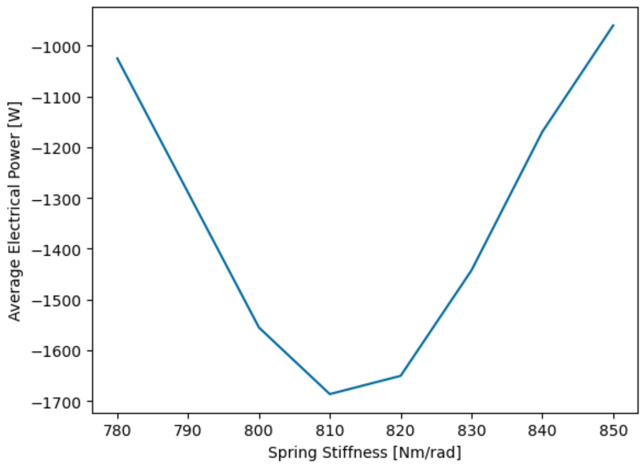

Fig. 10: Average electrical power vs. spring stiffness based on a frequency array with 50 components

Despite the same optimal stiffness value, the optimized electrical power is (inaccurately) significantly larger when completing the analysis using only 50 frequencies. Any design decisions made based on the 50-frequency study would lead to extreme overestimations of the required component parameters (e.g., rated generator power). The analysis completed in Section III supports a proper determination of the frequency array needed. Thus, the 150-frequency study leads to much more accurate electrical power results which can be used to make detailed design decisions with confidence.

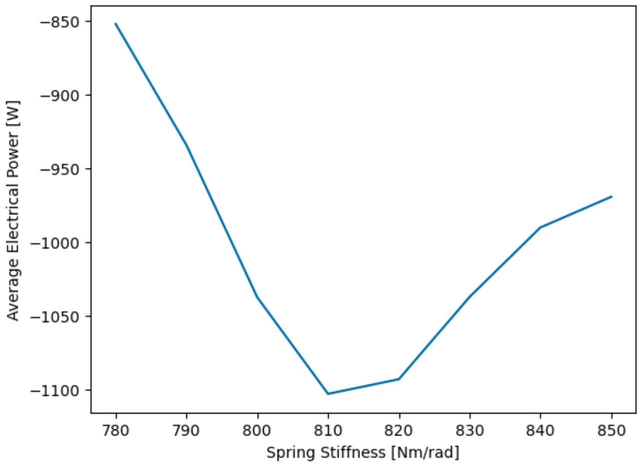

Fig. 11: Average electrical power vs. spring stiffness based on a frequency array with 150 components

V. Conclusion

The understanding and selection of the frequency array for any frequency-domain evaluation is paramount. An uninformed array selection can lead to significant errors in the resultant solutions. In particular, the range of the array and number of components can impact accuracy. Moreover, and specific to irregular wave conditions, the phase realization also impacts the resultant solution. This study exemplifies the impacts of the various aspects of frequency array selection and phase realizations and suggests the potential to avoid relatively significant errors by completing convergence studies to guide proper selection.

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