Towards wave-to-wire optimization enabled by differentiable BEM

Maha N. Haji
mhaji@umich.edu
Department of Mechanical Engineering
University of Michigan

January 23, 2026
Maynooth Workshop 2026

MECHANICAL ENGINEERING
UNIVERSITY OF MICHIGAN

SEA Lab
Symbiotic Engineering and Analysis Lab

Work featured in this presentation by

PhD graduate: Dr. Kapil Khanal

Systems Engineering, 2025

PhD candidate: Rebecca McCabe

5th year, Mechanical Engineering

PhD candidate: Collin Treacy

2nd year, Mechanical Engineering

Funding provided by:

Outline

- Need for multidisciplinary design optimization (MDO) in wave energy
- Differentiable BEM via MarineHydro.jl
- Towards differentiable wave-to-wire
- Scaling up differentiability to wave energy converter farms
- Conclusions

SEA Lab
Symbiotic Engineering and Analysis Lab

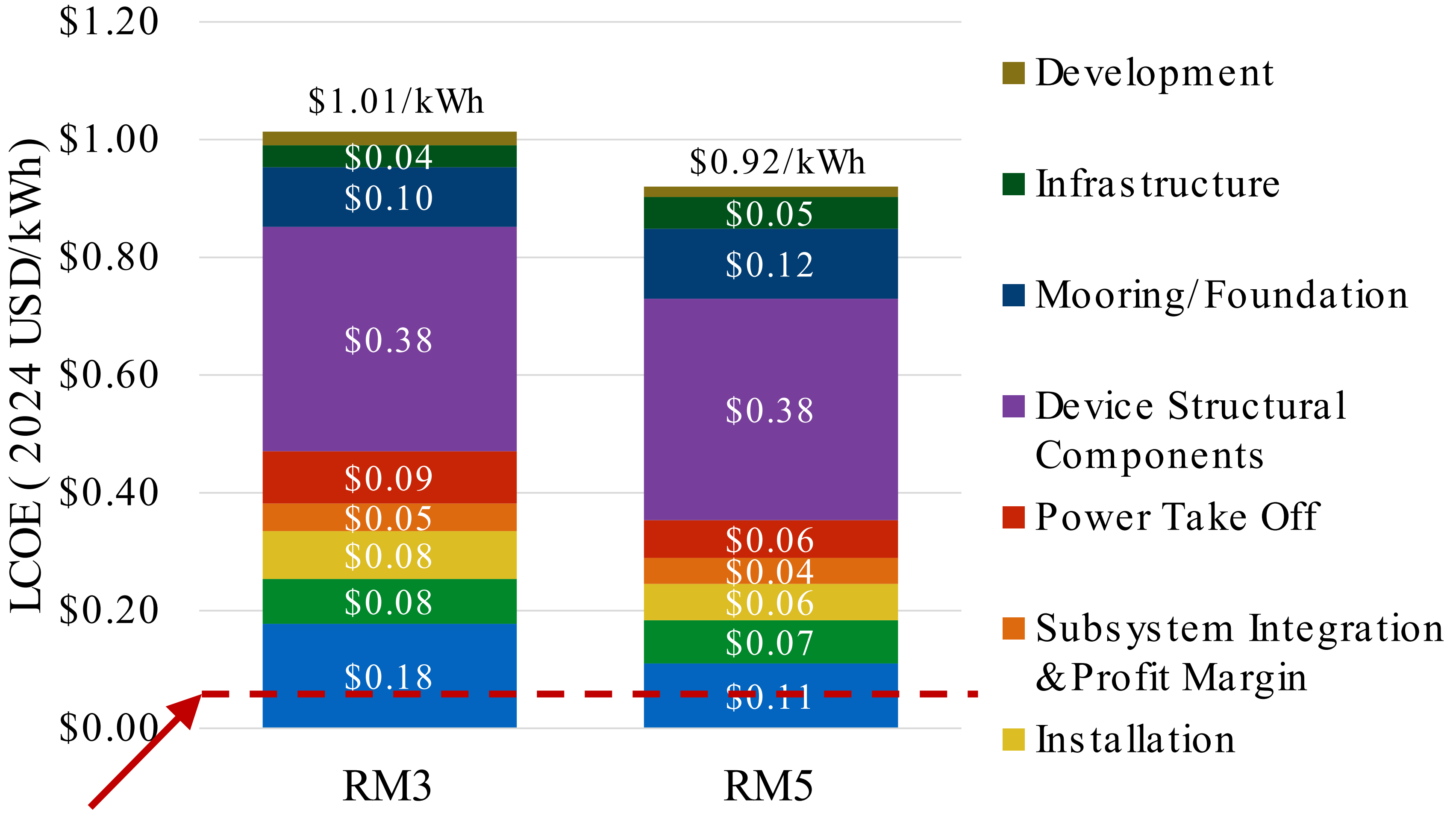
Wave energy challenges

- High levelized cost of energy (LCOE): ~$1/kWh
  - compared to wind ($0.039/kWh)
  - and solar ($0.040/kWh)

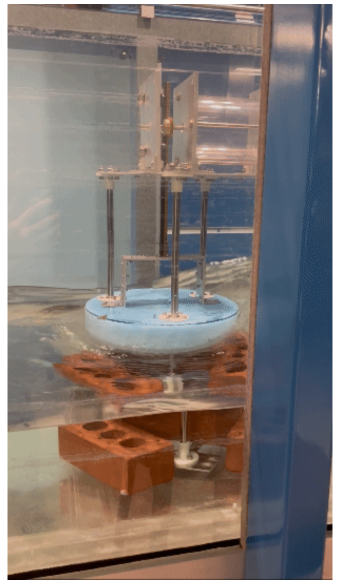
Cost reductions needed

- Complex multidisciplinary design: structures, controls, hydrodynamics, electronics, etc.

Traditional sequential design is limiting



Wind &solar



SEA Lab
Symbiotic Engineering and Analysis Lab

Traditional sequential WEC design

Traditional sequential WEC design has numerous disadvantages:

- Suboptimal solutions because it ignores interactions between disciplines.
- Results depend heavily on the optimization sequence.
- Adjustments in one discipline may require re-optimization of previous steps.

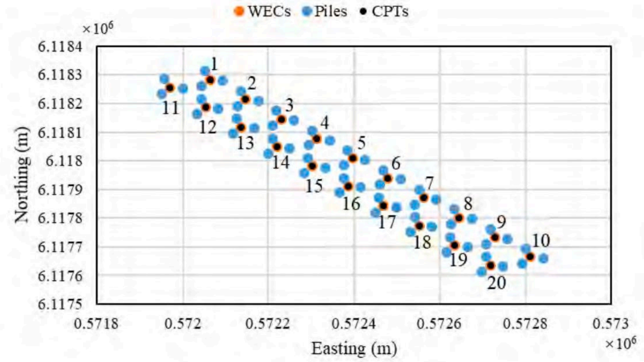
Multidisciplinary design optimization (MDO)

MDO can address these challenges.

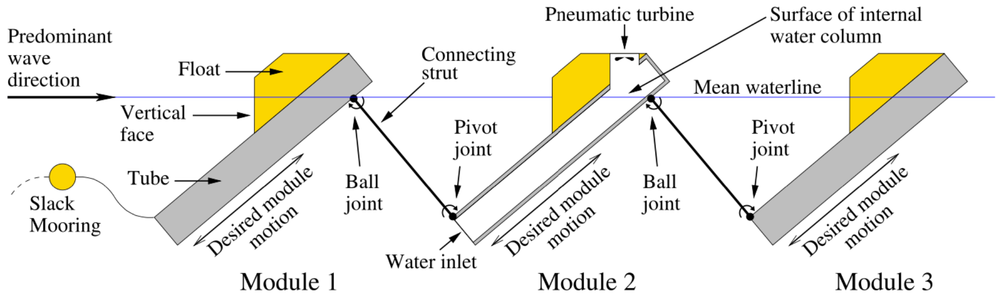
- Captures interactions between disciplines leading to globally optimal solutions.
- Simultaneous optimization reduces the need for iterative adjustments.

Existing approaches in WEC optimization

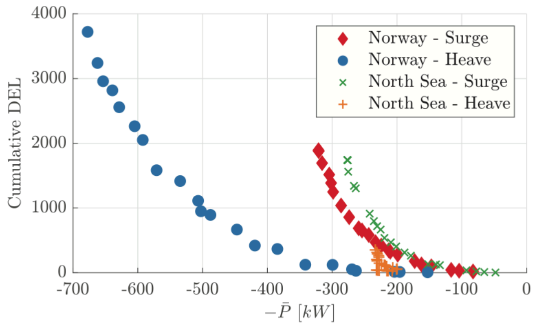
- Existing WEC optimization studies primarily focus on specific subsystems.
- MDO is widely used in aerospace and automotive industries and is emerging in offshore wind design.
- No studies explicitly consider MDO in WEC design, but a few could be considered to apply MDO in practice:



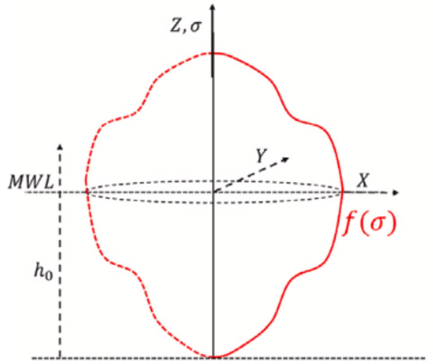
Optimized array layout to maximize power generation and minimize transmission cable length (Gaudin et al., 2021)



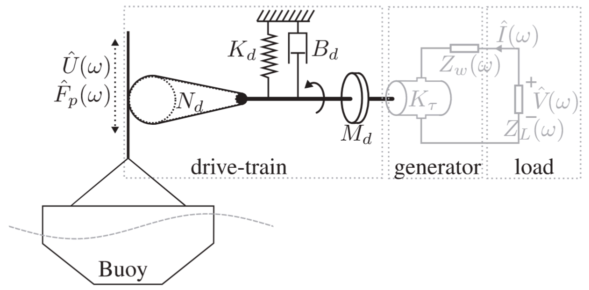
Optimized device geometry to maximize power generation and minimize joint fatigue damage (Cotton and Forehand, 2022)



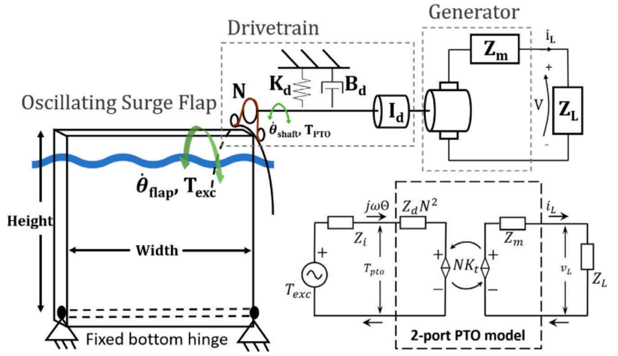
Optimized geometry and location to balance power production and structural reliability (Garcia-Teruel and Clark, 2021)



Co-optimized hull geometry and controls to maximize power (Abdulkadir and Abdelkhalik, 2024)



Co-optimized PTO and controls to maximize electric power (Ströfer et al., 2023)



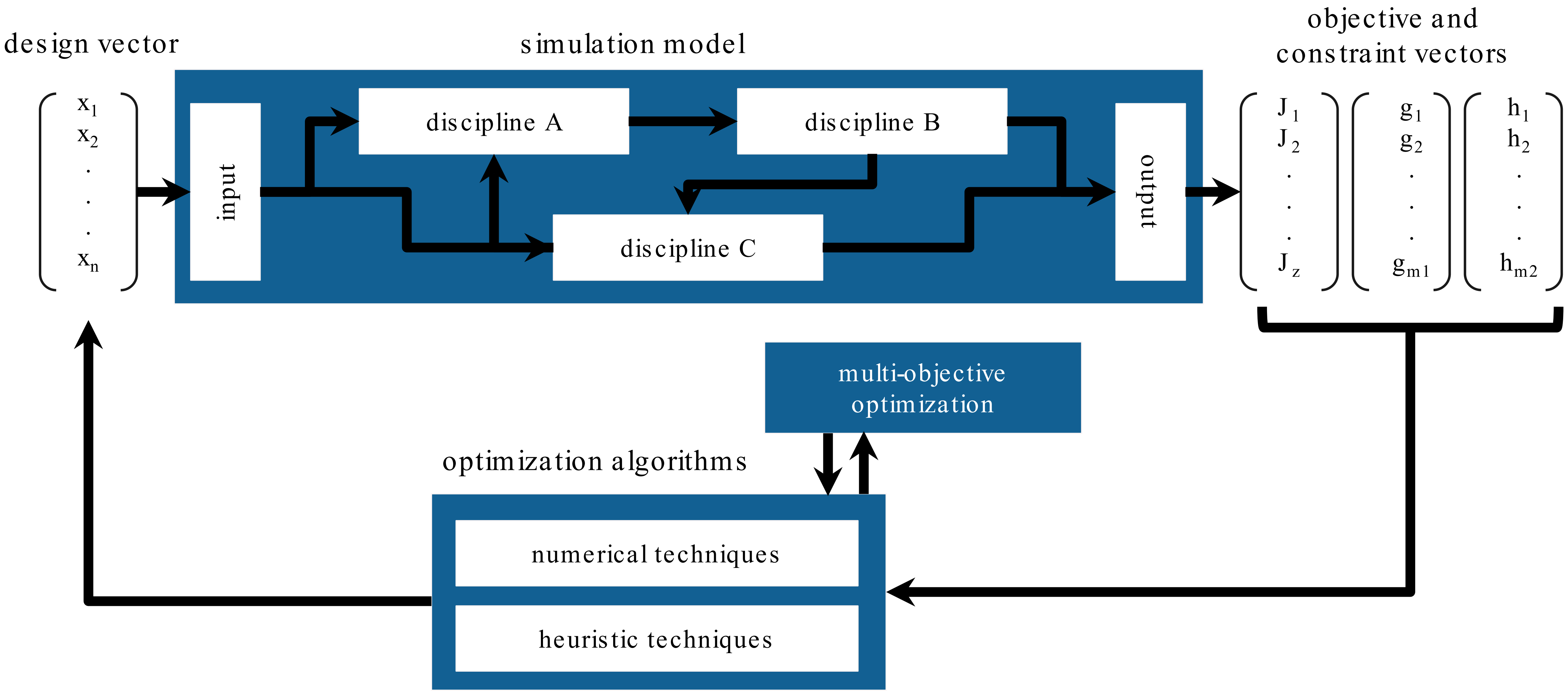
Co-optimized flap geometry, PTO, and controls to maximize electric power (Grasberger et al., 2024)

All used non-differentiable computationally intensive numerical models, and parameters sweeps or gradient-free heuristic optimization methods, limiting rapid design iteration.

SEA Lab
Symbiotic Engineering and Analysis Lab

Hydrodynamic modeling is the bottleneck!

- BEM is computationally costly for large-scale problems.
- Design optimization requires running simulations hundreds of times.
- Gradient-based optimization methods are often the fastest because they scale better with design variables.
- Current BEM solvers are not analytically differentiable, often resulting in the use of gradient-free or heuristic optimization methods which are also computationally intensive.



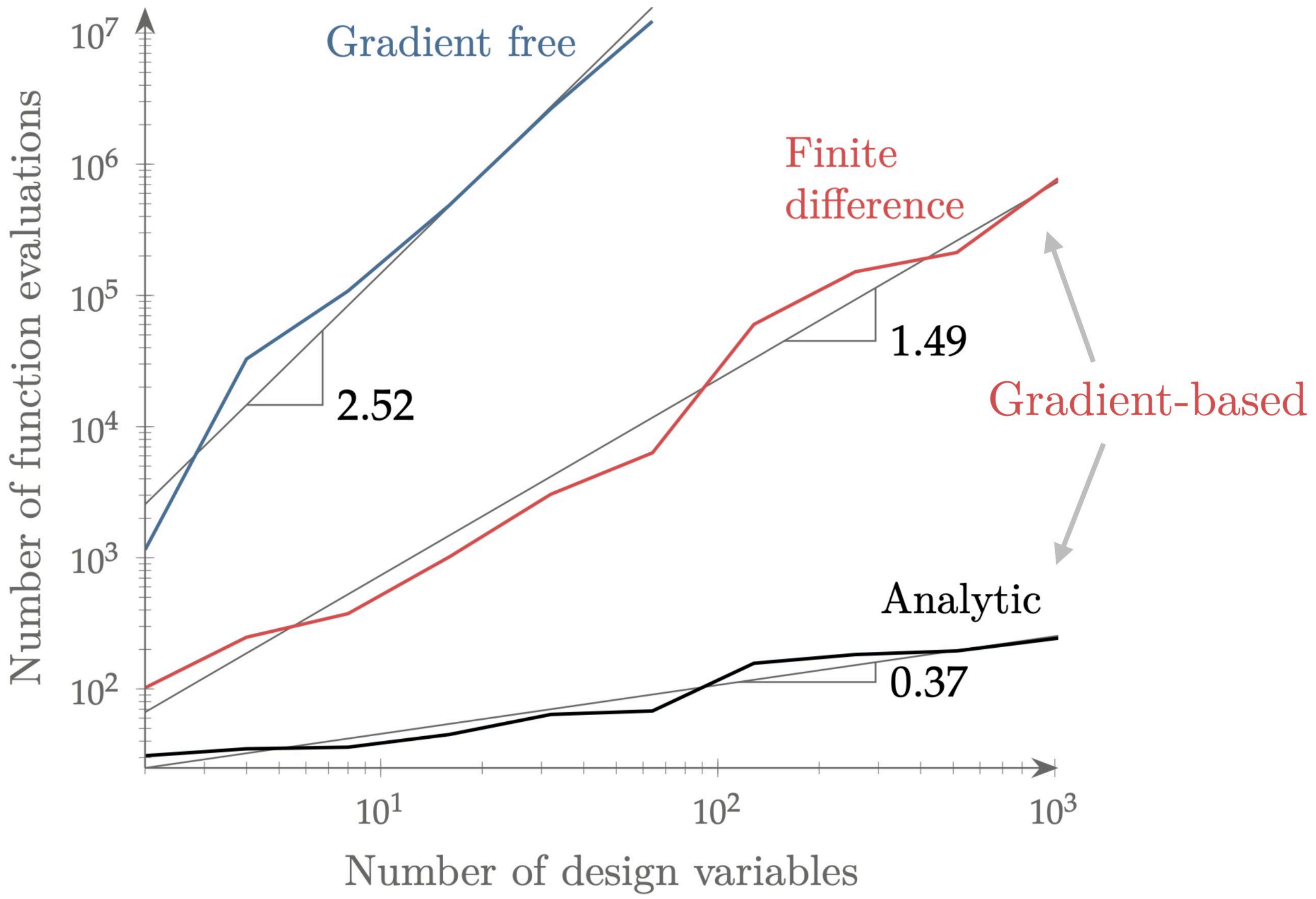
SEA Lab
Symbiotic Engineering and Analysis Lab

Differentiable programming with current BEM solvers

- Finite differences can suffer from inefficiencies and truncation errors.
- This method approximates the derivative as:

$$\frac{\partial \Phi}{\partial x} \approx \frac{\Phi(x+h) - \Phi(x)}{h}$$

  where h represents a small perturbation in each input variable, x.
- Each input perturbation requires two BEM solves to compute $\Phi(x + h)$ and $\Phi(x)$ and repeated for every input variable → high computational cost.
- In BEM solvers, finite differences are computationally expensive due to:
  - Evaluating the Green's function and resulting complex valued dense matrices.
  - Solving the discretized integral equation for each perturbation.

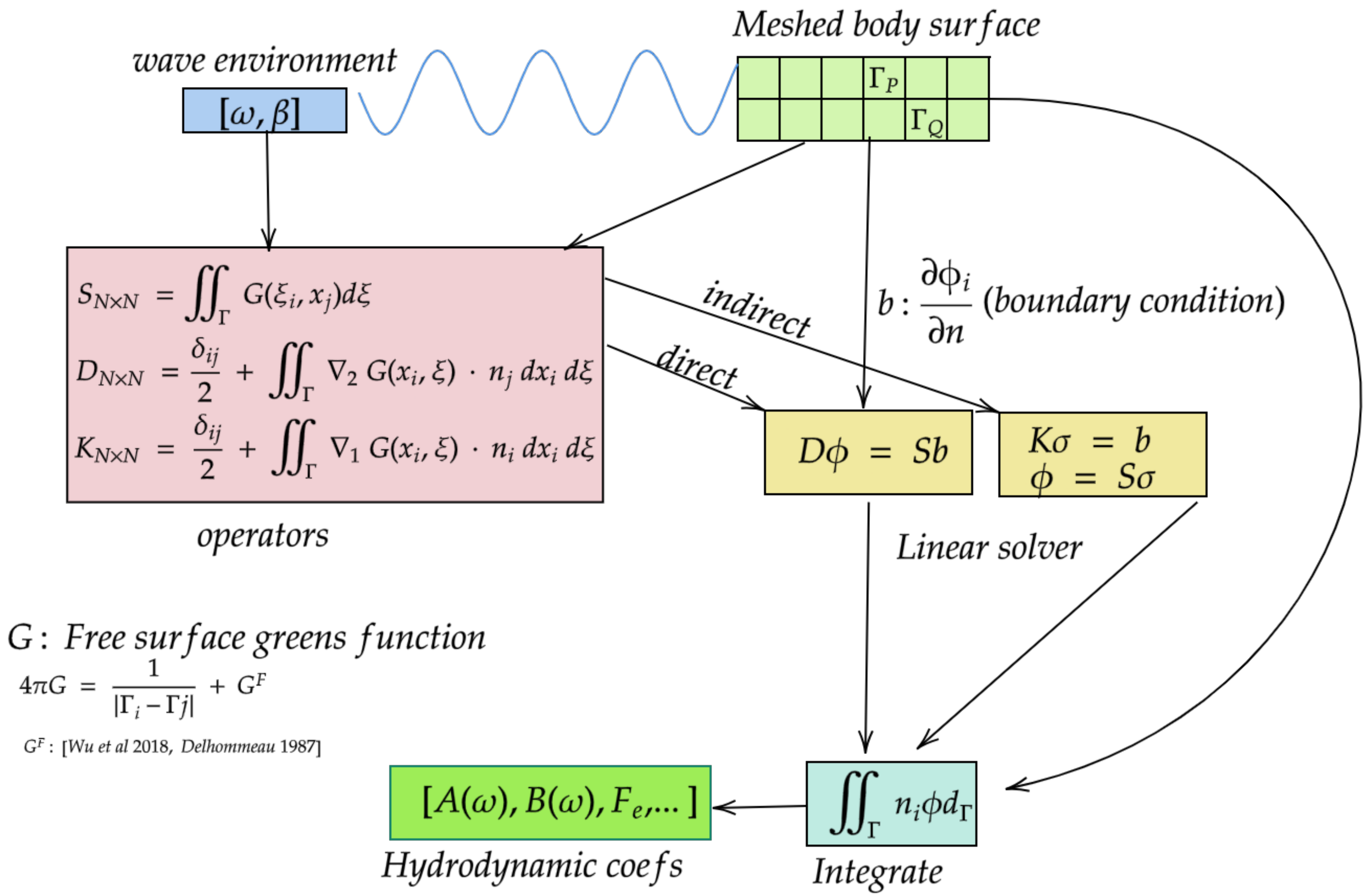


(Martins and Ning, 2022)

SEA Lab
Symbiotic Engineering and Analysis Lab

Introducing MarineHydro.jl

- Differentiable BEM solver.
- Implements both direct and indirect BEM formulations.
- Incorporates exact and surrogate Green's function expressions.
- 100% Julia (core hydrodynamics).
- ∂ifferentiable code - implements adjoint with automatic differentiation.



K. Khanal, C. Michélen Ströfer, M. Ancellin, and M. N. Haji, "Derivation of Discrete Adjoint Method for Sensitivity Analysis in Marine Energy Systems" Third Annual UMERC, Duluth, MN, August 7-9, 2024.

K. Khanal, C. Michélen Ströfer, M. Ancellin, and M. N. Haji, "Fully Differentiable Boundary Element Solver for Hydrodynamic Sensitivity Analysis of Wave-Structure Interactions" AOR, 163, 104707, 2025, https://doi.org/10.1016/j.apor.2025.104707.

SEA Lab
Symbiotic Engineering and Analysis Lab

Making of a Differentiable Solver

Minimization with Implicit State

- We aim to minimize a scalar objective: $J(\phi(\theta), \theta)$

- subject to the constraint: $D(\theta)\phi(\theta) - S(\theta)b(\phi) = 0$

  - D, S are n × n complex-valued dense matrices for n number of panels.
  - ϕ : vector of potentials on each panel.
  - θ: design variable(s).
  - J : objective depending on ϕ and θ.

- To compute the total derivative of J w.r.t. θ: $$\frac{dJ}{d\theta} = \frac{\partial J}{\partial \theta} + \frac{\partial J}{\partial \phi}\frac{\partial \phi}{\partial \theta}$$

- The challenge is computing $\frac{\partial \phi}{\partial \theta}$ since ϕ is only defined implicitly via the constraint.

SEA Lab
Symbiotic Engineering and Analysis Lab

Differentiation of the constraint

- Differentiate the constraint applying the product rule: $\frac{\partial D}{\partial \theta}\phi + D\frac{\partial \phi}{\partial \theta} = \frac{\partial S}{\partial \theta}b + S\frac{\partial b}{\partial \theta}$
- Solve for $\frac{\partial \phi}{\partial \theta}$: $\frac{\partial \phi}{\partial \theta} = D^{-1}\left(S\frac{\partial b}{\partial \theta} + \frac{\partial S}{\partial \theta}b - \frac{\partial D}{\partial \theta}\phi\right)$
- ⚠ But, finite difference approximation of $\frac{\partial \phi}{\partial \theta}$ is computationally expensive as it requires one full $\phi(\theta)$ solve per component of θ, the input!
- Instead define the adjoint variable λ as: $\lambda^T D = \frac{\partial J}{\partial \phi}$
- Combined with previous equation: $\left(\frac{\partial J}{\partial \phi}\right)\frac{\partial \phi}{\partial \theta} = \lambda^T\left(S\frac{\partial b}{\partial \theta} + \frac{\partial S}{\partial \theta}b - \frac{\partial D}{\partial \theta}\phi\right)$
- Leads to adjoint-based total derivative of J: $\frac{dJ}{d\theta} = \frac{\partial J}{\partial \theta} + \lambda^T\left(S\frac{\partial b}{\partial \theta} + \frac{\partial S}{\partial \theta}b - \frac{\partial D}{\partial \theta}\phi\right)$

Computational efficiency

- This avoids computing $\frac{\partial \phi}{\partial \theta}$ explicitly.
- Enables efficient gradient evaluation for large systems.
- Limits the number of linear solves to just two, whether problem has m = 10 or 10^9 design variables → scales O(1) with respect to design variables!
- Computational cost reduces from m × $O(n^3)$ → 2 ×$O(n^3)$, where n is the matrix size and m the number of design variables.
- Using a factorization-based solver (e.g., LU decomposition), the factorization A= LU can be reused for both solves, reducing complexity to $O(n^3)$.
- The adjoint matrix is the transpose of the forward matrix (D).

SEA Lab
Symbiotic Engineering and Analysis Lab

Towards fully differentiable wave-to-wire models

Wave Model

Wire Model

- High dimensional problem (R^n) requiring gradient-based methods for optimization of the entire system.

  → Requires end-to-end differentiability.

- More efficient gradient-based methods would enable use of high-fidelity solvers and full wave-to-wire models without simplifications.
- The nested optimization model is the most intuitive framework for this approach (Herber and Allison, 2019).

Nested formulation

Outer Objective (System-Level Objective):

WEC design optimization to maximize power

Nested wire model:

Control optimization to maximize power

SEA Lab
Symbiotic Engineering and Analysis Lab

Control co-design framework

Wave model: MarineHydro.jl

- Hydrodynamic forces (added mass, damping, excitation) are computed via a MarineHydro.jl
- Differentiability achieved using:
  - Forward-mode differentiation (when inputs are few, outputs are many).
  - Adjoint methods (semi-analytic adjoint).
- Outputs: Hydrodynamic sensitivities (Jacobian of hydrodynamic coefficients with respect to design parameters).

Wire model: WecOptTool

- Compute the optimal PTO force trajectory that maximizes total energy absorbed by the WEC over a time horizon using a pseudo-spectral method for trajectory optimization.
- Leveraging WecOptTool developed by Sandia National Laboratories.
- Optimal Control Problem (OCP) Transcribed to Nonlinear Programming (NLP) Problem:

Average electrical power delivered to the grid.

Governing equations of WEC dynamics.

Path constraints (e.g., maximum PTO force).

- Differentiating the wire model is differentiation of the optimization problem!

Differentiation of the wire model

- Differentiation through OCP of wire model:
  - Use Karush-Kuhn-Tucker (KKT) conditions to differentiate the inner optimization.
  - Post-optimality sensitivities are extracted from Lagrange multipliers.
  - This avoids brute-force finite differencing of the entire OCP.

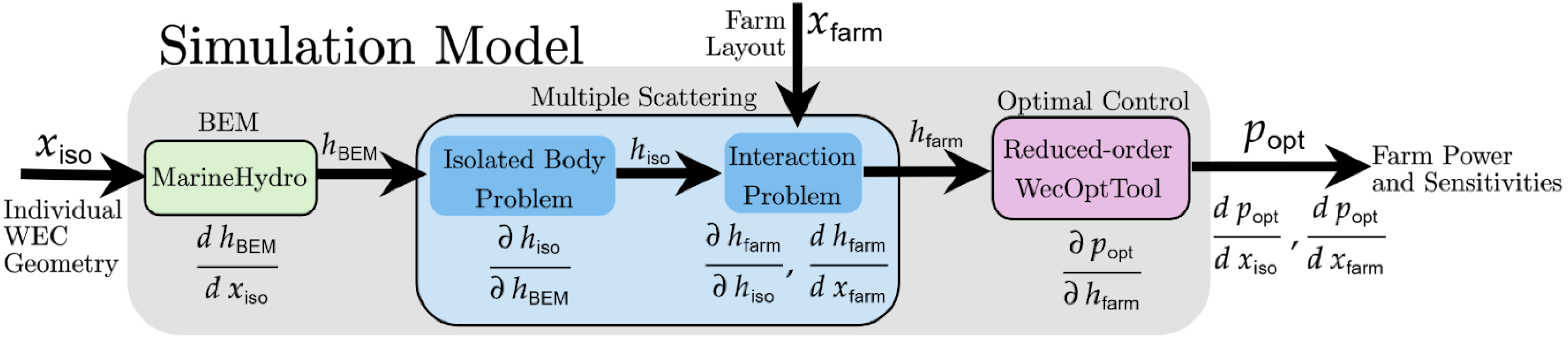
- Coupled Differentiation (Wave + Wire)
  - Sensitivities are coupled using the Unified Derivative Equations (UDE) framework in OpenMDAO.
  - UDE allows mixing adjoint, forward-mode, analytical, and finite difference derivatives for efficiency.
  - Total derivatives of system-level objectives (e.g., average power) are assembled systematically.

- Simplifications and Efficiency Gains
  - Reduces sensitivity calculations by evaluating derivatives of the Lagrangian at the optimum via Sobieski's method (Sobieszczanski-Sobieski et al., 2012), avoiding second-order derivatives.
  - Inequality constraints are enforced softly via penalties, keeping the optimization differentiable.
  - Chain rule assembly ensures scalability when combining derivatives from heterogeneous solvers.

K. Khanal, and M. N. Haji, "Differentiable Wave-to-Wire Model for Multidisciplinary Design Optimization of Wave Energy Converters" ASME IDETC-CIE, Anaheim, CA, August 17-20, 2025.

Scaling up for wave farm optimization?

- The computation cost of BEM and optimal control models (WecOptTool) scales rapidly with the # of WECs in a farm.
- Current research: either (1) limits # of WECs modeled, or (2) neglect hydrodynamic interactions altogether.
- Multiple scattering uses properties of individual devices and interaction theory to compute the hydrodynamic forces for the farm.
- Can we develop a differentiable version of multiple scattering method for efficient and accurate WEC farm modeling and optimization?
- Similarly, can we develop a reduced order model of WecOptTool via a modal decomposition to reduce computation time?

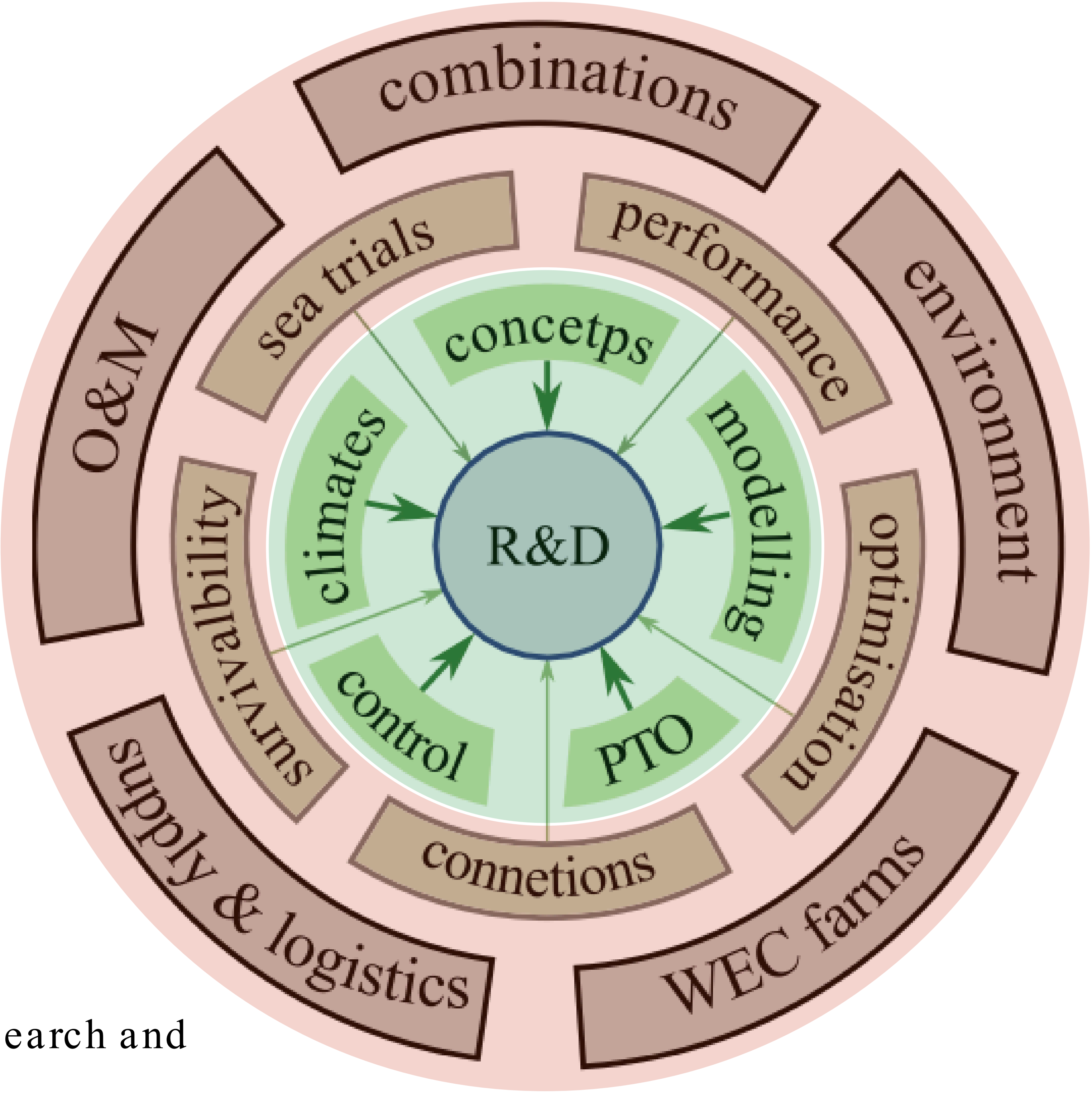


SEA Lab
Symbiotic Engineering and Analysis Lab

Goal is to drive advances in WEC R&D

- Current R&D focus: wave resource assessment, WEC concept developing, hydrodynamic modelling, PTO innovation and control design.
- Topics in the inner ring are well-studied.
- The R&D topics in the middle and outer rings are not yet fully understood.

MDO and CCD can help analyze topics in middle and outer rings and optimize designs.



Guo and Ringwood, "A review of wave energy technology from a research and commercial perspective," IET Renewable Power Generation, 2021

SEA Lab
Symbiotic Engineering and Analysis Lab

Thank you! Questions? Comments?

Maha N. Haji, mhaji@umich.edu

Symbiotic Engineering and Analysis Lab
Department of Mechanical Engineering
University of Michigan

All code available online at https://github.com/symbiotic-engineering

- K. Khanal, C. Michélen Ströfer, M. Ancellin, and M. N. Haji, "Derivation of Discrete Adjoint Method for Sensitivity Analysis in Marine Energy Systems" 3rd Annual University Marine Energy Research Community Conference, Duluth, MN, August 7-9, 2024.
- K. Khanal, C. Michélen Ströfer, M. Ancellin, and M. N. Haji, "Fully Differentiable Boundary Element Solver for Hydrodynamic Sensitivity Analysis of Wave-Structure Interactions" Applied Ocean Research, 163, 104707, 2025, https://doi.org/10.1016/j.apor.2025.104707.
- K. Khanal, and M. N. Haji, "Differentiable Wave-to-Wire Model for Multidisciplinary Design Optimization of Wave Energy Converters" ASME IDETC-CIE, Anaheim, CA, August 17-20, 2025.

MECHANICAL ENGINEERING
UNIVERSITY OF MICHIGAN

SEA Lab
Symbiotic Engineering and Analysis Lab