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Discrete Adjoint Method for Sensitivity Analysis in Marine Energy Systems

Kapil Khanal (a,b), Carlos A. Michelén Ströfer(b), Matthieu Ancellin(c), Maha Haji(a)

a. Cornell University, b. Sandia National Laboratory, c. Eurobios Mews Labs

Abstract

The optimal design of wave energy converters requires coupling of many different disciplines such as hydrodynamics, controls and geometry. Joint sensitivity calculation of all the disciplines is required for the gradient based optimization of the system objective. The adjoint method is the only viable method for calculating sensitivities with respect to a large number of input parameters at once as required in system design optimization problems. To this end, a discrete adjoint method is formulated, and the sensitivity of hydrodynamic coefficients is calculated for a point absorber system modeled as a sphere. The sensitivity are separated into real and imaginary sensitivities which one could interpret as the sensitivity for added mass and damping respectively. First, a differentiable boundary element method (BEM) based hydrodynamics code for fluid-structure interaction is developed in the Julia programming language. Automatic differentiation capabilities in Julia are then used to calculate the required partial derivatives for the adjoint equations. The accuracy of the automatic differentiation of Green's function and the resulting coefficients are compared with analytical derivation and the finite differences. The resulting sensitivities can be used in a large-scale gradient-based design optimization. The main contribution of this work is to formulate the discrete adjoint equations for the integral equations and modernize hydrodynamics BEM software to be able to provide necessary gradients. Thus, the method and formulation to get the sensitivities are discussed while the verification of the obtained gradients is an ongoing work.

Keywords: Sensitivity analysis; Automatic Differentiation, Discrete Adjoint Method, Julia

1. Introduction

The hydrodynamic coefficients , along with many other parameters are necessary for the calculation of electric power from a wave energy converter. A unified multidisciplinary approach to the design of WECs is necessary however this involves coupling the sensitivities of the electrical power with respect to all the design parameters. The change in shape/size or any other coupled analysis requires the hydrodynamic governing equation to be solved again using BEM for new state variables and the new objective evaluations. Design sensitivity analysis using BEM has a rich history in acoustics, electromagnetics [13] etc literature but the application to marine energy has been lacking. Although most of the theory of the integral equations are similar to other domains, the lack of adoption in hydrodynamics is most likely due to the mathematically complicated kernel function aka free surface Green's function[7]. Different complicated mathematical approximations of this Green's functions exist[10]. Approximation of the higher order derivative will have to be derived manually resulting in more mathematical complication. We use the recently derived global approximation for the Green's function as the kernel in the BIE [8]. This kernel is shown by the author to provide mostly accurate hydrodynamic coefficients for practical purposes. We use the automatic differentiation of this Green's function to estimate the gradient of added mass and damping with respect to a design parameter. Although accuracy decrease for the derivative of the Green's function is expected [5], we are interested in how much the gradients of the hydro-coefficients (added mass in this paper) themselves differ as they are of more practical use.

There are few ways to get the gradients of the hydro-coefficients discussed in structure, acoustics literatures [13] mainly; Finite difference method, Adjoint solver and the implicit differentiation of the integral equations with the design variables. In this paper, the derivatives of the BEM matrices are algorithmically computed with respect to the collocation points.One could also compute these derivatives with finite differences but the dense and complex matrices in BEM increases the cost and also since they are functions of wave frequency, extensive numerical study needs to be performed for each omega and mesh resolution. Additionally, we also explore the application of automatic differentiation in the BEM methods.

2. Methods

Existing open source BEM solvers [2] are already mature for hydrodynamic simulation and analysis purposes. These solvers are referred to as forward solvers. Most solvers are based on a reformulation of the linear boundary value problem for diffraction and radiation potentials . The associated Laplace equation, free- surface boundary condition, sommerfield radiation condition and kinematic boundary conditions is rewritten as boundary integral equations (BIEs) over the surface mesh of the floating body. These BIEs and the associated method (BEM) are well suited for wave problems [3] and solved via collocation points. This is usually solved by distributing the sources and dipoles over the surface of the body[4]. The BEM method from a shape optimization point of view, is very attractive as the mesh regeneration and the mesh change propagation across the domain (volumetric mesh) is not needed in contrast to the CFD based shape optimizations. Thus BEM is compatible for large scale simulation, sensitivity analysis and multidisciplinary optimization needed for complex marine energy systems. These systems level analysis require the availability of gradients from each of the subsystems. Unfortunately, existing hydrodynamic solvers do not provide the gradients right out of the box and have to rely on numerical differentiation. The errors and time complexity of numerical differentiation are difficult to ignore.

Additionally, only a few of these solvers are open source. In these solvers, performant code of the solvers and the user facing codes are separated inviting the complications for new users to contribute to the source codes and integrate the modern methods. Thus, a new solver that supports modern features such as the automatic differentiation, GPU kernels, with all the code in one language is created. Note that the geometry and meshing modules are not created yet. Traditionally, operator overloading can be used to create the differentiable code by employing AD tools such as tapenade etc however such an approach will be suboptimal as the implementation routines, parallelization, memory requirements etc of the forward solver significantly affects the efficiency of the adjoint solver [9] and it requires extensive familiarity with the language the solver is based on (Fortran). Additionally, with this new implementation, it's easier to extend the code to use the analytical gradients[5] to reduce the error, which magnifies with higher and higher order differentiation and for second order forces. As of now, no speed optimization or accuracy enhancement is done and only the framework and method of this new kind of solver is discussed.. The accuracy check for a hydro coefficient of simple hemisphere is shown below.

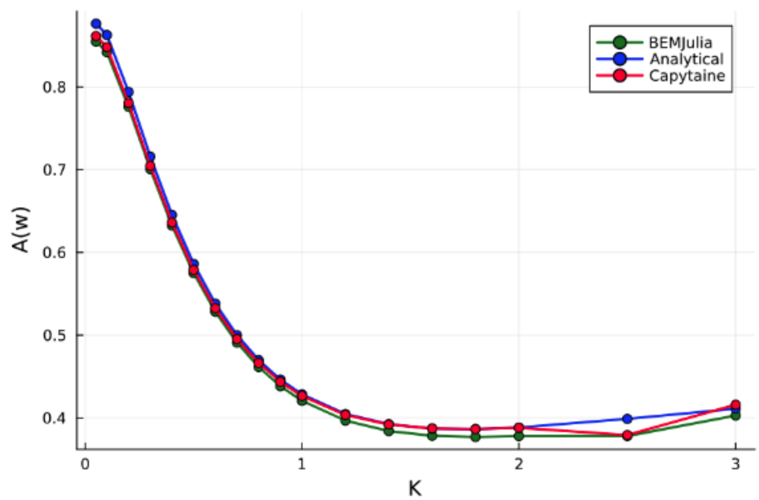


Figure 1: Comparison of added mass between BEMJulia, Capytaine and analytical results.

Discretized Boundary Integral Equation

The integral equation arises due to applying Green's theorem to calculate solution on the boundary/surface rather than the volume of the domain as the hydrodynamics domain (ocean) would be unbounded.

$$2\pi\phi(x) + \sum_{1}^{N} \int \phi(\xi; x) \frac{\delta G_2(\xi;x)}{\delta n_\xi} dA_\xi = \sum_{1}^{N} \int \frac{\delta\phi(\xi;x)}{\delta n_\xi} G_2(\xi; x) dA_\xi \quad (1)$$

The numerical integration in the integral equation can be separated

$$S_{ij} = \int\int_{A_j} G(x; \xi) dA_\xi \text{ and } D_{ij} = \frac{\delta_{ij}}{2} + \int\int_{A_j} \frac{G(x;\xi)}{\partial \xi} . n_\xi dA_\xi$$

The solution of the discrete integral equation is obtained via the resolution of the linear system (see equation 3). The green's function is split into $G = rankine + reflectedRankine + wave$. Each of these terms are added to satisfy the boundary conditions. These terms have different approximations and calculate individual influence coefficients. Once the rankine terms and the wave term for each panel are calculated they are to be integrated and added to get the final S and D (or K for indirect IE formulation) matrices. In the next section, we discuss how we calculate the sensitivity of the pressure on each collocation point given the normal derivative of the potential (normal velocity) is available for each of the panels.

Differentiation of the Integral Equation with respect to the collocation points

The differentiation of the boundary integral equation can be directly done with respect to the design variable (for example, shape parameter) or with respect to the collocation point. Both are immediately useful for shape design sensitivity analysis. The goal here is to get the gradients of the added mass (or any scalar objective like electrical power) with respect to the collocation and then to the vertices such that an external module can move the vertices to deform the boundary while keeping it suitable for the linear potential flow analysis. Depending on the size of the floating body and number of panels, the computational cost and the errors add up significantly. In contrast, differentiating with respect to the collocation point abstracts away the geometry generation and makes it more general to any kind of deformation and provides more control over the boundary of the shape. Naively differentiating the hydrodynamic solver results in many linear solves, as many as there are design variables. An adjoint variable method is adopted to reduce the number of the linear solves to only 2 independent of the number of the design variables[1] equivalent to solving another adjoint boundary value problem for one objective function. Thus even though for a slightly slow hydro-coefficient evaluation, the gain in speed from this solver is huge for the design optimization and sensitivity analysis. The computational cost reduction is from 'P' linear solves to 2 linear solves (*O(N^3)*), N being the dimension of the square asymmetric dense matrices and P being the number of design variables changing the surface mesh of the floating bodies. Automatic differentiation is usually designed for explicit operators such as Green's function evaluations. Since, we have to differentiate through the direct linear solver or sometimes through an iterative process (GMRES), the adjoint state method/reverse mode automatic differentiation needs to be formulated [11]. For general scalar objective $J(\phi, \theta)$ gradient with respect to parameters, the total jacobian using chain rule can be calculated as:

$$\frac{d(J)}{d\theta} = \frac{\partial(J)}{\partial\theta} + \frac{\partial(J)}{\partial\phi}\frac{\partial(\phi)}{\partial\theta} \quad (2)$$

Assume J is the added mass of the floating body. For the direct BEM following linear system has to be resolved.

$$D(\theta)\,\phi = S(\theta)\,b \quad (3)$$

where $D_{n\times n}$, $S_{n\times n}$ are asymmetric complex valued dense square matrices and are functions of collocation points. $\theta_{n\times 3}$ is the collocation points. $\phi_{n\times 3}$ is the potential and $b_{n\times 1}$ is the boundary condition at each panel and the n is the number of panels in the floating body. Applying a linear perturbation on both sides and expanding using forward mode differentiation for matrix multiplication,

$$\frac{d(J)}{d\theta} = \frac{\delta(J)}{\delta\theta} + \frac{\delta(J)}{\delta\phi}\left(D^{-1}\left(\frac{\partial b}{\partial\theta}S + b\frac{\partial S}{\partial\theta} - \phi\frac{\partial D}{\partial\theta}\right)\right)$$

With slight re-arrangement, we get a new adjoint linear system ; solvable by same routine as above

$$\lambda^T = \frac{\delta(J)}{\delta\phi}D^{-1} \quad (4)$$

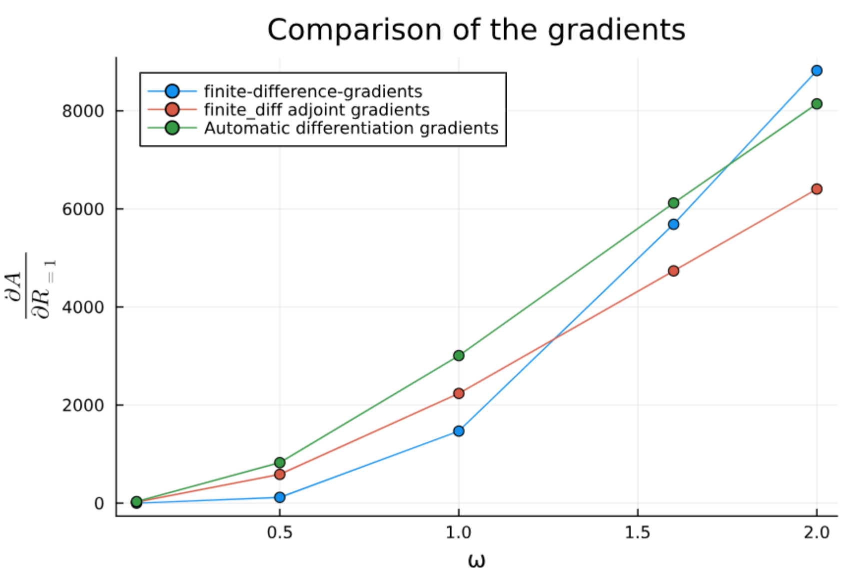
Solving this additional linear system and plugging in on the total derivative provides a gradient with respect to all the collocation points at once.

$$\frac{d(J)}{d\theta} = \frac{\delta(J)}{\delta\theta} + \lambda^T\left(\frac{\partial b}{\partial\theta}S + b\frac{\partial S}{\partial\theta} - \phi\frac{\partial D}{\partial\theta}\right)$$

Individual matrix derivatives can be obtained through automatic differentiation or finite difference. Here Julia's automatic differentiation capabilities via Zygote [12] are utilized to obtain

$$\frac{\partial D}{\partial\theta} = \frac{\partial}{\partial\theta}\int\int(G_2\hat{n})$$ and

$$\frac{\partial S}{\partial\theta} = \frac{\partial}{\partial\theta}\int\int(G_2)$$



The plot on the right compares the gradients of added mass with respect to the radius for a unit hemisphere with finite difference, Discrete adjoint via finite difference partials and automatic differentiation partials..

Three different implementations where , a) the gradients are via finite differences, second where only the matrix partials are obtained via finite-differences and the automatic differentiation based gradients.

The gradients in this current implementation **do have high error and seem inaccurate**. Some of the error is expected since the underlying green's function and its gradients are approximated[5]. Additionally, more error in low frequency(% wise) could be due to high oscillations and the mathematical singularity of the kernel function near zero. For low frequency the non-dimensionalized input to the function becomes close to zero and hence high error in the evaluation of the function and its gradients. Constant BEM panels are piecewise continuous functions and thus may cause some errors in the differentiation and we expect some errors due to the meshing resolution as well . Thus, a mesh convergence for the coefficient and the derivative may need to be performed. Although these gradients are inaccurate, they are still of practical use as the general trend and the comparable magnitude suggests that these inexact gradients are still valuable for optimizers and sensitivity analysis. For example, the panels of the sphere with high sensitivity values (absolute) can be deformed to minimize/ maximize the hydrodynamic coefficients. Thus, this solver and crude approximation of gradient is still useful for early stage parametric design/optimization studies and BEM error analysis. Note that the gradient computations are also very slow, probably due to the complicated computational graph obtained by the existing implementation of the algorithm, especially the rankine integration algorithms. To conclude, there is quite a bit of work remaining to improve the accuracy of the gradients as well as improving the forward BEM solver as well.

3. Future Work

The gradients obtained should be numerically verified against the analytical results. Since we suspect some errors on the gradients obtained, we plan to do more numerical/gradient calculation checks and error analysis of the results obtained. Another major aspect is to integrate analytical gradients when possible and make them compatible with chain rules in the Julia AD system. The gradients then will be used to couple multidisciplinary modules of a wave energy converter system and use gradient based optimization.

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Appendix A.

A.1. Accuracy of rankine only added mass obtained from AD.

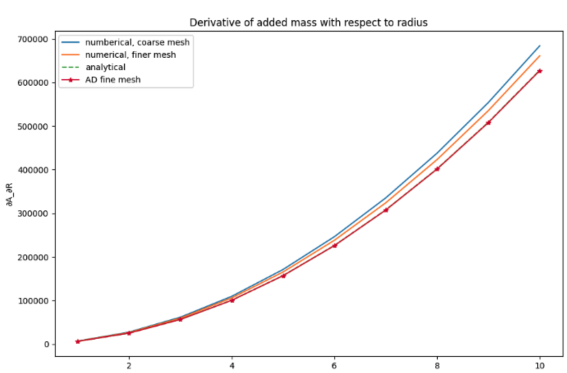

Figure 3: Analytical rankine added mass with numerical and automatic differentiation for a submerged sphere without a free surface

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