Differentiable Hydrodynamics

for Optimization and Sensitivity Analysis

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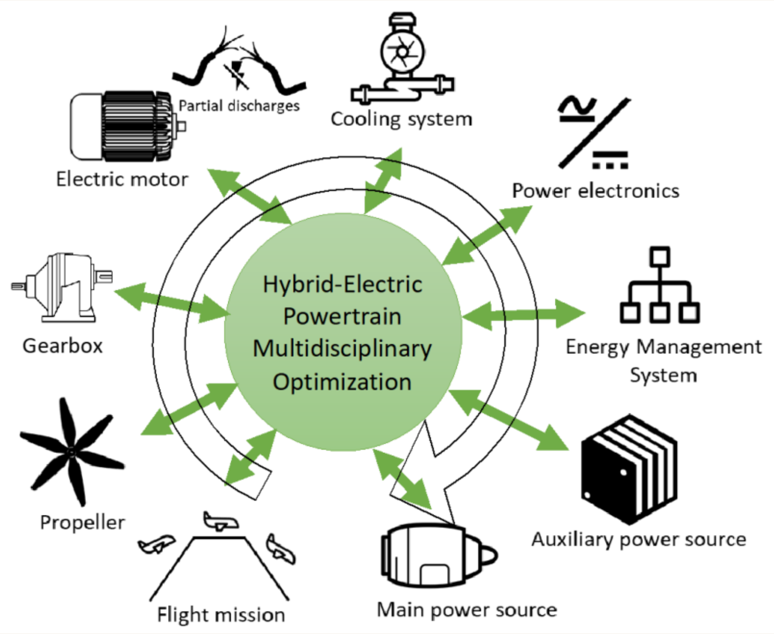
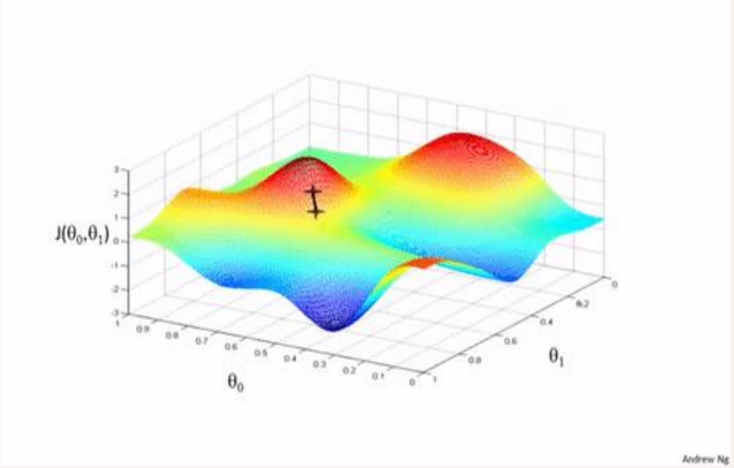
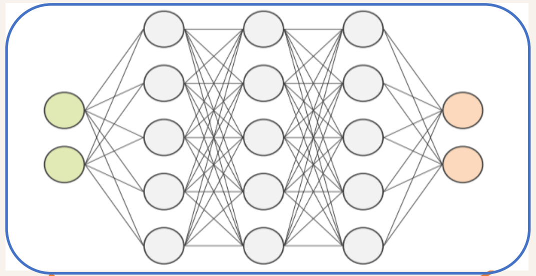
Why ∇Hydrodynamics?

- Adjoint-based optimization
  - Large multi-disciplinary optimization (MDO)
  - Non-parametric geometry optimization
- Control co-design
- Physics-Informed Machine Learning







$$\boldsymbol{u}\cdot\nabla\boldsymbol{u} - \nu\nabla^2\boldsymbol{u} + \nabla\cdot\boldsymbol{\tau} + \nabla p - \boldsymbol{g} = \boldsymbol{0}$$
$$\nabla\cdot\boldsymbol{u} = 0$$

Automatic Differentiation

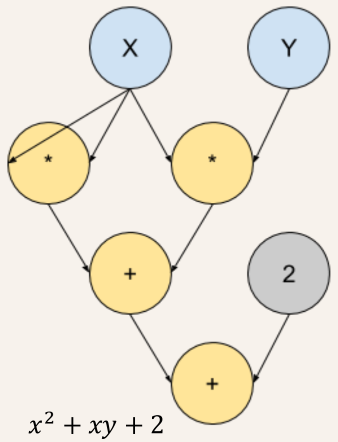
$$f' = \frac{f(x+h)-f(x)}{h}$$

Finite Difference:

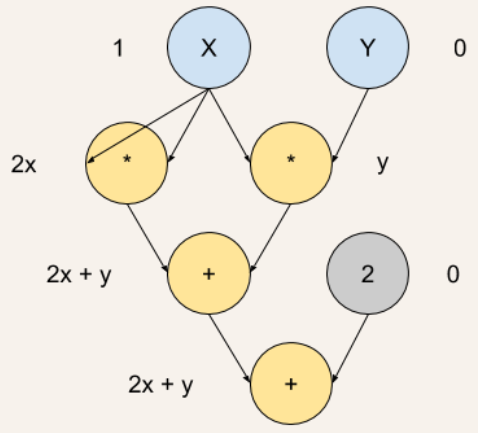
- *Cost = n_inputs x function*

Automatic Differentiation:

- *Cost = 2 x function*



Computational Graph



Derivative w.r.t x

Boundary Element Method

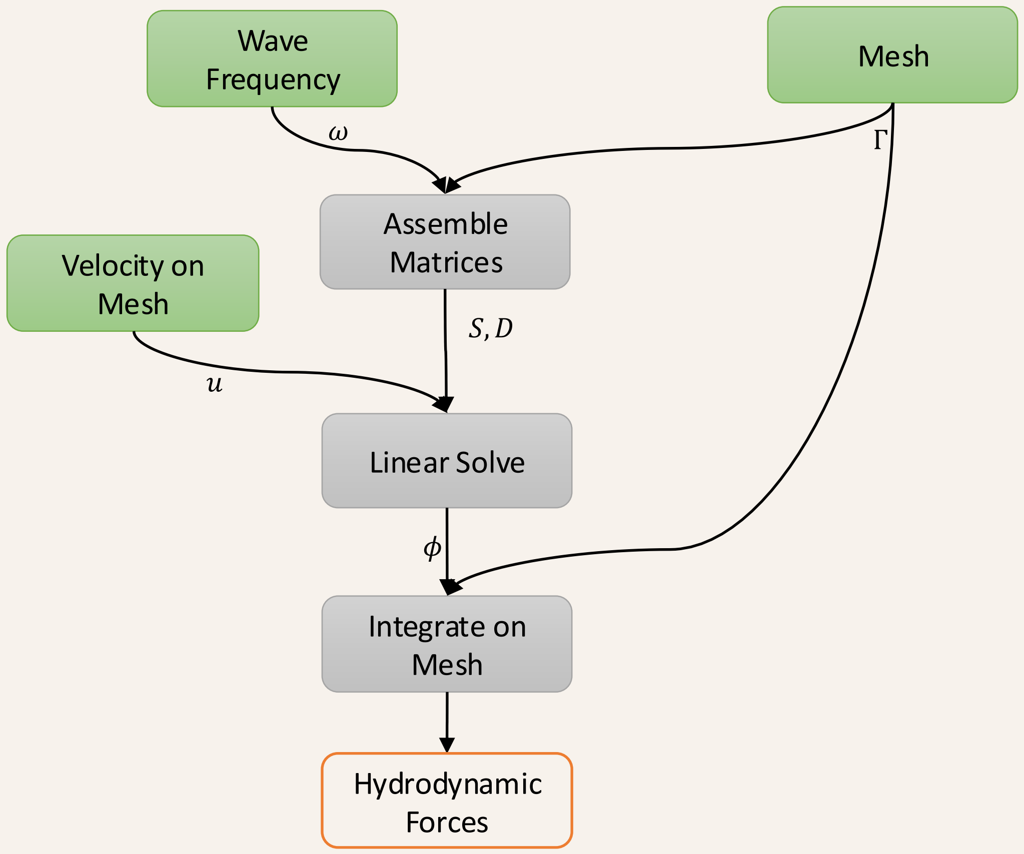
$$S_{ij} = \iint_{\Gamma_j} G(x_i, \xi) ds_\xi$$

$$D_{ij} = \frac{\delta_{ij}}{2} + \iint_{\Gamma_j} \nabla_\xi G(x_i, \xi) \cdot n_j ds_\xi$$

} Assemble Matrices

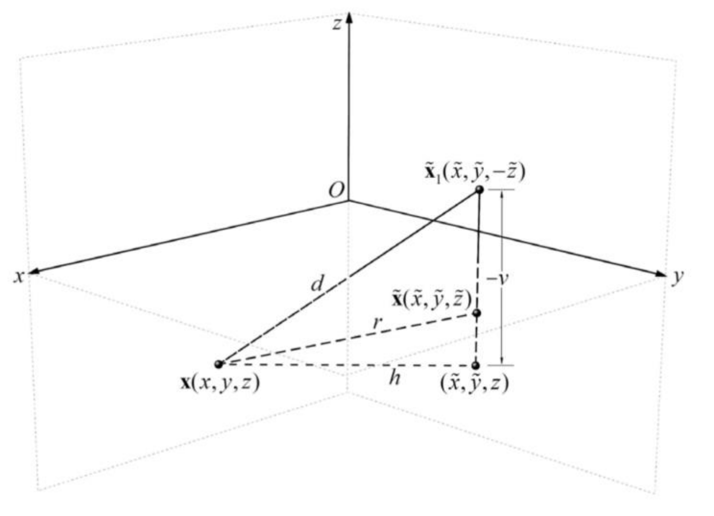
$$D\varphi = Su$$

} Linear Solve



Boundary Element Method

$$4\pi G(x,\xi) = -\frac{1}{r} - \frac{1}{d} + L(x,\xi) + 2\pi\left[\widetilde{H}_0(h) - iJ_0(h)\right]e^{v}$$



$$L^a \equiv -\frac{1}{d} + \frac{2P}{1+d^3} + 2\rho(1-\rho)^3 R \tag{33a}$$

where P and R are defined by (22b) and (26b) as

$$P \equiv e^v\left(\log\frac{-v}{2} + \gamma - 2d^2\right) + d^2 - v \tag{33b}$$

$$R \equiv (1-\beta)A - \beta B - \frac{\alpha C}{1+6\alpha\rho(1-\rho)} + \beta(1-\beta)D. \tag{33c}$$

Here, $\gamma = 0.577\ldots$ is Euler's constant, and α, β, ρ are defined by (15) and (21a). Moreover, the polynomials $A(\rho)$, $B(\rho)$, $C(\rho)$ and $D(\rho)$ in (33c) are defined as

$$\begin{aligned} A \equiv{} & 1.21 - 13.328\rho + 215.896\rho^2 - 1763.96\rho^3 + 8418.94\rho^4 \\ & - 24314.21\rho^5 + 42002.57\rho^6 \\ & - 41592.9\rho^7 + 21859\rho^8 - 4838.6\rho^9 \end{aligned} \tag{33d}$$

$$\begin{aligned} B \equiv{} & 0.938 + 5.373\rho - 67.92\rho^2 + 796.534\rho^3 - 4780.77\rho^4 \\ & + 17137.74\rho^5 - 36618.81\rho^6 + 44894.06\rho^7 \\ & - 29030.24\rho^8 + 7671.22\rho^9 \end{aligned} \tag{33e}$$

$$\begin{aligned} C \equiv{} & 1.268 - 9.747\rho + 209.653\rho^2 - 1397.89\rho^3 + 5155.67\rho^4 \\ & - 9844.35\rho^5 + 9136.4\rho^6 - 3272.62\rho^7 \end{aligned} \tag{33f}$$

$$\begin{aligned} D \equiv{} & 0.632 - 40.97\rho + 667.16\rho^2 - 6072.07\rho^3 + 31127.39\rho^4 \\ & - 96293.05\rho^5 + 181856.75\rho^6 - 205690.43\rho^7 \\ & + 128170.2\rho^8 - 33744.6\rho^9. \end{aligned} \tag{33g}$$

Wu, Huiyu, et al. ***"A global approximation to the green function for diffraction-radiation of regular water waves in deep water."*** MARINE VII: proceedings of the VII International Conference on Computational Methods in Marine Engineering. CIMNE, 2017.

Automatic Differentiation of BEM

- *implicitAD.jl*
- Custom gradients

```
function bem_program(radius,omega =1.3 ,dof = [0,0,1])
    mesh = differentiableMesh(radius) #fd
    wavenumber = omega^2 / 9.8
    S,D = assemble_matrices(mesh,wavenumber)
    BC = neumanBC(mesh.normals,dof,omega)
    ϕ = implicit_linear(D,S*BC)
    A = added_mass(ϕ,mesh.normals,mesh.areas,omega,dof)
    return A
end

# gradient with respect to mesh, omega and dof
Zygote.gradient(X -> bem_program(X[1], X[2], X[3]), [r, w, heave])
```

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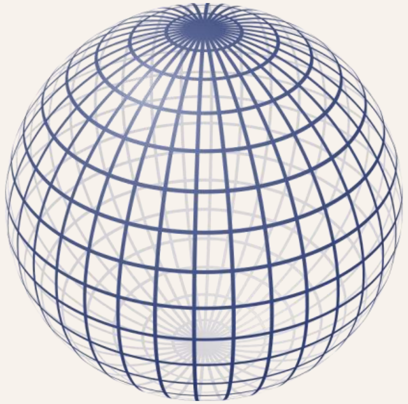
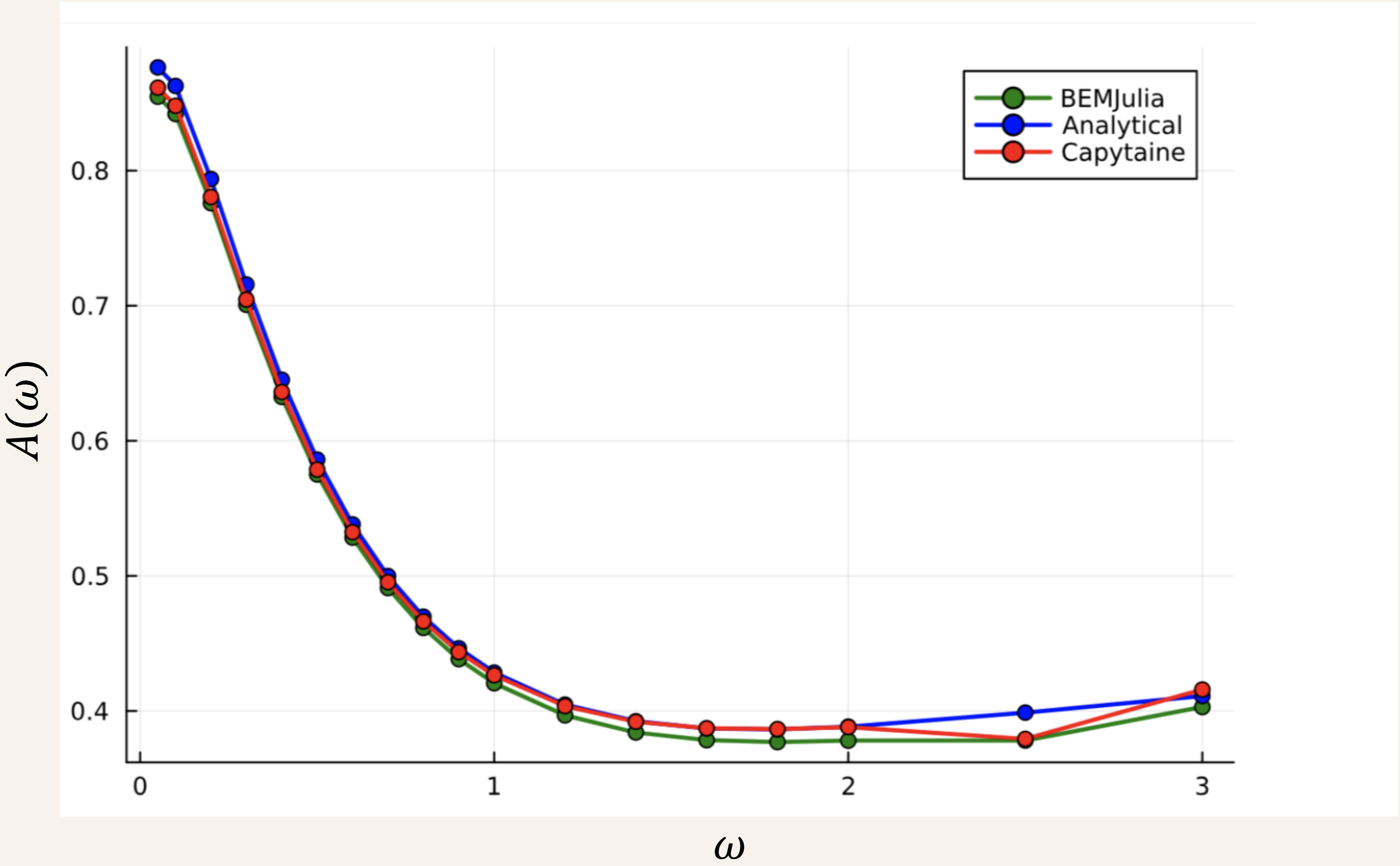
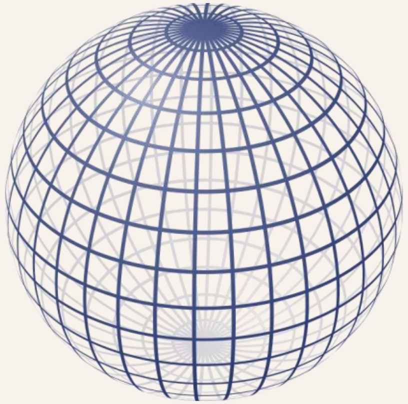
Preliminary Results

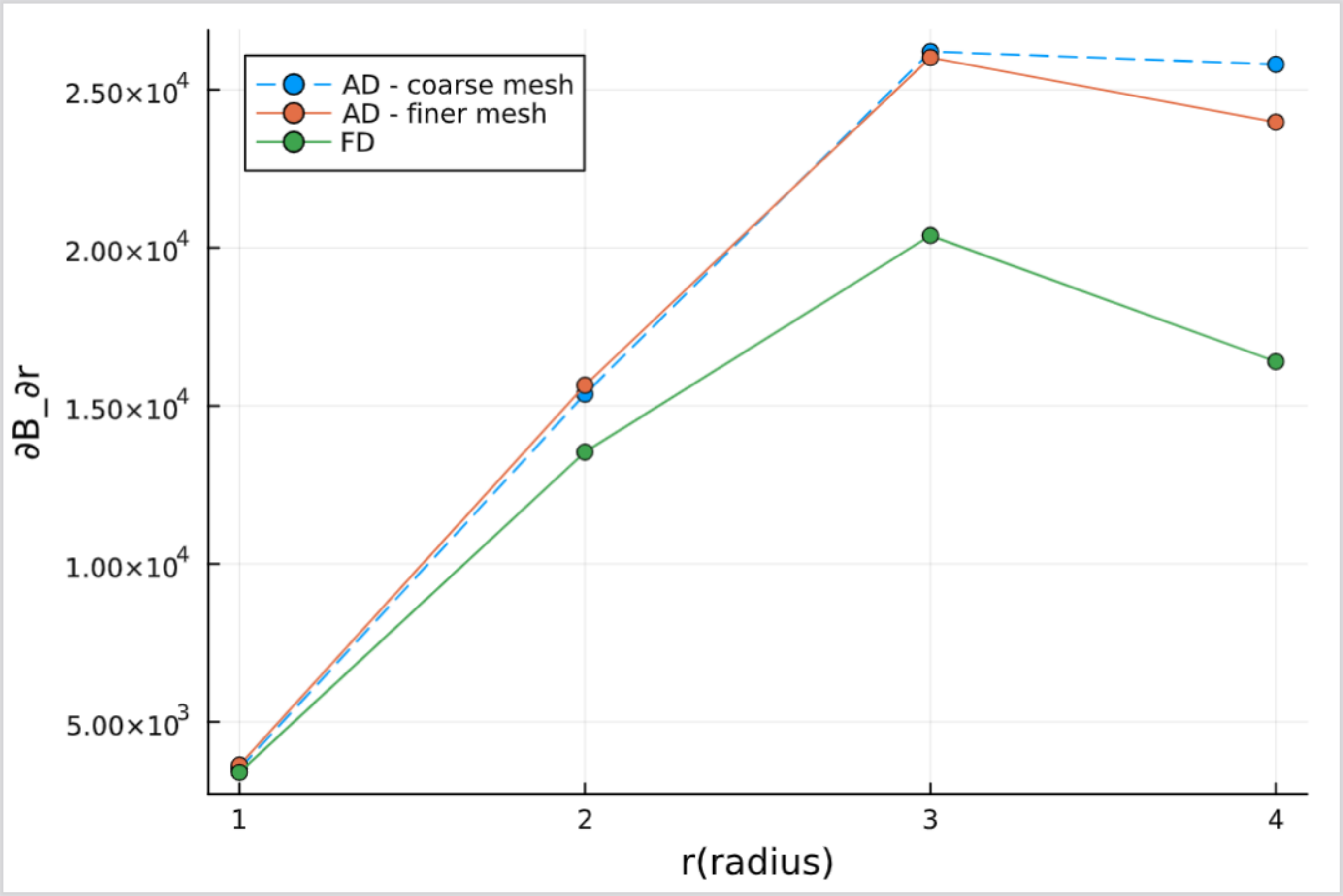
Added Mass




BEMJulia
Analytical
Capytaine
0.8
0.7
0.6
0.5
0.4
0
1
2
3
$A(\omega)$
ω

Gradient of Added Mass w.r.t. Radius





Next Steps

- Finish verification of gradients
- Demonstrate use in optimization study
- Release: Code structure & documentation
- Parallelization & GPUs

Questions?

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