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Development and Validation of an Inverted Pendulum Wave Energy Converter Model

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umerc + OREC 2025

SNL 399589

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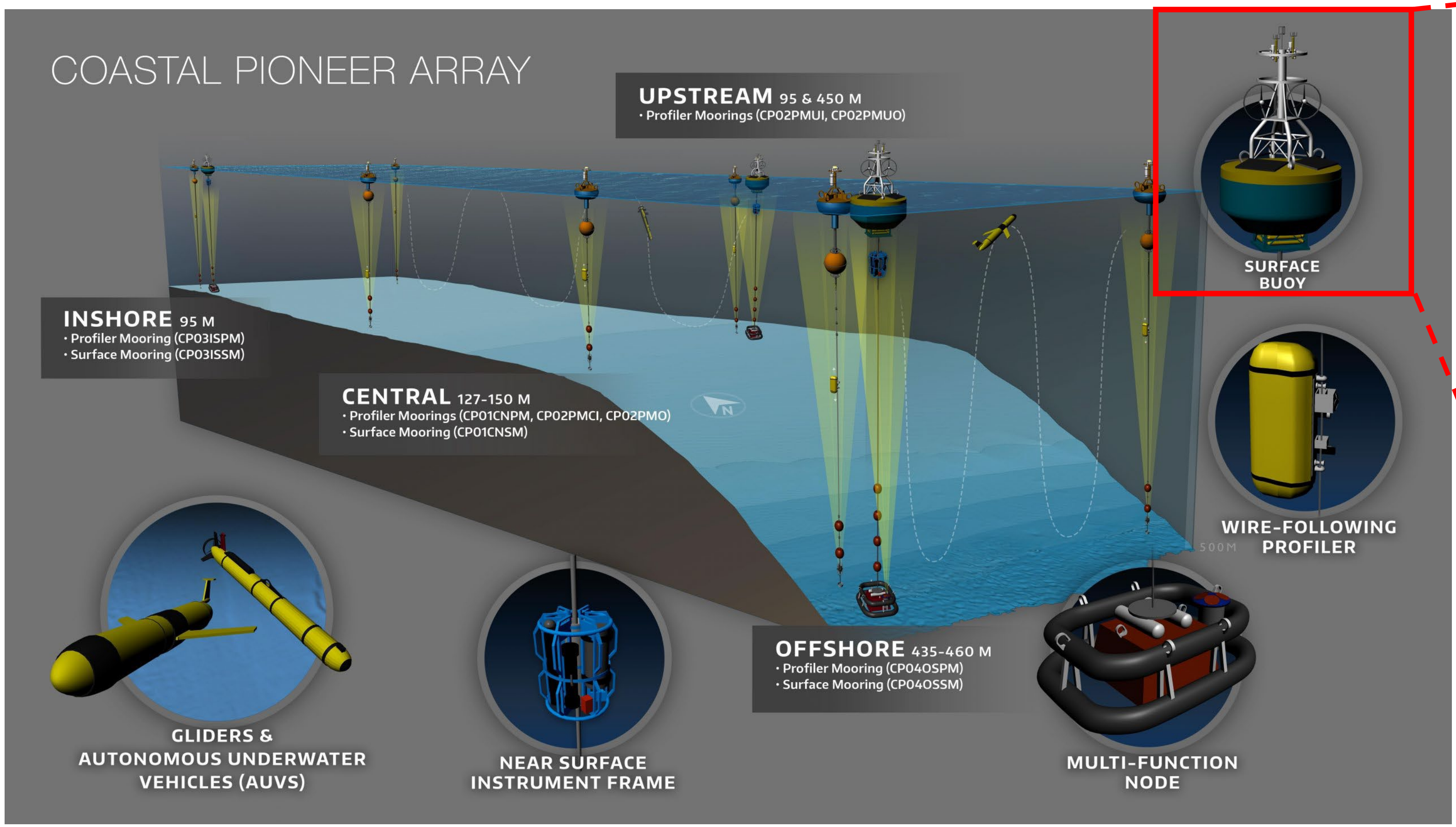
Introduction: Coastal Pioneer Array



Pioneer oceanographic buoy

"Coastal Pioneer New England Shelf Array." Ocean Observatories Initiative, oceanobservatories.org/array/coastal-pioneer-array/. Accessed 16 July 2025.

Motivation: Reliable Power for Buoys

Current power supply can only generate **70%** of power demand [1]

Need sustainable energy resource for *continuous* operation

Solution: Incorporate wave energy converter

Pioneer Buoy deployed in the ocean

Coe, Ryan G., Lee, Jantzen et al. Pioneer WEC concept design report. United States: N. p., 2023. Web. doi:10.2172/2280833.

Inverted Pendulum WEC Design

Sandia National Labs Pioneer Array [4]

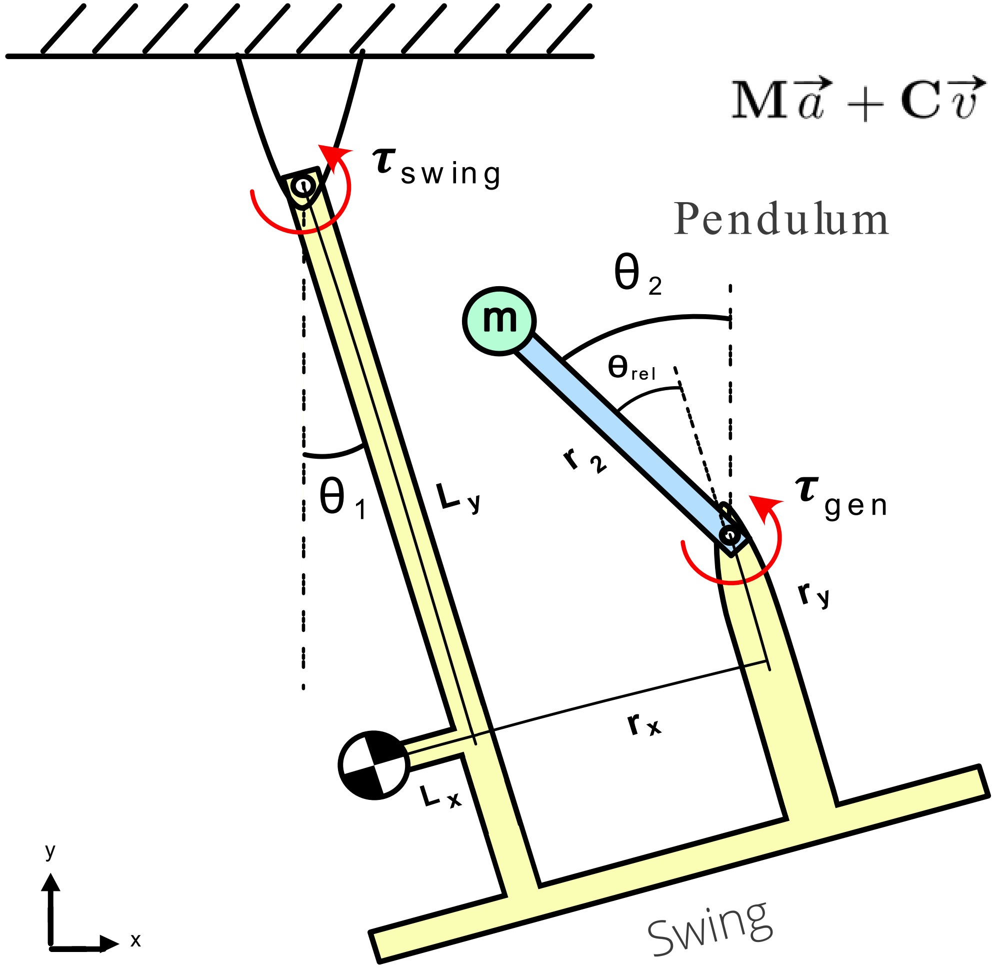
Objective

Develop nonlinear and linear models to predict system dynamics

Assess range of applicability for linear model

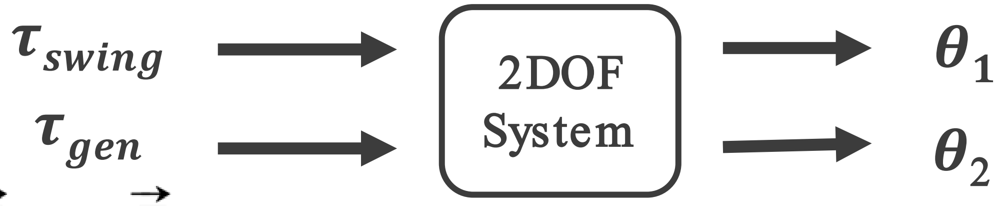
Numerical Model Derivation

$$\mathbf{M}\vec{a} + \mathbf{C}\vec{v} + \mathbf{K}\vec{x} = \vec{F}_{\text{exc}} + \vec{F}_{\text{col}}$$



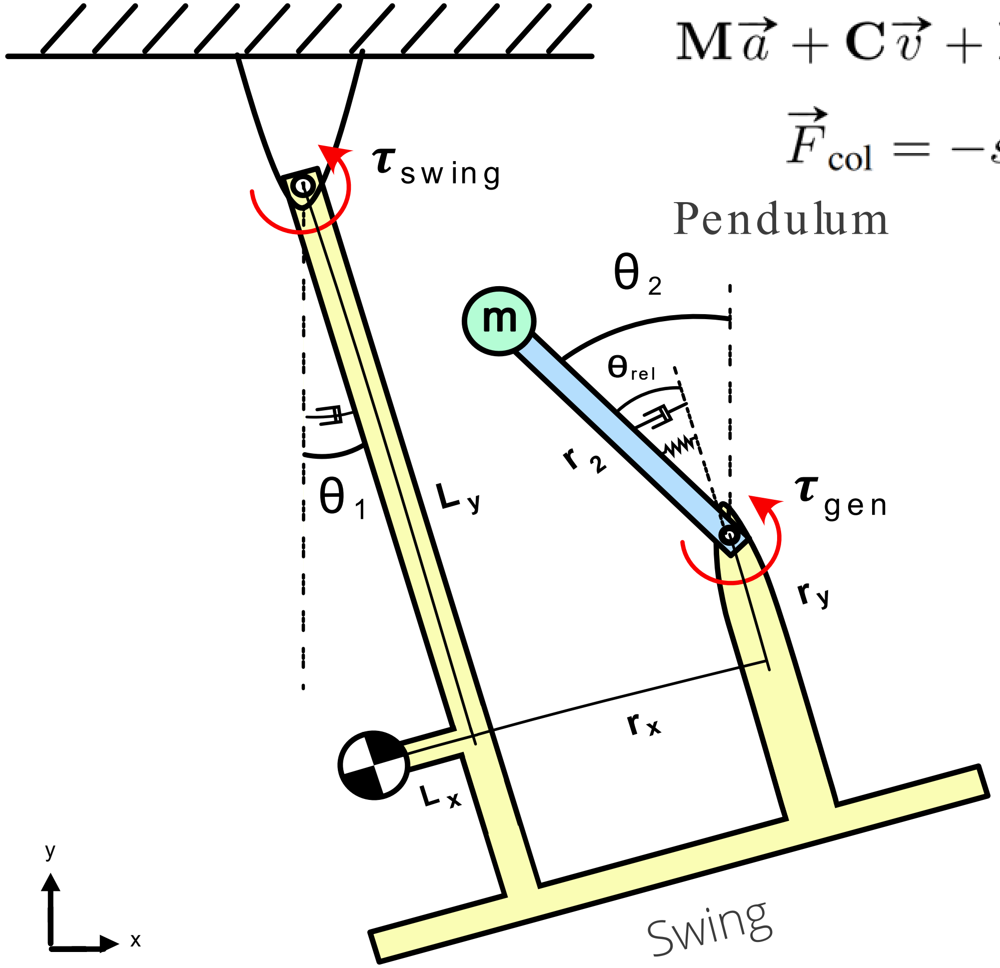
Governing Equations of Motion Derivation

Numerical Model Derivation



$$\mathbf{M}\vec{a} + \mathbf{C}\vec{v} + \mathbf{K}\vec{x} = \vec{F}_{\mathrm{exc}} + \vec{F}_{\mathrm{col}}$$

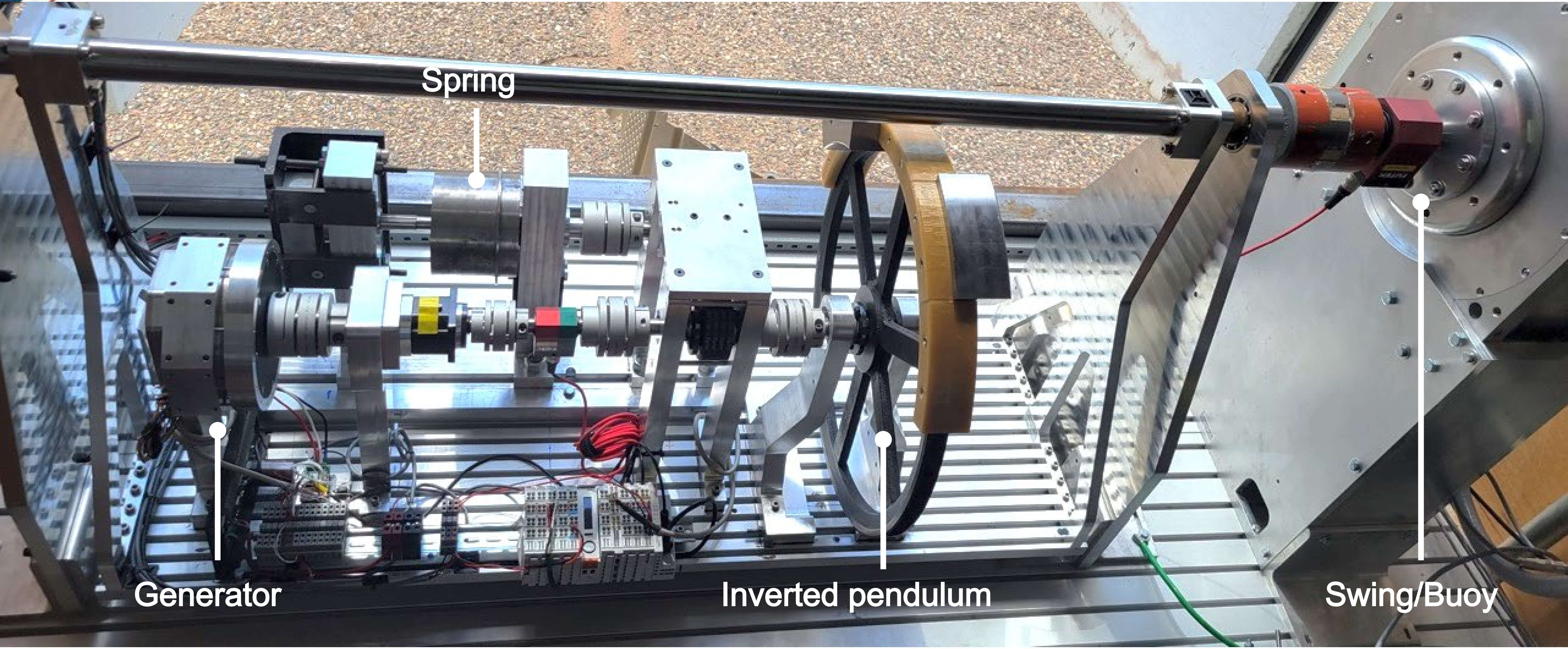
$$\vec{F}_{\mathrm{col}} = -sign(\vec{v}) * \mu N$$



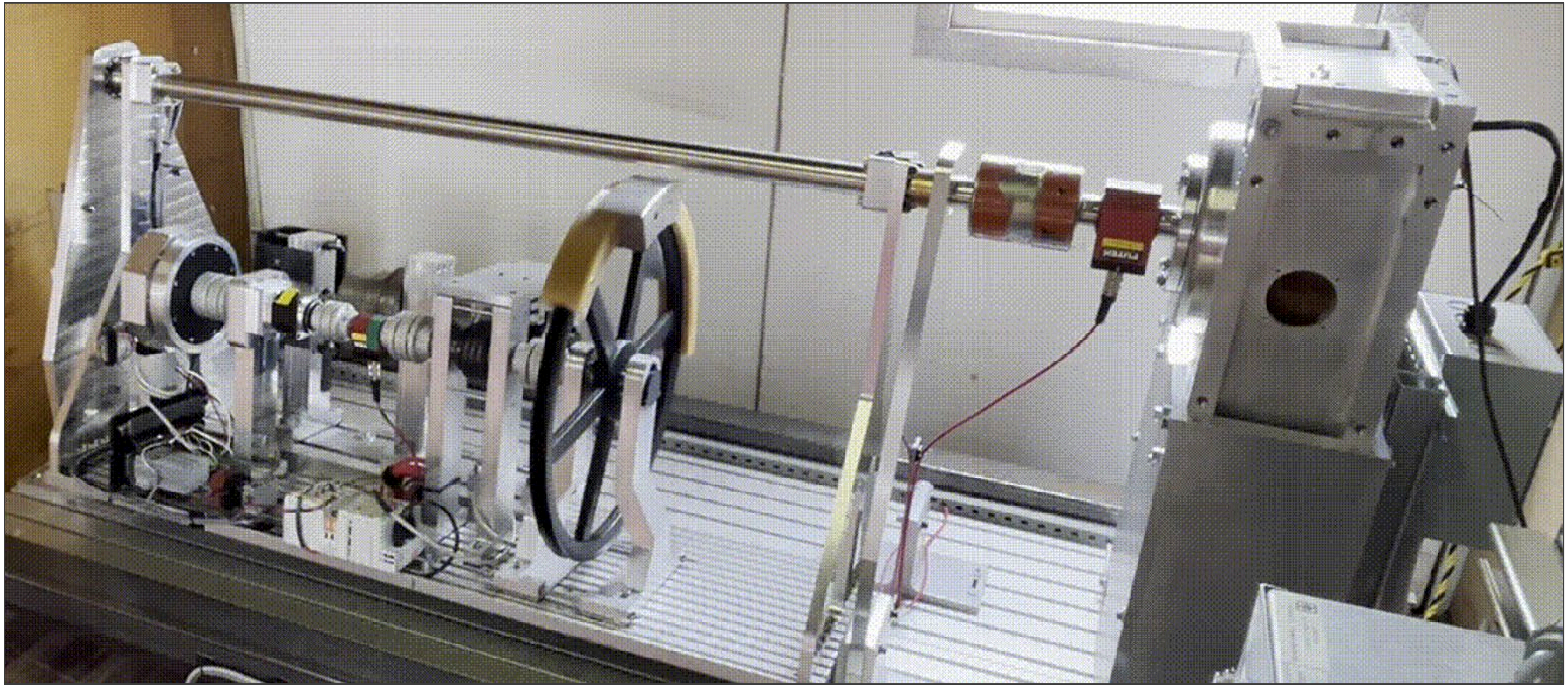
Governing Equations of Motion Derivation

MATLAB® Simscape Multibody Simulation

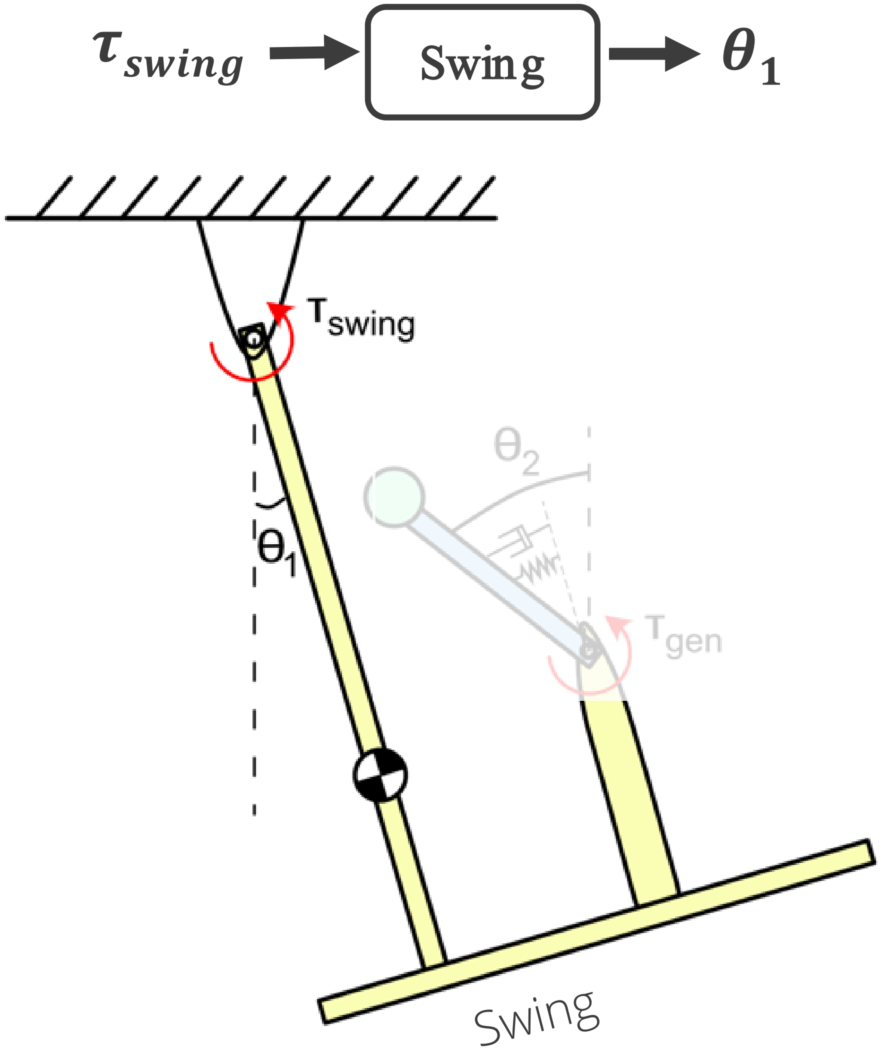
Experimental Test Setup



Experiment Motion Demonstration



1DOF Swing Joint Experimental Data

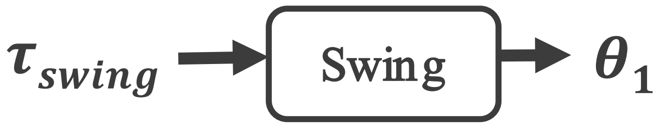


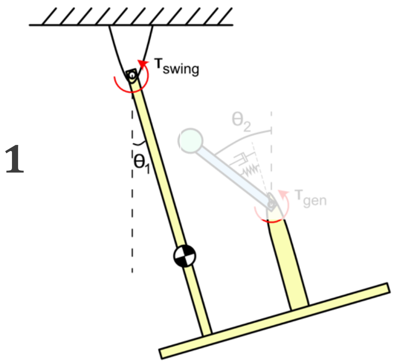
Rotation at Swing Joint under Applied τ_{swing}

---- Numerical estimates

—— Experimental Data

1DOF Swing Joint Model Analysis





Linear model obtained via small angle approx.

$$sin\theta \approx \theta$$
$$cos\theta \approx 1$$

RMS of Linear against Nonlinear Swing model

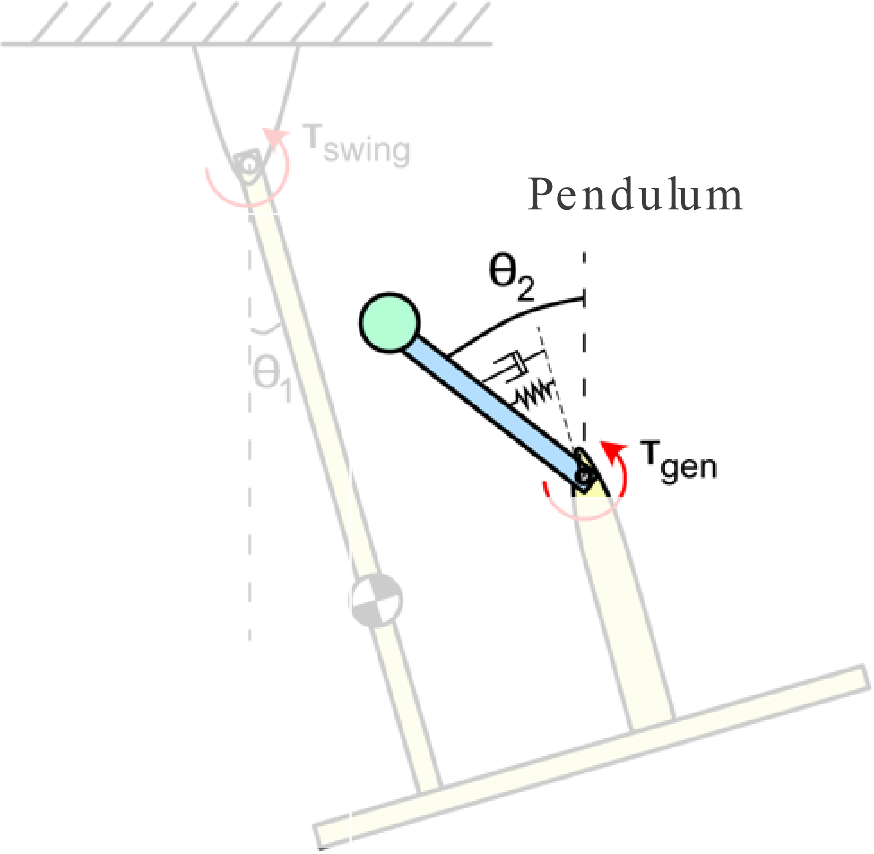
Resonant Frequency ≈ 0.77Hz

Total Harmonic Distortion of Nonlinear Swing model

Lower amp./freq.

1DOF Pendulum Experimental Data

τ_{pend} → Pend. → θ_2



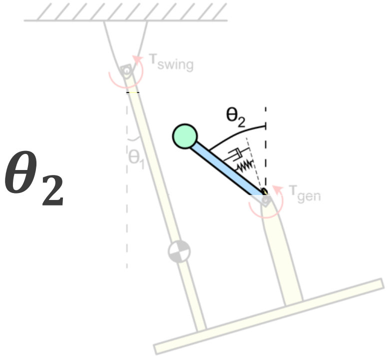
Rotation at Pend. Joint under Applied τ_{gen}

---- Numerical estimates

—— Experimental Data

1DOF Pendulum Joint Model Analysis

τ_{pend} → Pend. → θ_2



Linear model obtained via small angle approx.

$$\sin\theta \approx \theta$$
$$\cos\theta \approx 1$$

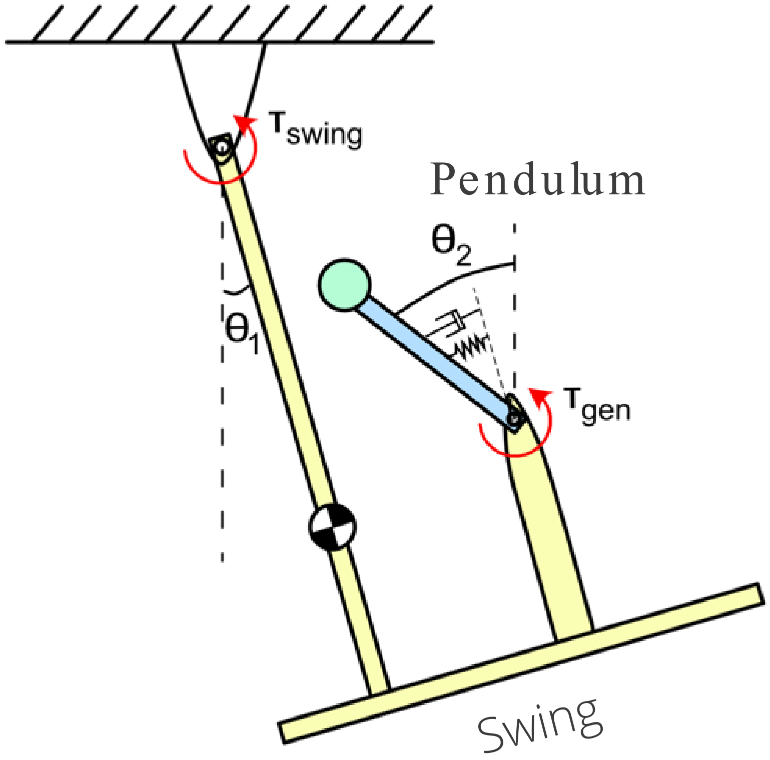
Lower amp./freq.

RMS of Linear against Nonlinear Pend. model

Total Harmonic Distortion of Nonlinear model

Resonant Frequency ≈ 0.35Hz

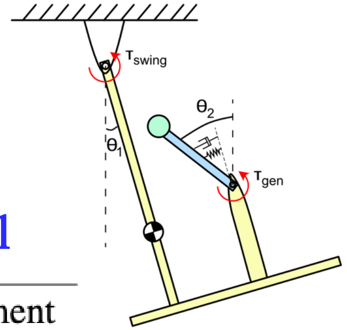
2DOF System Experimental Data



---- Numerical estimates

—— Experimental Data

Comparison of Nonlinear and Linear Models

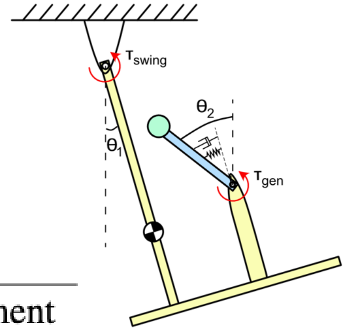


Nonlinear Swing model

		Trained Experiments Average	Validation Experiment
1DOF Swing Model	Nonlinear Model RMS for Swing Angle [deg]	0.41	0.23
	Linear Model RMS for Swing Angle [deg]	3.13	1.18
	Nonlinear Model RMS for Swing Velocity [deg/s]	2.35	1.24
	Linear Model RMS for Swing Velocity [deg/s]	14.09	5.29
1DOF Pendulum Model	Nonlinear Model RMS for Pend. Angle [deg]	4.28	
	Linear Model RMS for Pend. Angle [deg]	8.83	8.82
	Nonlinear Model RMS for Pend. Velocity [deg/s]	9.11	9.60
	Linear Model RMS for Pend. Velocity [deg/s]	24.99	24.97
2DOF System Model	Nonlinear Model RMS for Swing Angle [deg]	0.37	0.42
	Linear Model RMS for Swing Angle [deg]	1.76	1.96
	Nonlinear Model RMS for Swing Velocity [deg/s]	1.51	1.91
	Linear Model RMS for Swing Velocity [deg/s]	7.53	8.41
	Nonlinear Model RMS for Pend. Angle [deg]	3.39	3.06
	Linear Model RMS for Pend. Angle [deg]	12.89	13.30
	Nonlinear Model RMS for Pend. Velocity [deg/s]	7.05	7.48
	Linear Model RMS for Pend. Velocity [deg/s]	37.71	39.65

Linear Swing model

Comparison of Nonlinear and Linear Models



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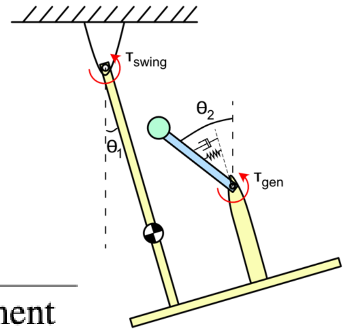
Nonlinear Swing model

Linear Swing model

Comparison of Nonlinear and Linear Models



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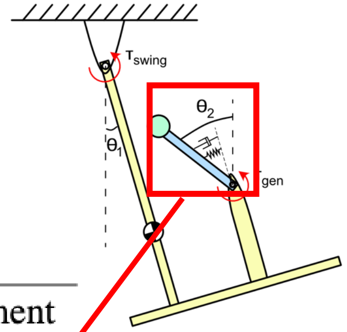
Nonlinear Swing model

Linear Swing model

Comparison of Nonlinear and Linear Models



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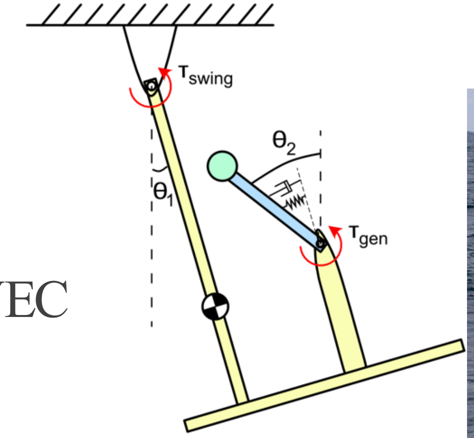
Nonlinear spring stiffness

Conclusions



Develop linear and nonlinear models of Inverted Pend. WEC

Coulomb friction dominates in near-zero frequencies and amplitudes

Linear model shows more disagreement with nonlinear model for high amplitudes/resonant frequencies

RMS of Linear against Nonlinear Swing model

THD of Nonlinear Swing model

Lower amp./freq.

Res freq.

Acknowledgements

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Thank you for your attention!

Questions? Email me at ejl2401@stanford.edu

Appendix

6DOF LAMP Testing of the WEC

Citation

[1] Coe, Ryan G., Lee, Jantzen, Bacelli, Giorgio, Spencer, Steven J., Dullea, Kevin, Plueddemann, Albert J., Buffitt, Derek, Reine, John, Peters, Donald, Spinneken, Johannes, Hamilton, Andrew, Sabet, Sahand, Husain, Salman, Jenne, Dale, Korde, Umesh, Muglia, Mike, Taylor, Trip, and Wade, Eric. Pioneer WEC concept design report. United States: N. p., 2023. Web. doi:10.2172/2280833.

[2] I.A. Antoniadis, V. Georgoutsos, A. Paradeisiotis, Fully enclosed multi-axis inertial reaction mechanisms for wave energy conversion, Journal of Ocean Engineering and Science,

Volume 2, Issue 1, 2017, Pages 5-17, ISSN 2468-0133, https://doi.org/10.1016/j.joes.2017.02.003.

[3] Aref Afsharfard, Inwon Lee, Kyung Chun Kim, Study application of an unmoored ocean wave energy harvester with harmonic and random excitation, Energy Conversion and Management, Volume 293, 2023, 117535, ISSN 0196-8904, https://doi.org/10.1016/j.enconman.2023.117535.

[4] YouTube. (2023, November 29). *The Pioneer Array: Harnessing Wave Power to collect Ocean Data* YouTube. https://www.youtube.com/watch?v=PsNvlGnm59o

[5] Olejnik, P., Ayankoso, S. Friction modelling and the use of a physics-informed neural network for estimating frictional torque characteristics. Meccanica 58, 1885–1908 (2023). https://doi.org/10.1007/s11012-023-01716-8