Modeling and predicting OSWEC behavior using dynamic mode decomposition

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Oscillating Surge Wave Energy Converter

Maine Marine Composites, 2014
OSWEC System

\[ \eta(t) \]

\[ F_t \]

\[ F_h \]

\[ F_s \]

\[ \tau_t \]

\[ \theta, \dot{\theta} \]
Dynamic Mode Decomposition (DMD)

\[
\frac{dx}{dt} = Ax \quad \rightarrow \quad x_{k+1} = Ax_k
\]

1. Data

State 1

State 2

State 3

\[ t \quad \rightarrow \]

2. Arrange

\[
X = \begin{bmatrix}
x_1 & x_2 & \cdots & x_{m-1} \\
\end{bmatrix}
\]

\[
X' \approx AX
\]

3. Future State Prediction

\[
A = X'X^+ \\
A = \begin{bmatrix}
\end{bmatrix}
\]

State 1

State 2

State 3

\[ t \quad \rightarrow \]
Time-Delay Coordinates

\[ X_{\text{aug}} = \begin{bmatrix} x_1 & x_2 & \cdots & x_{m-1} \\ x_2 & x_3 & \cdots & x_m \end{bmatrix} \]
Data Analysis Pipeline

**Generate Data**
- Data source
- Input wave

**Preprocess Data**
- Choose states
- Normalize
- Partition training and testing data

**DMD**
- Run DMD on training data

**Evaluate Performance**
\[ \varepsilon = \frac{\|X_{DMD} - X\|_2}{\|X\|_2} \]
- Training
- Testing
Cases

**Case 1: Weakly nonlinear flap**

- Data source: WEC-Sim, nonlinear flap
- Sea State: Regular
- $H = 2\ m$, $T = 8\ s$
Cases

Case 1: Weakly nonlinear flap
- Data source: WEC-Sim, nonlinear flap
- Sea State: Regular
- $H = 2\,\text{m},\, T = 8\,\text{s}$

Case 2: Irregular wave field
- Data source: WEC-Sim, nonlinear flap
- Sea State: Irregular, real spectrum
- $H_s = 0.94\,\text{m},\, T_p = 5.8\,\text{s}$
Case 1: Weakly nonlinear dynamics

\[
\dot{\theta} = \frac{1}{\tau_0} \left( F_{s} - F_{h} - P_{1} - P_{2} - P_{3} - P_{4} - P_{5} \right)
\]

\[
\epsilon
\]

Time [s]
Case 2: Irregular wave field

\( \eta \quad \bar{\theta}_{\text{train}} \quad \bar{\theta}_{\text{test}} \)
Summary and Future Work

• DMD shows promise for creating a fast, accurate model for OSWECs
  – Weakly nonlinear dynamics
  – Irregular waves

• Next Steps
  – Further investigation on irregular wave case
  – Using DMD model in for control optimization
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Model Predictive Control

![Diagram of Model Predictive Control with DMD](image)

Schwenzer, et al., 2021
Total Least Squares DMD

\[ x_{k+1} = A x_k \]
Semi-Analytical Model

Swell

Wind

Combined Sea State

TF

$\mu_1(\omega), v_1(\omega), \tau_{e,1}(\omega)$

TF

$\mu_2(\omega), v_2(\omega), \tau_{e,2}(\omega)$

$\eta [m]$

$\theta [^\circ]$

$\tau_h [kNm]$

$t/T_{max}$

Time [s]
WEC-Sim

\[ m\ddot{X}(t) = F_e(t) + F_r(t) + F_v(t) + F_{PTO}(t) + F_B(t) + F_{md}(t) + F_m(t) \]

- Frequency domain analysis (BEM) for hydrodynamic parameters
- Time domain solver
- Capabilities
  - Weakly nonlinear flap dynamics
  - Irregular sea state
    - Real spectrum
Cases

Case 1: Noisy signal

- Data source: Semi-analytical model
- Sea State: Polychromatic (2 frequencies)
- Noise: Added white Gaussian noise
- $H_1 = 0.7 \text{ m}$, $H_2 = 0.3 \text{ m}$, $T_1 = 8 \text{ s}$, $T_2 = 2.55 \text{ s}$
Case 1: Noisy Dynamics

SNR=70
SNR=40
SNR=30
Case 1: Noisy Dynamics

SNR=40

SNR=30
Case 3: Irregular wave field

\[ \eta \]

\[ \bar{\tau}_h, \text{train} \]

\[ \bar{\tau}_h, \text{test} \]