WEC modeling in irregular waves using Sparse Identification of Nonlinear Dynamics (SINDy)

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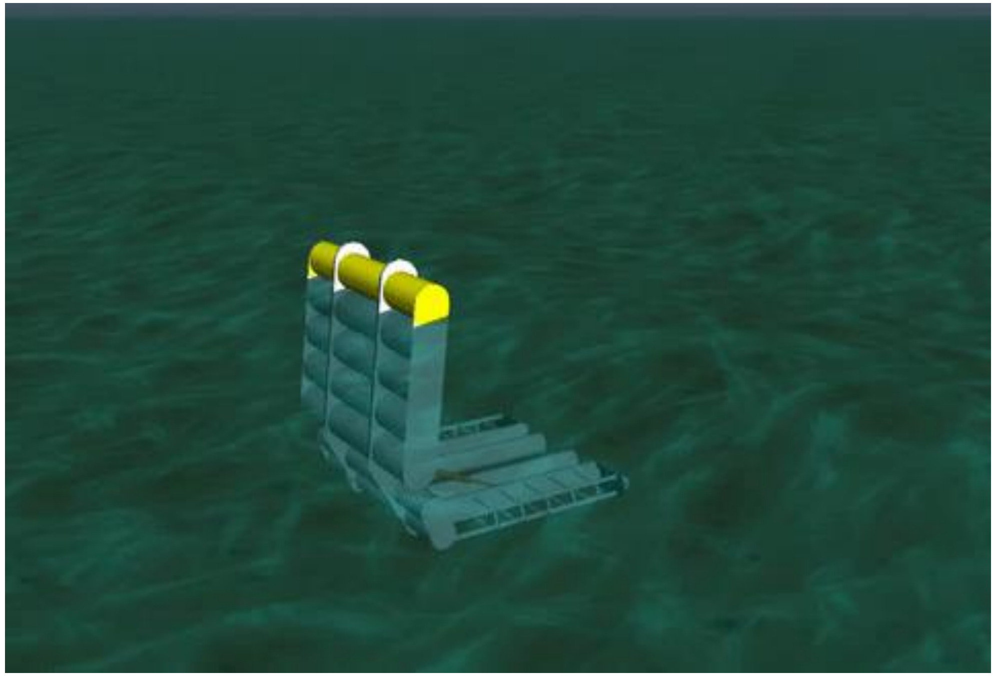
Oscillating Surge Wave Energy Converter (OSWEC)

Modeling challenges:

- OSWEC behavior in irregular waves is high-dimensional and complex
  - Changing sea states
  - Non-periodic
  - Stochastic
- Time-domain models have limitations
  - Requires significant computation time
  - Requires knowledge of wave field



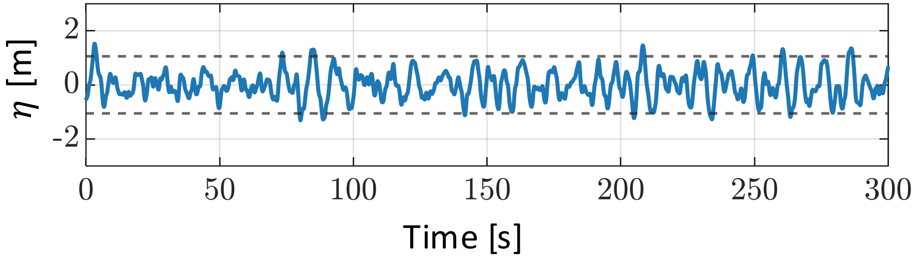
Maine Marine Composites, 2014

Goal: Use **data-driven methods** to build **generalizable** models for OSWEC behavior in **irregular** seas in the **time domain**

Methods: Workflow

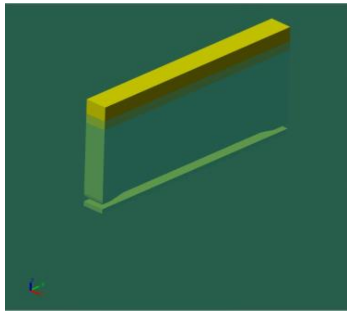
SWIFT Data

- Buoy data from real seas
- H_s = 2-3 m
- T_p = 6-10 s



Wave elevation data →

WEC-Sim



- 18 m wide flap
- 10.9 m water depth
- PTO Damping

↓ Training data

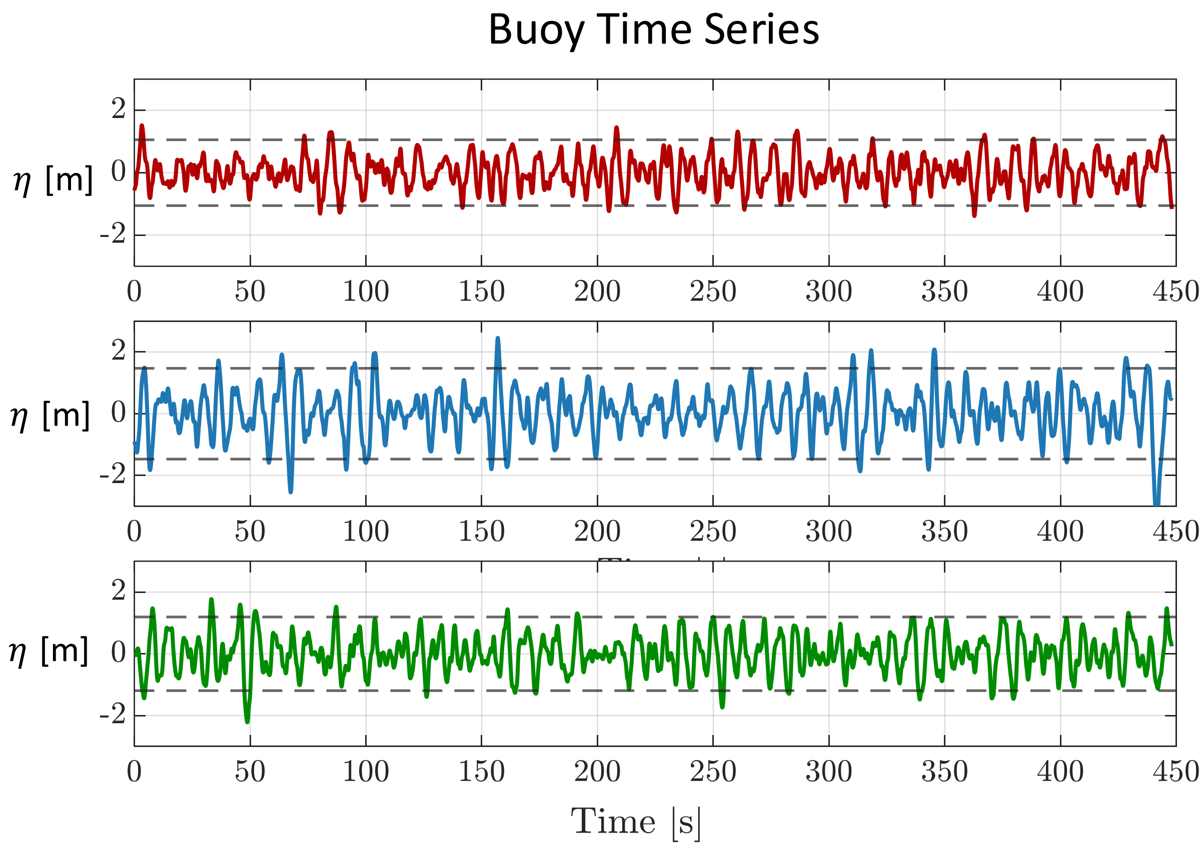
SINDy

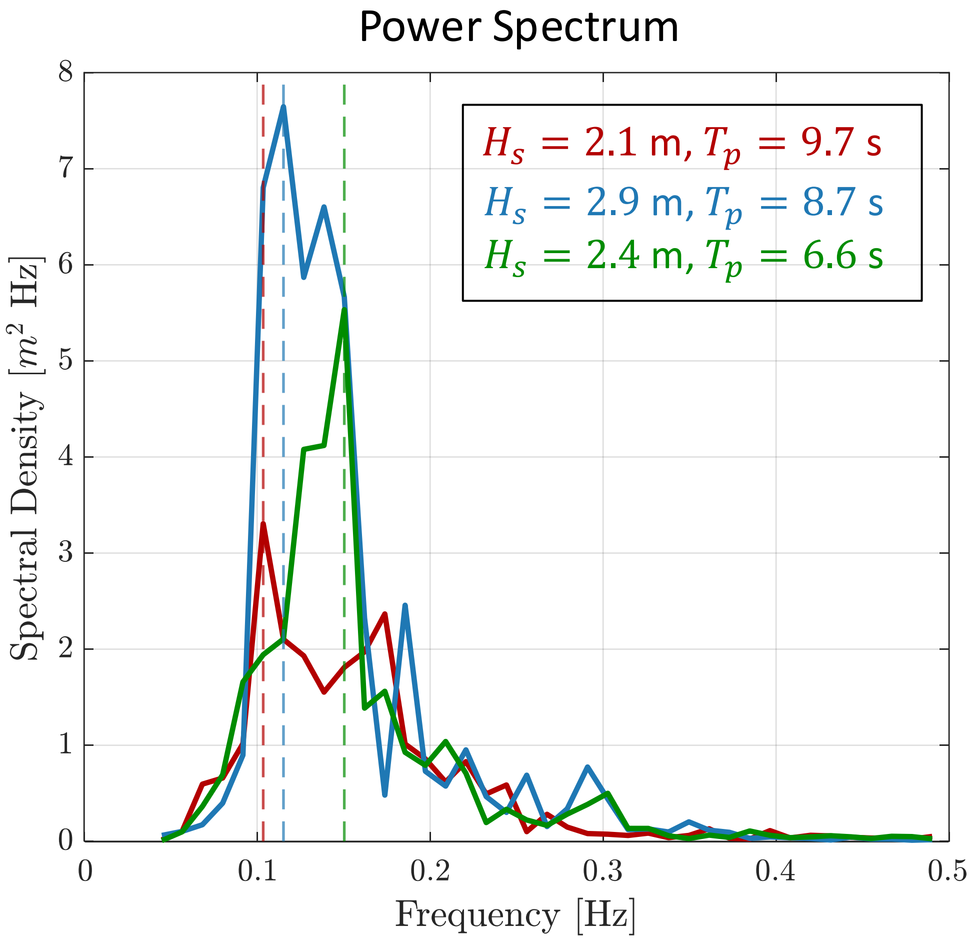
$$\ddot{\theta} = f^N(\theta, \dot{\theta}, F_x, M, \eta)$$

- 60 second training time
- Both linear (N = 1) and nonlinear (N = 3) models

Thomson, Jim. "Wave breaking dissipation observed with “SWIFT” drifters." Journal of Atmospheric and Oceanic Technology 29.12 (2012): 1866-1882.

Methods: SWIFT Data





Methods: SINDy workflow

Step 1: Collect data & choose state variables

$$\ddot{\theta} = \Lambda^{N}(\theta, \dot{\theta}, F_x, M, \eta)\xi$$

Modelled Variable (from data)

Function Library (from data)

maximum polynomial order

States (from data)

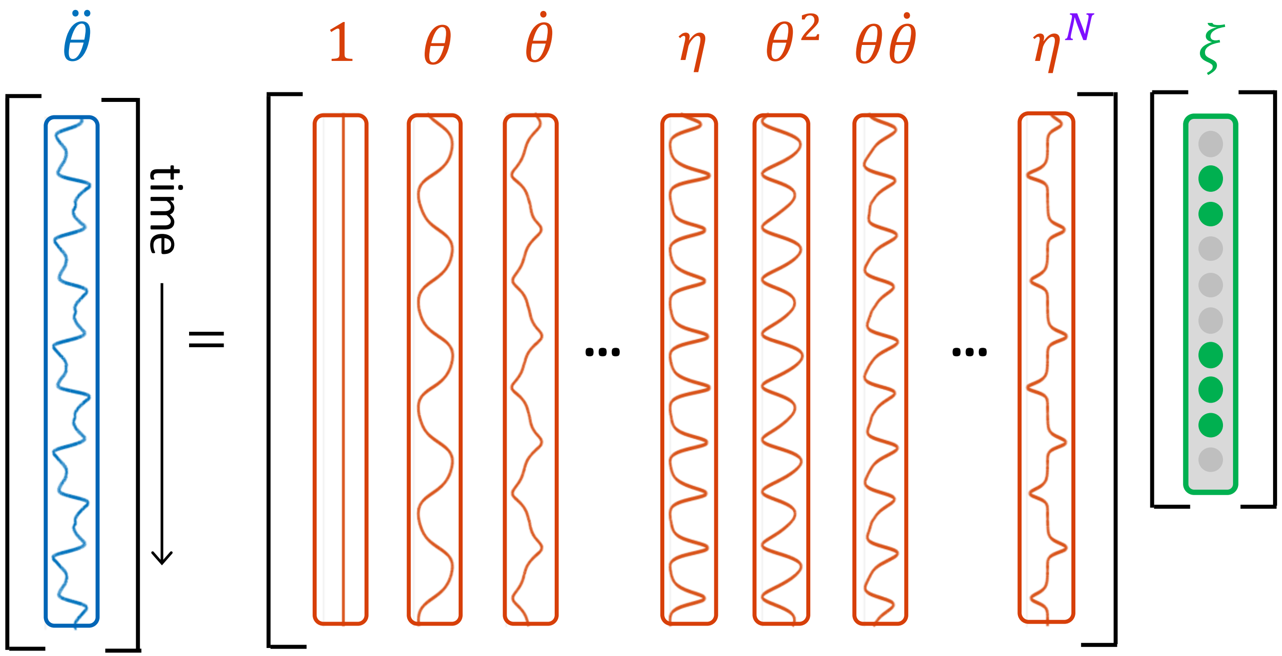
function weights (what SINDy solves for)

Methods: SINDy workflow

Step 1: Collect data & choose state variables

$$\ddot{\theta} = \Lambda^{N}(\theta, \dot{\theta}, F_x, M, \eta)\xi$$

Step 2: Build matrix equation

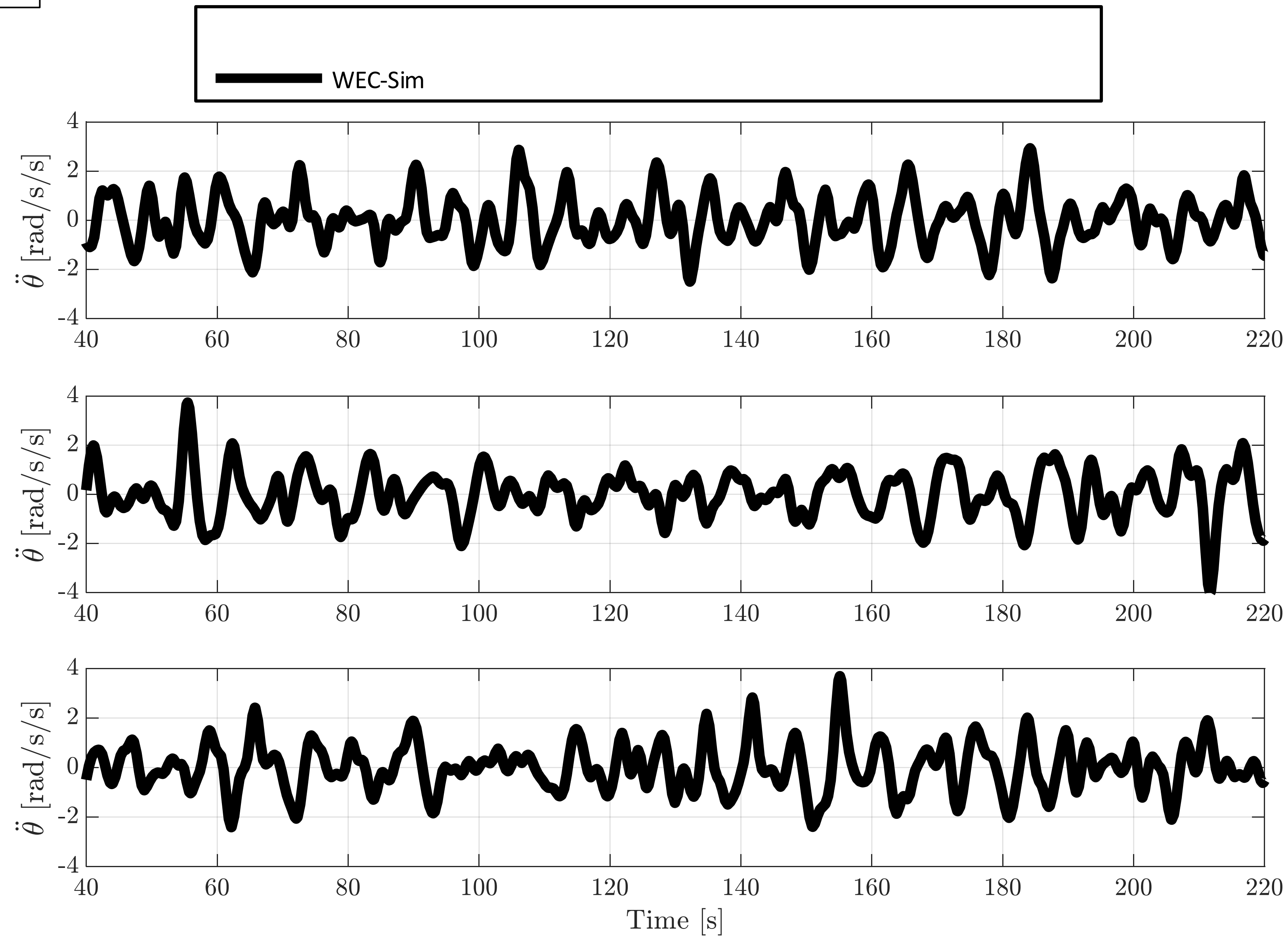


Step 3: Choose parameters, solve for ξ

$$\min_{\xi}\left[\|\ddot{\theta} - \Lambda\,\xi\|_2 + \lambda\|\xi\|_1\right]$$

Results: SINDy Model

$H_s = 2.1$ m, $T_p = 9.7$ s
$H_s = 2.9$ m, $T_p = 8.7$ s
$H_s = 2.4$ m, $T_p = 6.6$ s



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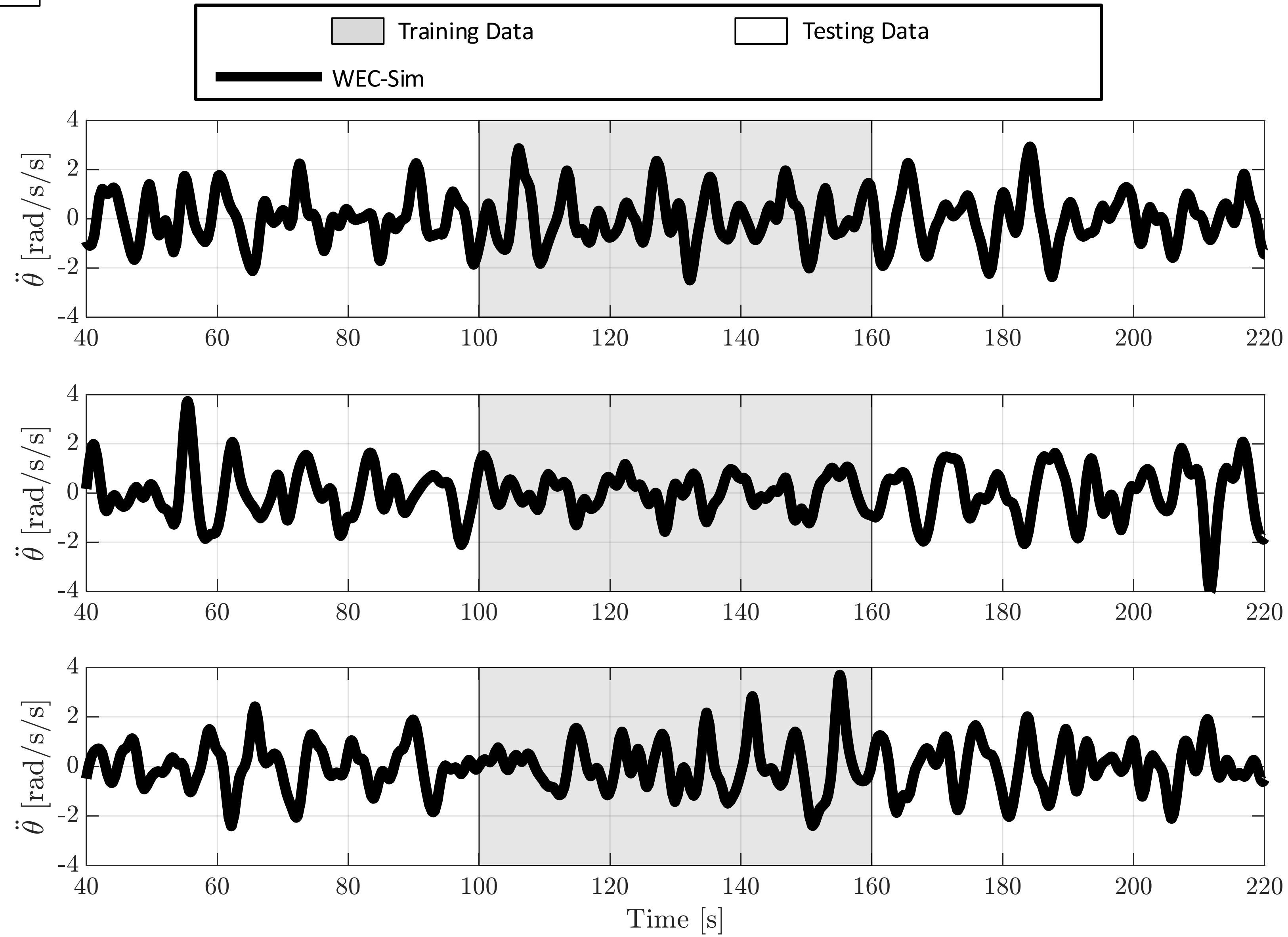

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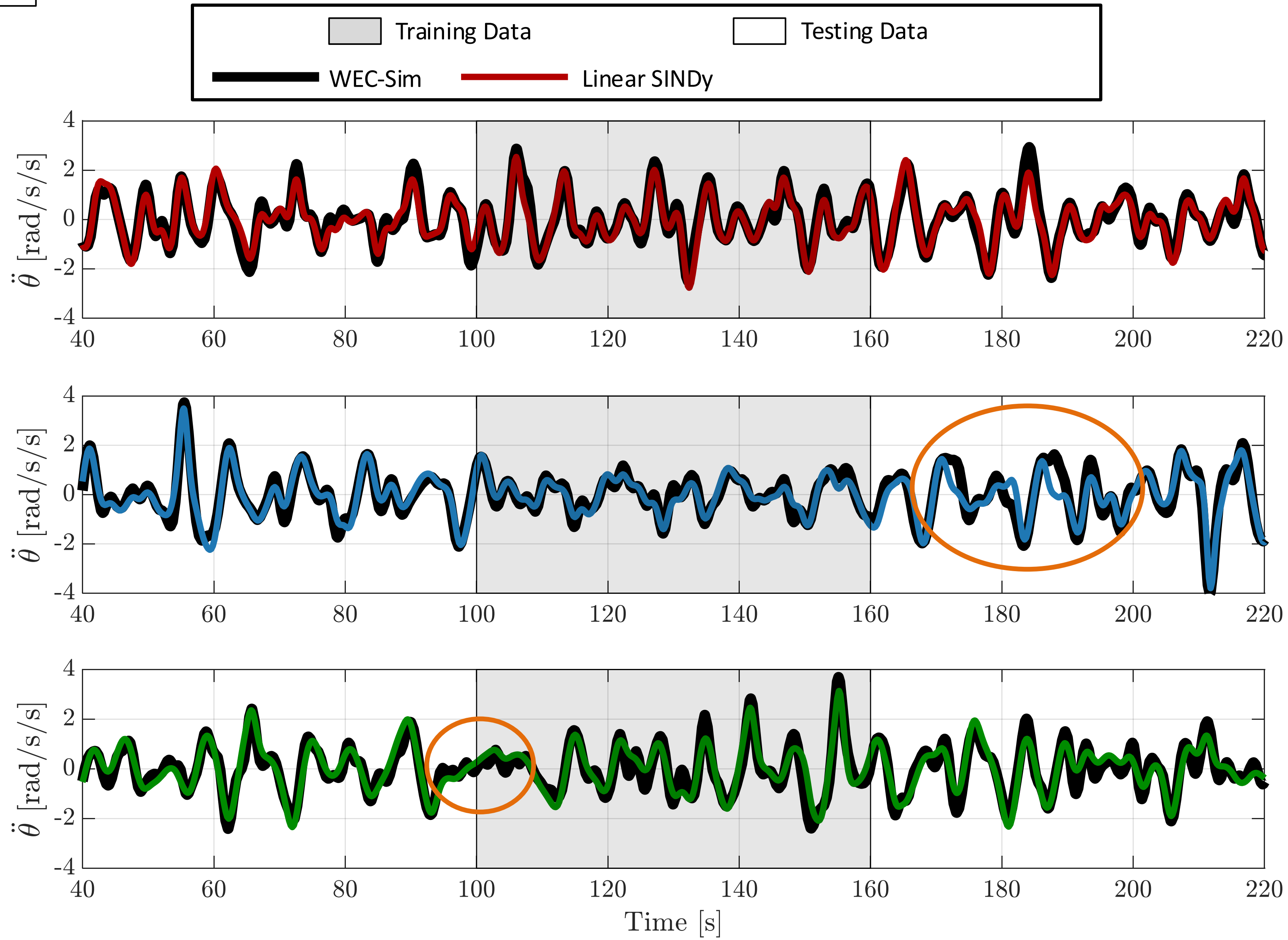

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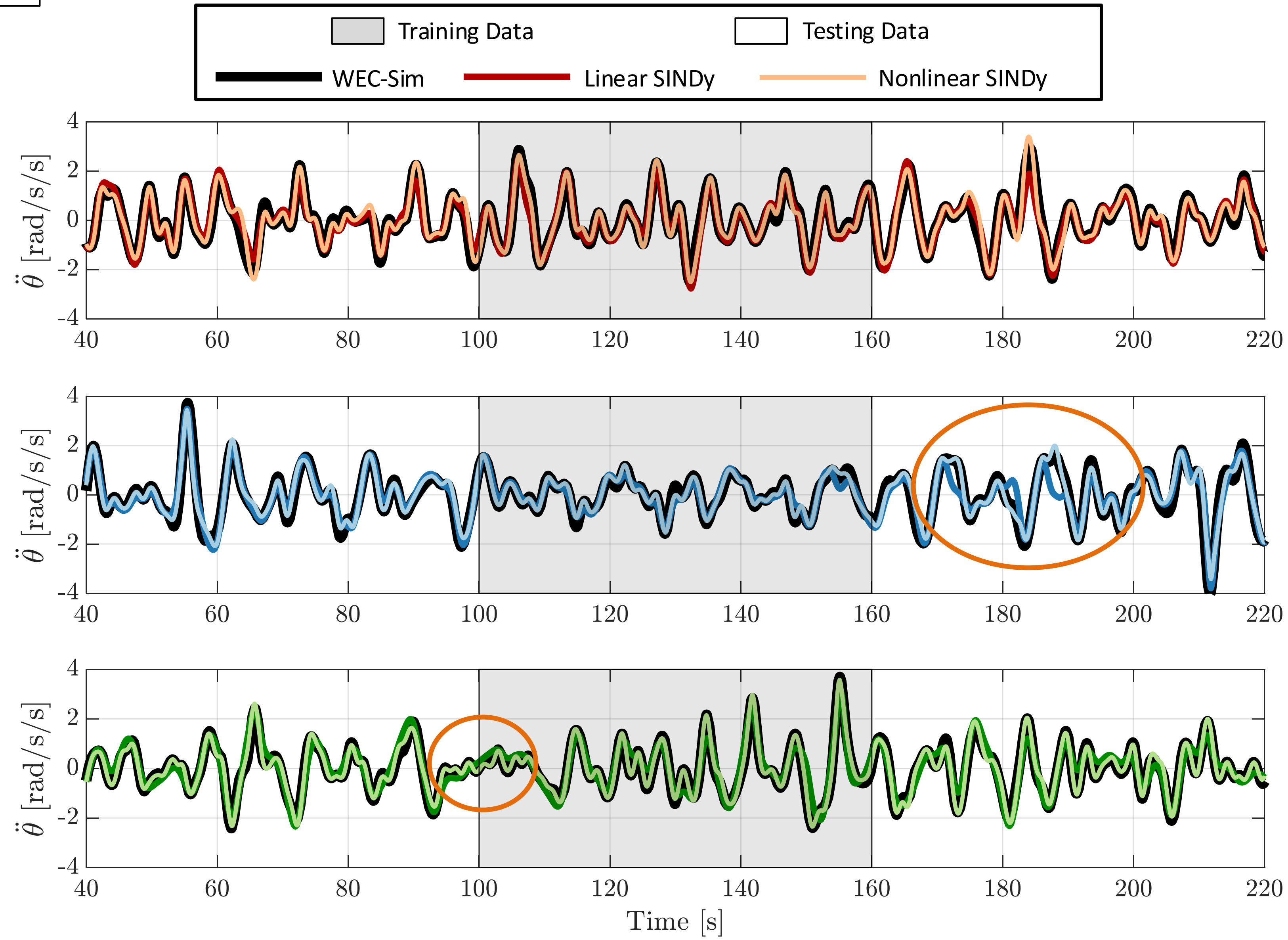

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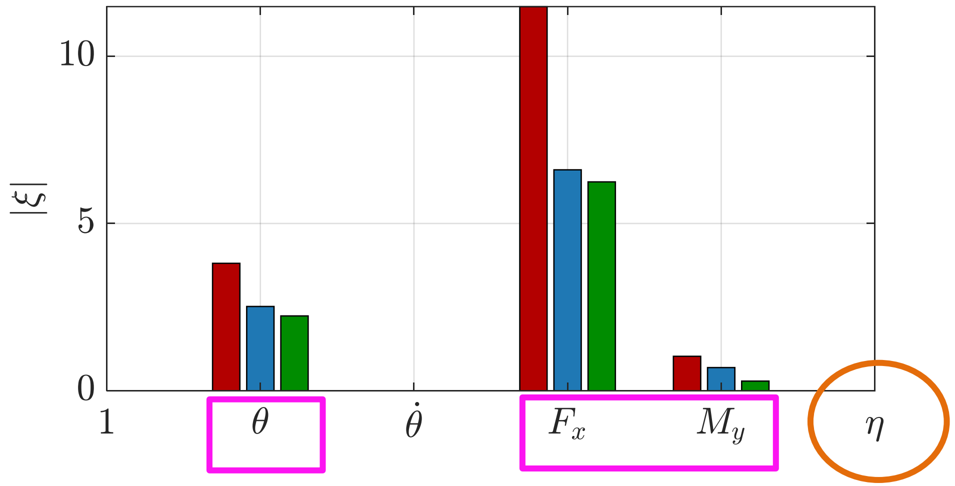
Results: SINDy Model

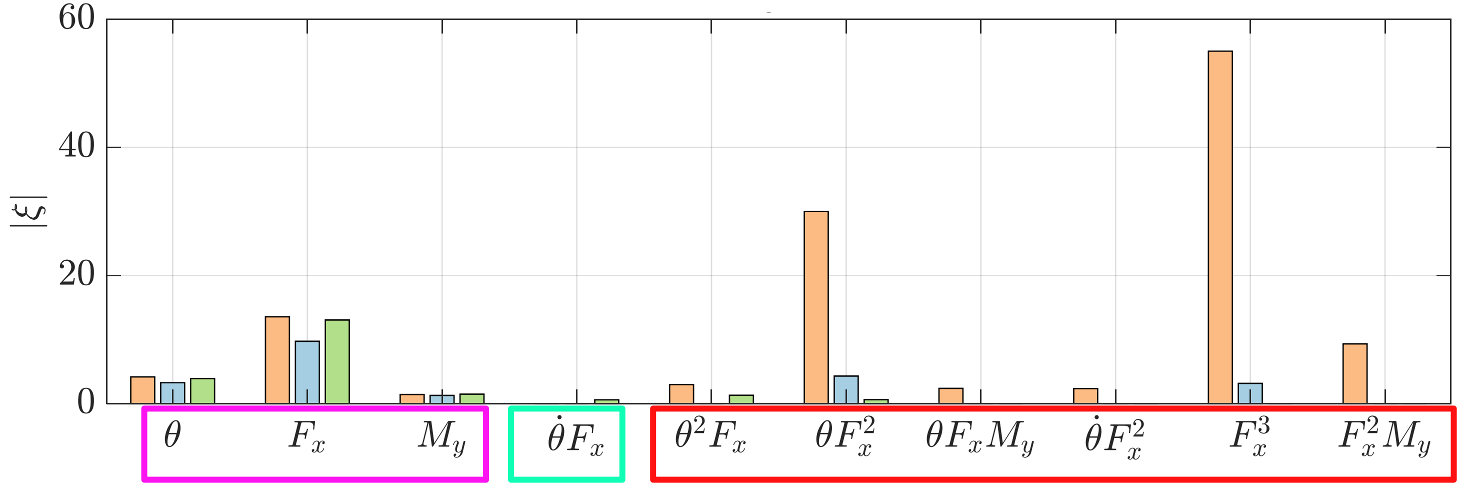
$$\ddot{\theta}_{SINDy} = \Lambda^{N}(\theta, \dot{\theta}, F_x, M, \eta)\xi$$

$$\varepsilon = \left. \frac{\left\| \ddot{\theta} - \ddot{\theta}_{SINDy} \right\|_2}{\left\| \ddot{\theta} \right\|_2} \right|_{30\,s}$$

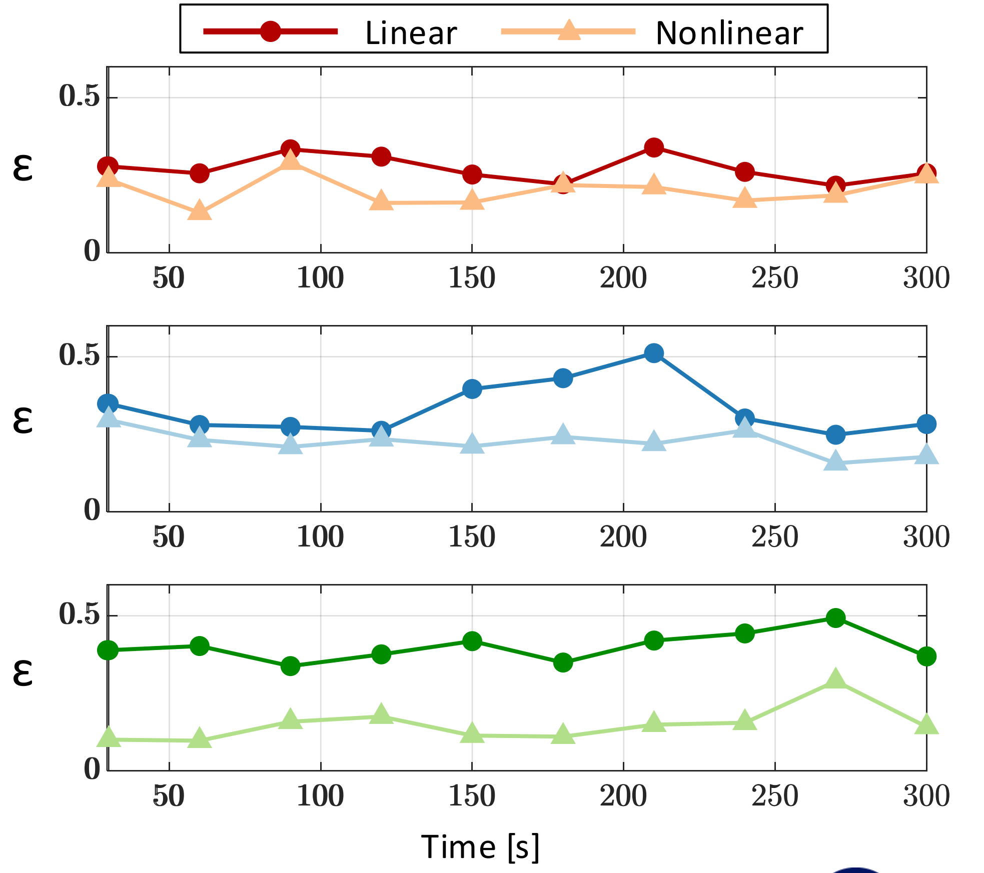
Function Weights – Linear (N=1)



Function Weights – Nonlinear (N=3)



Error



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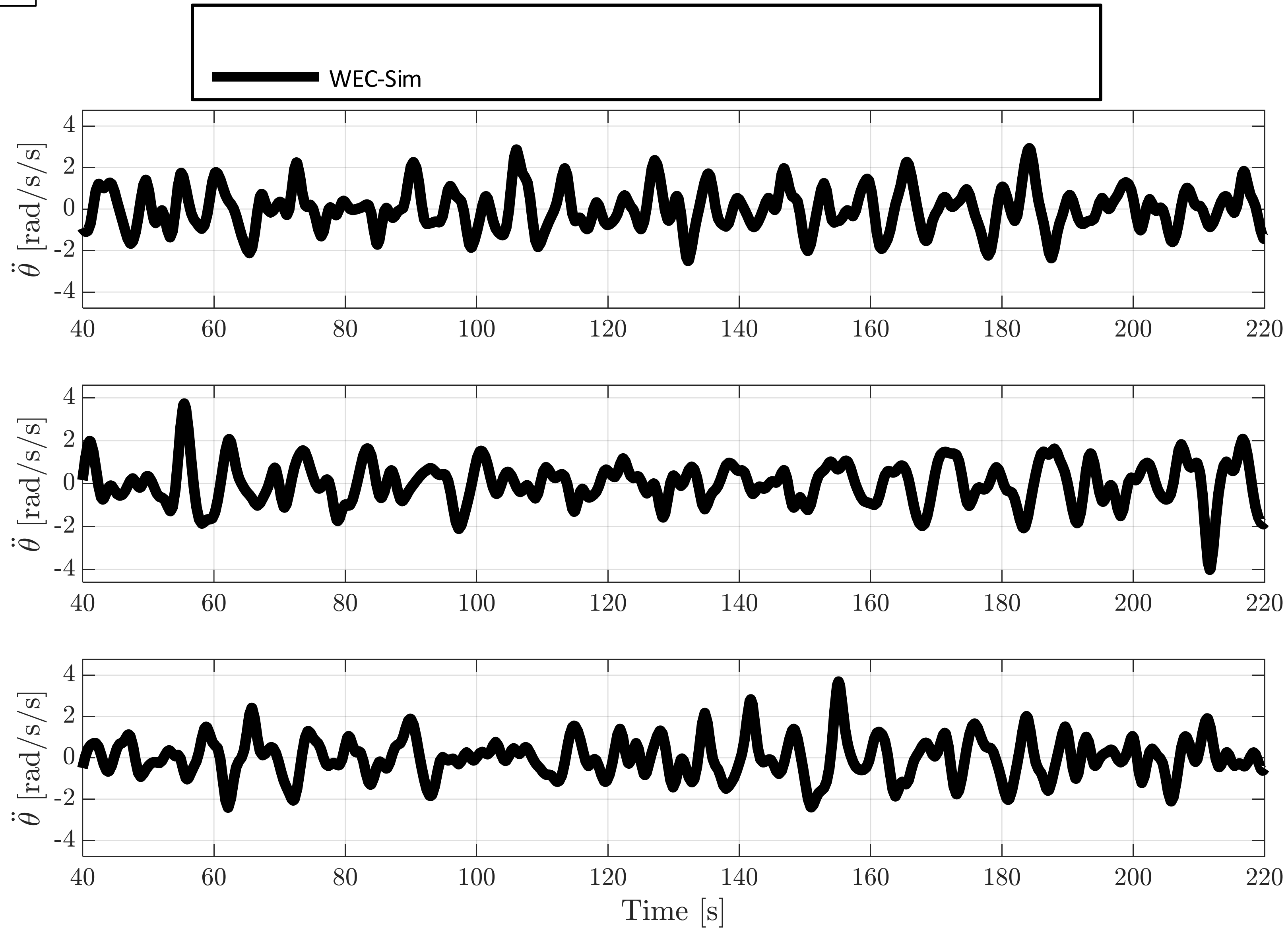
Results: Generalization



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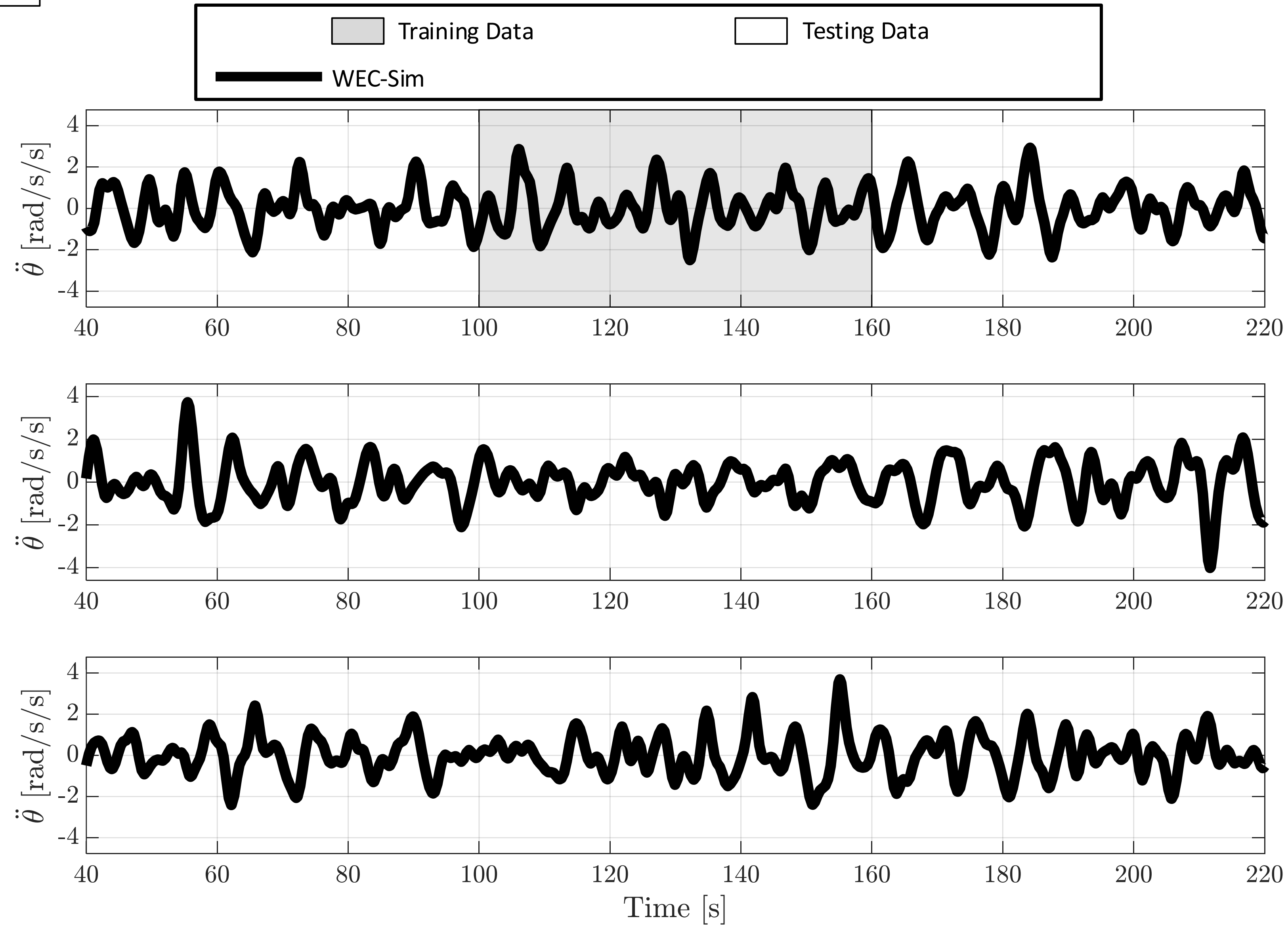

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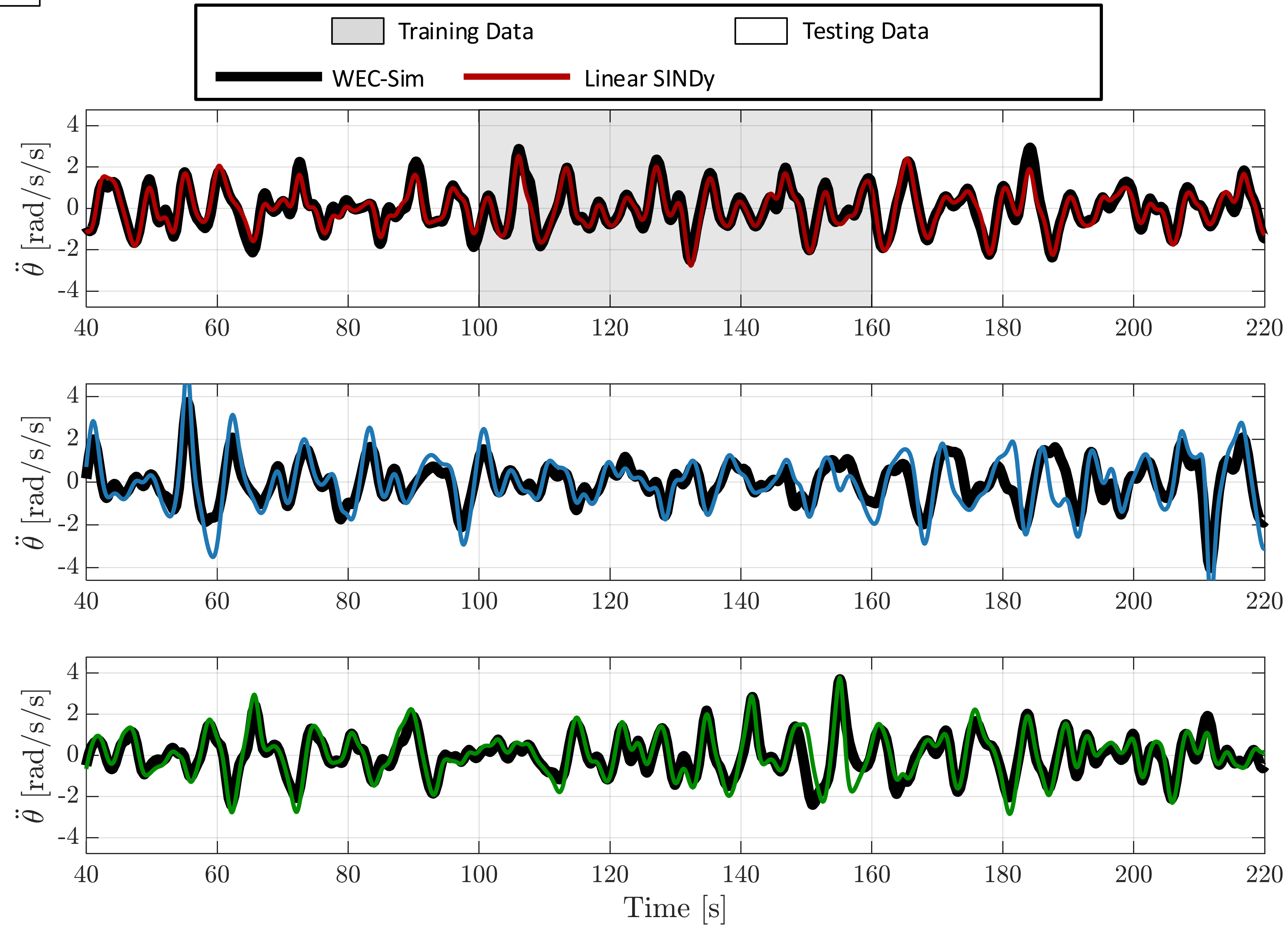

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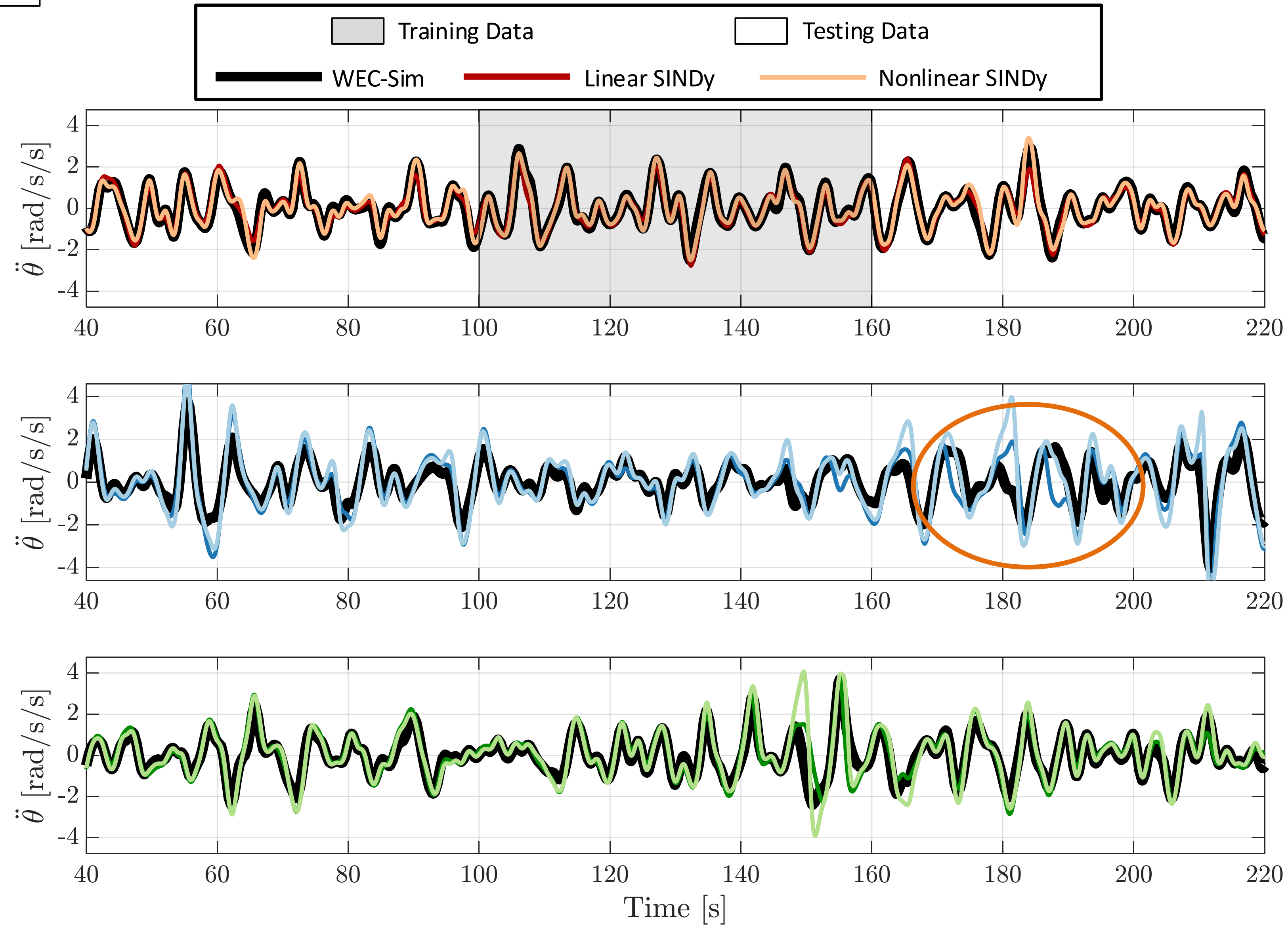
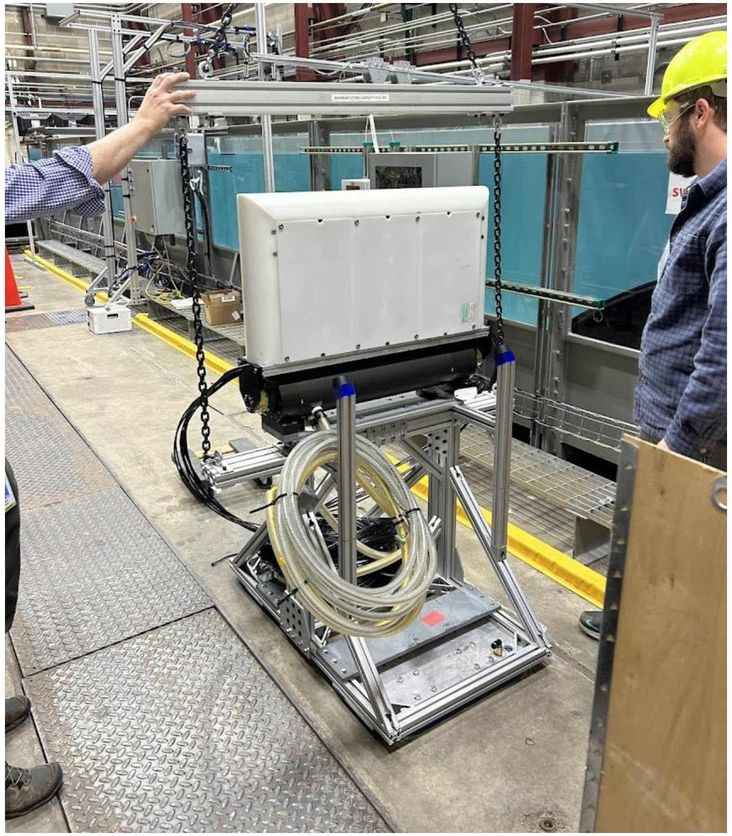


Conclusions

- SINDy can create models for OSWEC acceleration in irregular waves
  - Without using incident wave field
  - Both linear and nonlinear models capture dynamics well
  - Variety of sea states
- Composition of nonlinear models is mostly cubic
- Linear models could be slightly more generalizable than nonlinear models

Future work: Experimental Comparison

Experimental Device



SWEL Wave Tank at NREL

Acknowledgements

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Colleagues at MREL

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- Naval Facilities Engineering Command (NAVFAC)
- National Science Foundation Graduate Research Fellowship Program

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pmec.us

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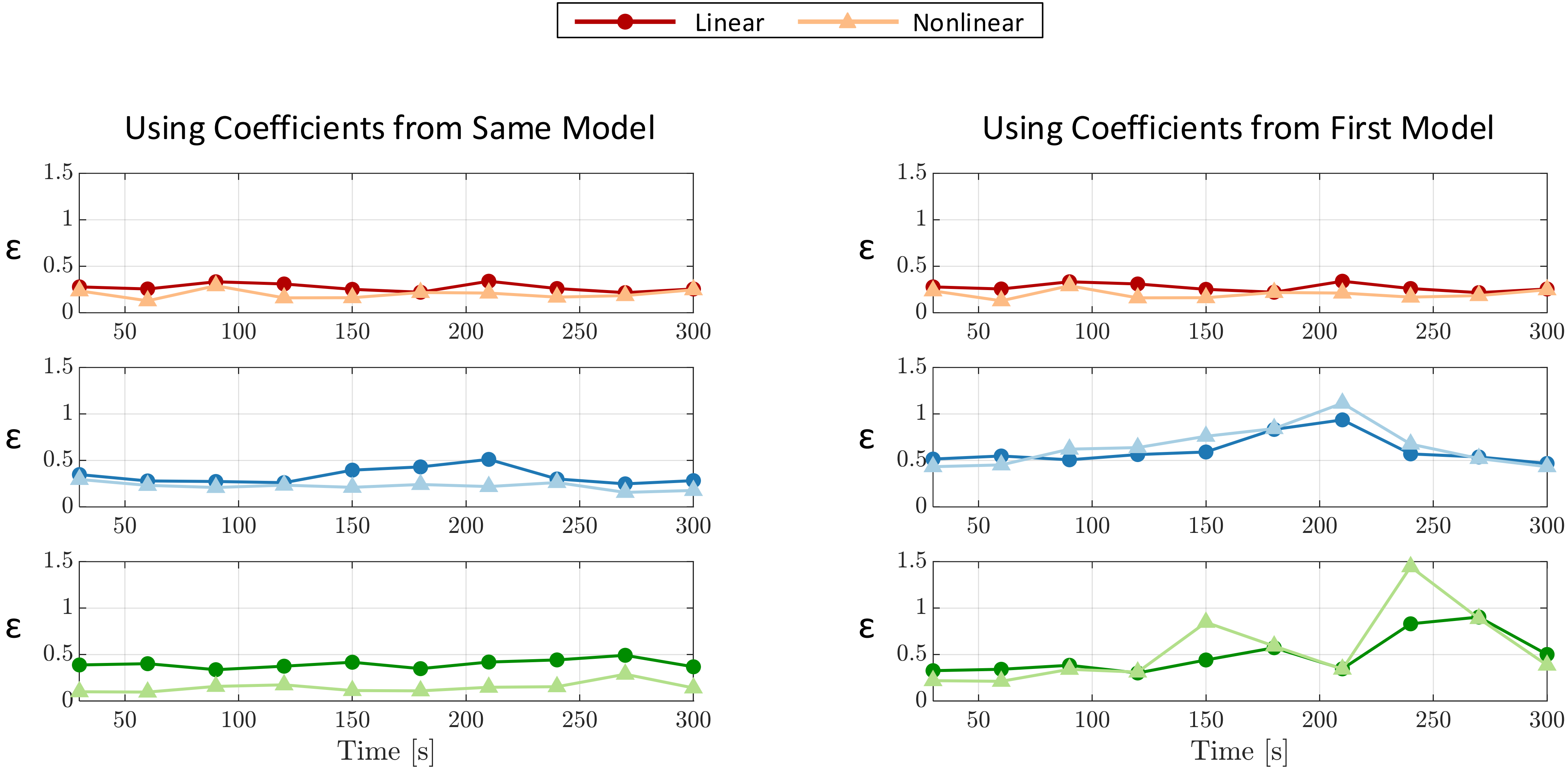
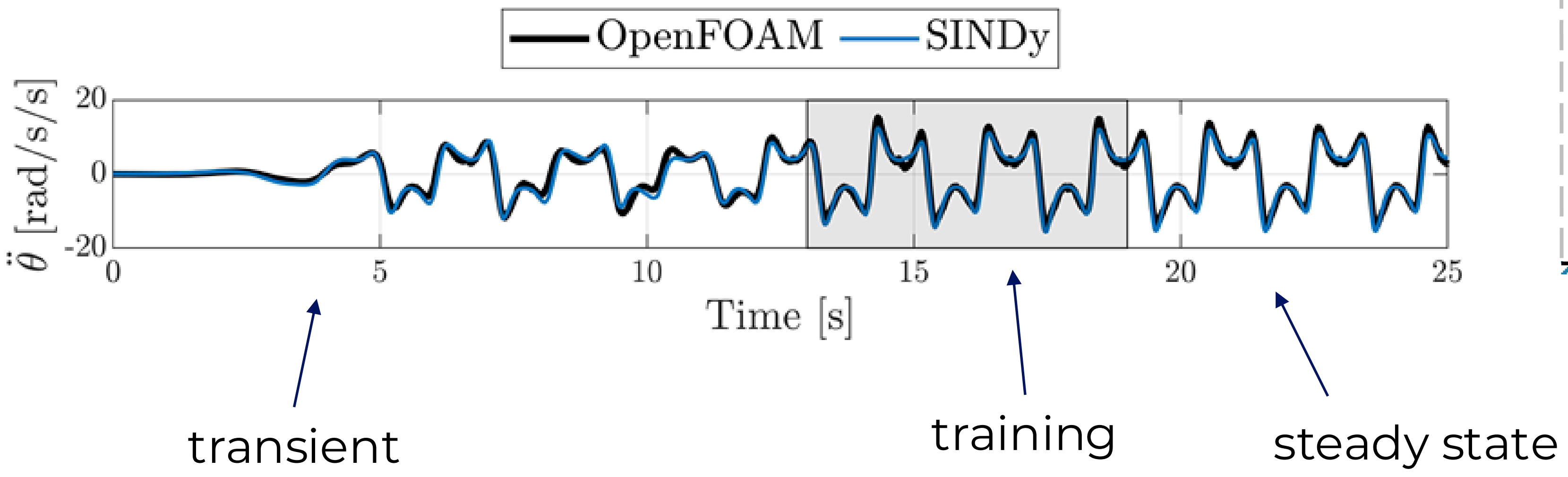
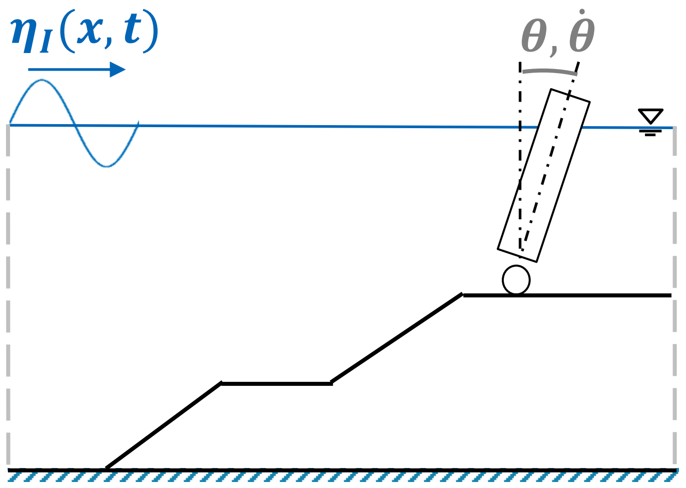


Ex: Nonlinear kinematics in regular waves

$$\ddot{\theta} = -8.7\theta + 1.8\dot{\theta} + 4.3\theta^3 - 8.2\theta^2\dot{\theta} - 2.4\theta\dot{\theta}^2$$





Based on experiments run at Queen's University, Schmitt & Elsaesser, 2015

Lydon, Brittany, Brian Polagye, and Steven Brunton. **"Nonlinear WEC modeling using Sparse Identification of Nonlinear Dynamics (SINDy)."** Proceedings of the European Wave and Tidal Energy Conference. Vol. 15. 2023.

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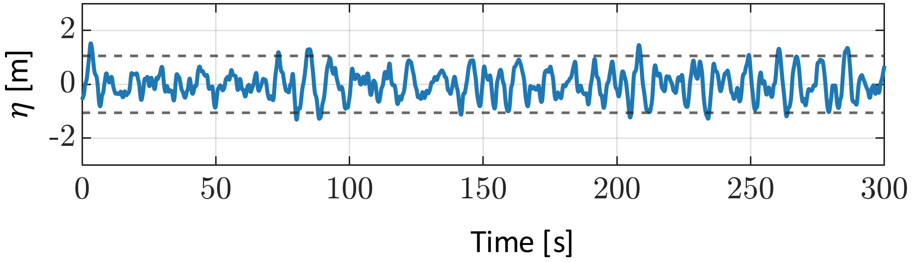

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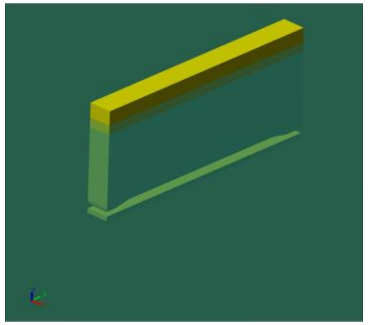

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Sparse Identification of Nonlinear Dynamics (SINDy)

Main idea: Generate parsimonious nonlinear reduced order models using only data

$$\frac{d}{dt}\boldsymbol{x} = f(\boldsymbol{x}, t)$$

dynamics (from data) → $\dot{\boldsymbol{X}} = \boldsymbol{\Lambda}(\boldsymbol{X})\boldsymbol{\xi}$ ← function weights (what SINDy solves for)

nonlinear

sparse

nonlinear function library

states (from data)

$$\min_{\boldsymbol{\xi}} \left[\left\| \dot{\boldsymbol{X}} - \boldsymbol{\Lambda}(\boldsymbol{X})\boldsymbol{\xi} \right\|_2 + \lambda \left\| \boldsymbol{\xi} \right\|_1 \right]$$

promotes accuracy

promotes sparsity