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Wave Energy Absorption by Floating Piezoelectric Plates

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Highlights

Using modes, the solution for wave forcing of multiple floating piezelectric plates is found. An energy balance relation is derived from first principles, and a numerical simulation is performed in the time domain.

1 Introduction

The efficient extraction of energy from ocean waves remains an ongoing challenge despite the large amount of energy available. Many different designs of wave energy converters (WEC) have been proposed. Recently, there has been significant interest in WEC that uses flexure to convert energy starting with Renzi (2016). Recent work includes Zheng et al. (2021), Collins et al. (2021), Zheng et al. (2022). The key to modelling the absorption of energy from flexure is to include an imaginary part in the stiffness in the frequency domain formulation. This appears for piezoelectric wave energy converters, for example. We consider here a solution which is based on the modal expansion and the calculation of the added mass and damping. This method is more standard in ocean engineering and allows the application of well-known methods and computational techniques. We also derive from first principles an energy absorption equation.

2 Equations of motion

We are modelling an elastic plate floating on the water surface that consists of layers of material, some of which are piezoelectric, following the work of Renzi (2016). This leads to a formulation in which the plate has complex stiffness. The imaginary part of this stiffness corresponds to the absorption of energy proportional to bending. The exact form of this energy removal is not essential; for example, the current formulation is identical to that for viscoelastic plates. Water is assumed to be homogeneous, inviscid, and incompressible, and its motion is irrotational and time-harmonic with an angular frequency ω. Therefore, the water velocity field is defined as

$$\Phi(x, z, t) = \mathrm{Re}\{\phi(x, y, z)\mathrm{e}^{-\mathrm{i}\omega t}\}, \tag{1}$$

where Re denotes the real part, t is time, and the scalar function ϕ is a complex-valued velocity potential. The displacement of the plate is $w(x, y)$. The velocity potential satisfies Laplace's equation and the impermeable-bed condition. Except under the plate, the free surface condition is $\partial_z \phi = \alpha\phi$ where $\alpha = \omega^2/g$.

Assuming the plate remains in contact with the water, ϕ is coupled w by

$$\partial_z \phi = \alpha w, \qquad \text{and} \qquad \phi = w + (\beta_r + i\beta_i)\partial_x^4 w - \alpha\gamma w, \tag{2}$$

for $-L < x < L$ (under the plate) and $z = 0$. Here γ is the mass per unit area of the plate scaled with respect to the water density, and $\beta = \beta_r + i\beta_i$ is the complex rigidity of the plate scaled with respect to the water density. We assume free boundary conditions $\partial_x^2 w = \partial_x^3 w = 0$. Figure 1 shows a schematic diagram of the problem.

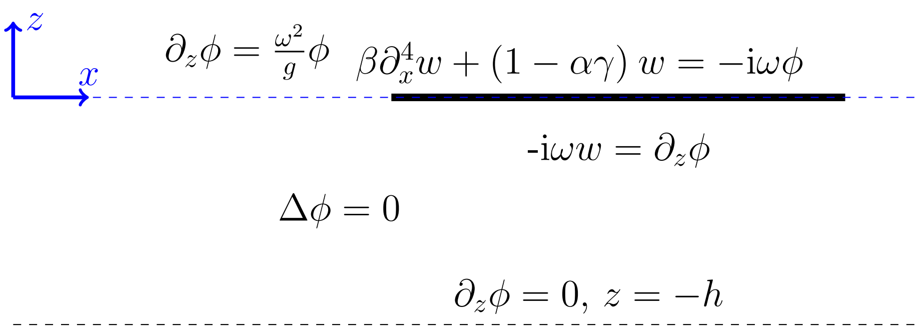

Figure 1: Schematic diagram showing the equations of motion.

The plate movement is expanded in the modes of free vibration. This method is standard in ocean engineering, but it is not widely applied to wave energy conversion. The method does not require a solution of a compact dispersion equation like eigenfunction matching, and it is much more general and flexible in its application. The expansion is

$$w(x) = \sum_{j=1}^{\infty} u_j w_j(x), \tag{3}$$

with coefficients u_j, where the modes w_j, $j = 1, 2, \ldots$, satisfy the eigenvalue problem for the biharmonic operator,

$$\partial_x^4 w_j = \mu_j^4 w_j, \qquad \text{for} \qquad -L < x < L, \tag{4}$$

together with the free-edge conditions (or other conditions such as clamped), where μ_j^4 are the eigenvalues numbered in ascending order of their magnitude. The modes represent the free vibrational modes of the plate in vacuo. We form an orthonormal basis from these modes. By linearity, the velocity potential is expanded as

$$\phi(x,z) = \phi^{\mathrm{in}}(x,z) + \phi^{\mathrm{di}}(x,z) - \sum_{j=1}^{\infty} u_j \phi_j^{\mathrm{ra}}(x,z), \tag{5}$$

where ϕ^{in} is the incident wave. We solve for the diffracted ϕ^{di} and radiation potentials ϕ_j^{ra}, using a Green function. A linear system for the modal weights is

$$\left(\mathbf{K} + \mathbf{C} - \alpha\mathbf{M} - \omega^2\mathbf{A}(\omega) - \mathrm{i}\omega\mathbf{B}(\omega)\right)\boldsymbol{u} = \mathbf{f}(\omega). \tag{6}$$

Here $\mathbf{K}$, $\mathbf{M}$ and $\mathbf{C}$ are stiffness, mass, and hydrostatic-restoring matrices, respectively. They are given by

$$\mathbf{K} = \lceil \beta \mu_j^4 \rfloor_j, \qquad \mathbf{M} = \gamma \mathbf{I}, \qquad \text{and} \qquad \mathbf{C} = \mathbf{I}, \tag{7}$$

where $\lceil c_i \rfloor_i$ denotes a diagonal matrix with diagonal entries c_i, and $\mathbf{I}$ is the identity matrix. Moreover, the elements of the real-valued added mass matrix $\mathbf{A} = [A_{ij}]_{ij}$ and real-valued damping matrix $\mathbf{B} = [B_{ij}]_{ij}$ are defined as

$$\omega^2 A_{ij} + \mathrm{i}\omega B_{ij} = -\int_{-L}^{L} \phi_j^{\mathrm{ra}} w_i \,\mathrm{d}S, \tag{8}$$

and the elements of the forcing vector $\mathbf{f} = [f_i]_i$ are given by

$$f_i = \int_{-L}^{L} \left(\phi^{\mathrm{in}} + \phi^{\mathrm{di}}\right) w_i \,\mathrm{d}S. \tag{9}$$

2.1 Multiple plates

We extend this formulation to the case of two plates. A linear system for the modal weights is obtained as before

$$\left(\begin{pmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{pmatrix} + \begin{pmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{pmatrix} - \alpha \begin{pmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{pmatrix} - \omega^2 \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{pmatrix} - \mathrm{i}\omega \begin{pmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{pmatrix} \right) \begin{pmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{f}(\omega)_1 \\ \mathbf{f}(\omega)_2 \end{pmatrix}, \tag{10}$$

where $\mathbf{K}$, $\mathbf{M}$, and $\mathbf{C}$ are as previously and the added mass and damping elements are

$$\omega^2 A_{ij}^{mn} + \mathrm{i}\omega B_{ij}^{mn} = -\int_{-L}^{L} \phi_{jn}^{\mathrm{ra}} w_{im} \,\mathrm{d}S, \tag{11}$$

and the elements of the forcing vector $\mathbf{f} = [f_i]_j$ are given by

$$f_i^n = \int_{-L}^{L} \left(\phi^{\mathrm{i}} + \phi^{\mathrm{di}}\right) w_{in} \,\mathrm{d}S. \tag{12}$$

where m and n are indices for the two plates.

3 Energy Balance

The system is subject to incident wave energy and radiated and transmitted energy. We can derive the energy balance by considering the following integral

$$\iint_{\Omega} \phi^* \Delta\phi - \phi \Delta \phi^* \mathrm{d}S. \tag{13}$$

This leads to

$$c_g \left(1 - |R|^2 - |T|^2\right) = \frac{1}{\omega} \int_{x=-L}^{L} \mathrm{Im}(\beta) \left|\partial_x^2 w\right|^2 \,\mathrm{d}x, \tag{14}$$

where $c_g = \frac{1}{2\omega}\left(\tan(kh) + kh \sec(kh)^2\right)$ is the group velocity. The right-hand side is the power absorption from the flexible plate while the left-hand side is the rate of energy loss at infinity, i.e. the difference between the incident wave power and the transmitted and reflected wave power. This matches the formula in Renzi (2016) derived from the piezoelectric equations.

4 Results and Conclusion

An example simulation in the time domain is given in Figure 2 showing energy absorption. The numerical method can be seen to be well suited to analyse piezoelectic WEC. More examples will be presented in the workshop.

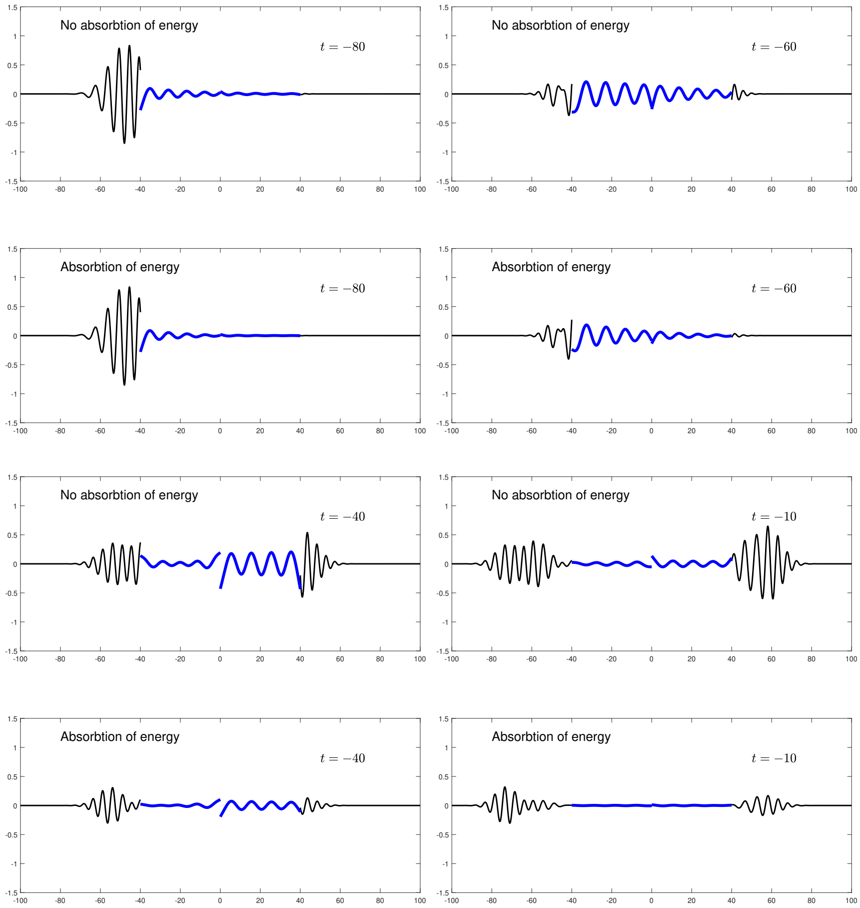

Figure 2: Time dependent motion for two floating elastic plates subject to an incident wave pulse. The top figures are for non-energy absorption $\beta = 10$ and the bottom is for energy absorption $\beta = 10 - 2i$. The absorption of energy is clearly apparent.

References

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