Tidal Power Arrays and the Coriolis Force with Array Design Considerations

by

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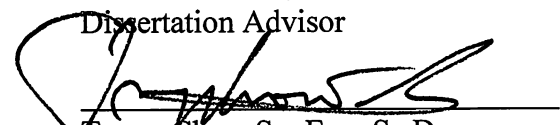**Tidal Power Arrays and the Coriolis Force with Array Design Considerations**

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This dissertation was prepared under the direction of the candidate's dissertation advisor, Dr. Manhar Dhanak, Department of Ocean and Mechanical Engineering, and has been approved by all members of the supervisory committee. It was submitted to the faculty of the College of Engineering and Computer Science and was accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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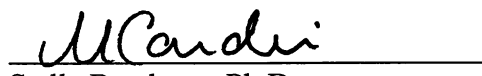
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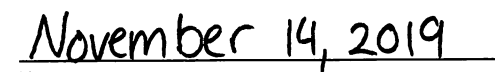
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Vita

Victoria Miglietta began her research in ocean energy at the University of Hawaii with Dr. Alex Malahoff. From 2008 to 2011 she studied tidal energy in Cook Strait New Zealand, focusing on resource assessment and development feasibility. Miglietta completed her doctoral research under the supervision of Dr. Manhar Dhanak and, Dr. Pallaniswamy Ananthakrishnan from 2013 to 2019. Her interests are in large offshore structures and resource utilization. Miglietta has a Bachelor of Science in Earth Science with a minor in physics from the University of California at Santa Cruz, and a Master's degree in Ocean & Resources Engineering from the University of Hawaii at Manoa. She is a licensed engineer, a seasoned scuba diver and a mother of three. For leisure Mrs. Miglietta enjoys seafood and the outdoors with her family, and she also enjoys reading nonfiction.

Acknowledgements

Abstract

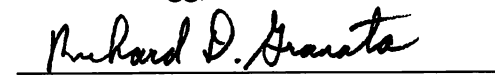|	
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Tidal currents are a renewable energy resource and the work presented is in the field of harnessing tidal currents for electrical power generation. The main objective of the research is to provide information on rotational flow effects, caused by the spinning of the earth, around obstacles on the sea floor, in support of developing robust design of an underwater turbine array. This research looks at a gravity based linear array, a single turbine deep, with its largest dimension several kilometers long. The primary goal is to model a Taylor column above a linear array. The Taylor column has closed streamlines or stagnant flows inside of it and the flows around the column accelerate asymmetrically. The layout design of the array is intended to minimize the effect of the stagnant flows by predicting the location where closed streamlines could develop. The design is for the array and not for a turbine. Also, the locations where the energetic flows through the array have the longest periods are identified.

Numerical modeling with ANSYS Fluent failed repeatedly to accurately model rotational effects around an obstacle with a minimal relative current so as to form a

Taylor column. Instead, Johnson's (1982) analytical solutions for quasigeostrophic flows over elongated topography are used to study how the blocking parameter influences streamlines with changes in velocity typical of a tidal change. The streamlines illustrate the location over an array where the flows are accelerated and also where closed streamlines form.

While Taylor columns are not expected to form distinctively in the short slack tides, the results indicate that rotational flows manifest on the southern end of an obstruction oriented N-S with an eastward flow. Viewing an array as the obstruction, the south end would experience drops of velocity with the changing tide first. The study was done in the f-plane, meaning the Coriolis parameter was held constant. The turbines attempting to harness the current acceleration around a Taylor column would physically need to be placed opposite from the side of the array experiencing low Rossby numbers first. Such turbines would need to be sensitive to currents <1m/s.

Dedication

I dedicate this work to my husband Mario and my three children Joseph, Michelle and Chiara.

Tidal Power Arrays and the Coriolis Force with Array Design Considerations

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Nomenclature

1) Blocking parameter- a dimensionless number used in the analysis of rotational flows, see Equation 2.

2) Coriolis force – a fictious force caused by the rotation of the Earth. In the rotating reference frame of Earth, the Coriolis force is an inertial force which can be observed in the northern hemisphere in the motion of objects as a deflection to the right, and in the southern hemisphere the deflection is to the left.

3) Megawatt - million watts. A commercial power facility will typically produce electricity on the scale of several megawatts for continuous periods.

4) Geostrophic- an equilibrium state which describes a condition where the Coriolis force and the pressure gradient balance each other out.

5) Quasigeostrophic- flow in which the Coriolis force balances the pressure gradient and small inertial forces.

6) Rossby number – a dimensionless number which represents the ratio of inertial forces to rotational forces, see Equation 1.

7) Taylor column- best described as a physical 'shadow' to oncoming flow, the Taylor column forms above and possibly below an obstruction in quasigeostrophic flow and acts as a barrier to flow. It can appear as closed streamlines or a stagnant region however the physical dynamics of the interior of a Taylor column are not well understood.

Introduction

A renewed focus on the ocean as a source of renewable energy was apparent in popular news publications and published research beginning in approximately 2006. The Department of Interior created the Bureau of Ocean Energy Management (BOEM) in 2010 when the Minerals Management Service was reorganized after the Deepwater Horizon oil spill. The BOEM manages offshore leases in the United States exclusive economic zone (EEZ) and last year the revenue from renewable ocean energy leases surpassed those of offshore oil leases.

Various designs of electrical power turbine generators have been developed for ocean tides and many prototypes have been tested. Furthermore, locations of strong tidal currents have been sought out and identified worldwide. No commercial scale power operation of these new designs has yet been developed. The old style of a tidal barrage has been the only municipal scale operating design for tidal current power. Hesitation to duplicate tidal barrage operations are rooted in the environmental impact of a tidal barrage, which are similar to those of dams. The new turbine generators being designed are developed as stand-alone turbines, which are expected to be deployed in arrays, or "fields," much like wind turbine generators.

This research was on the engineering design of a turbine array in an unsteady or oscillating tidal flow. The design is for the array, which can be composed of turbines chosen by a developer. No turbine designs were analyzed in this study. The current speed variation is considered for utilization in electricity production and not the head

changes associated with the tides. This study investigated how the obstruction to flow

caused by the array affects the incident current's flow pattern, and how the array's layout

could be designed to make the array more robust.

The Coriolis force and its effects were incorporated into the study. Typically, the

Coriolis force is minimal and is neglected in engineering analysis. However, in this study,

we took the Coriolis force into consideration and analyzed its effects on flows at low

Rossby numbers as a primary focus of the study. The Coriolis parameter, given by

$$f = 2\Omega \sin(\theta)$$

Equation 1:The Coriolis parameter , f , which varies with latitude θ. Ω is the angular velocity of the earth which is 7.2921E-5 rad/sec.

The higher the latitude, the larger the Coriolis parameter and the greater the effects of

Earth's rotation on a body or parcel of fluid. Along with the Coriolis parameter the

dimensionless Rossby number is used to describe the significance of rotational effects.

The Rossby number is the ratio of inertial forces to the Coriolis force, given by:

$$R_0 = U/fL$$

Equation 2: Rossby number is a dimensionless value where U is the current velocity, f, is the Coriolis parameter and L is the charactersitic length, the longest horizonal dimension.

Background

Tidal current power is an ocean renewable energy which gains the support of public and private funds for research and development as oil prices rise. Also, remote coastal communities desiring to limit their dependence on imported diesel are investing in developing technologies which harness tidal current power. Renewable energy sources are much less concentrated than conventional fossil fuel. For this reason, a power plant using renewable energy is much larger than a conventional power plant. For example, Florida's Crystal River coal power plant sits on approximately 19km^2 (4,695 acres) and produces approximately 2,280 MW, or 120MW/km^2. Furthermore, California's San Gorgonio wind farm produces 358MW and sits on approximately 181km^2 (44,726 acres), or just under 2MW/km^2. A tidal power plant would be expected to produce more than that of wind because of the high density of water, yet significantly less than that of coal. However, the size of the plant would be comparable to a wind farm. A power plant needs to produce a minimum of hundreds of megawatts to match the economy/scale of existing power plants. This is why the nominal dimension of 10,000m was held as a constant dimension (length) for the array, as its maximum dimension. The array foundation was assumed to be gravity based, or bottom mounted.

Most of the ocean floor can be described as featureless, sometimes described as a flat abyss. An array spanning 10,000m in length is comparable to a small seamount. Research on circulation around seamounts was reviewed to better understand how an oscillating tidal flow would behave around a feature as large as a renewable energy

installation on the seafloor. It is assumed that during periods of peak tidal currents an array of turbines would spin continuously and produce electricity. Keeping the array producing electricity for the longest periods and delivering maximum power from the array would be primary goals for its operators.
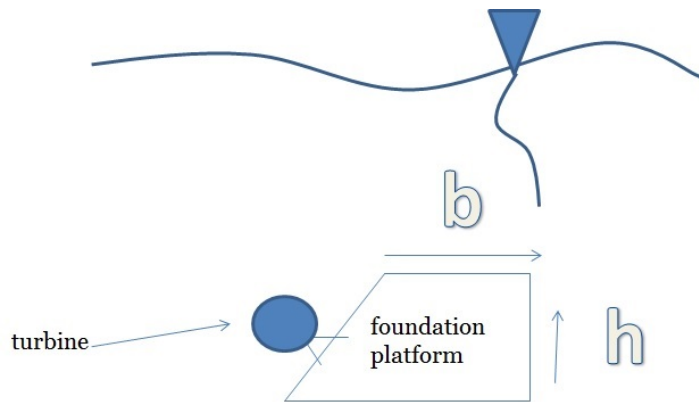
Figure 1 A schematic of a tidal current turbine attached to a static foundation platform.
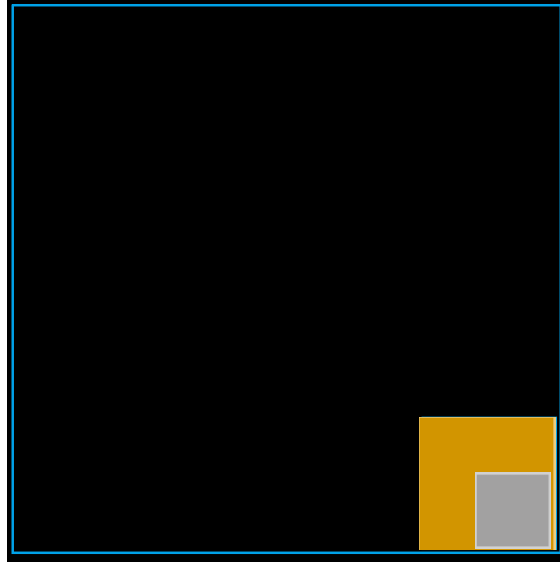
Figure 2 The concentration of energy from renewable sources is much smaller than fossil fuels which is why renewable energy operations are so large. Here the squares are representative of the amount of power produced per square kilometer of a power plants footprint on a map. Coal power is represented by the largest square with geothermal and wind energy shown as the smaller two squares on the bottom right.

While most information about the new turbine technologies is not available to the public, it is known that most begin producing power when currents flow through them at approximately 1m/s (1.94 knots); this is known as the cut-in speed. This in itself is a requirement that is met only by the strongest tidal current flows. Locations with high tidal currents which can be identified as potential sites for tidal energy are, Puget Sound in Washington State, Cobscook Bay in Maine, New York City's East River, Canada's Bay of Fundy, New Zealand's Cook Strait and the Gimsøystraumen Strait in Norway.

Here speeds below 1m/s are investigated. A better understanding of the slack tide period and how flows move around an obstruction is the aim.

Furthermore, it is the author's opinion that a successful tidal power array would not interfere with marine traffic. For this reason, it is assumed that every part of a tidal

power system would be a minimum of 15m below the surface. This is typically the maximum draft assumed for cargo tankers. It is assumed that the dynamics related to Ekman transport would be minimal at these depths, so Ekman layer effects will be ignored.

Literature Review

A preliminary research focus was how previous numerical models of flows on the scale of hundreds of meters to kilometers were set-up. Research quickly brought attention to the work done on the effect of seamounts and seamount chains on ocean circulation. Interestingly, this research follows decades of studies on seamounts, which are natural, large seafloor structures. Roden (1987) wrote a chapter, "Effects of Seamounts and Seamount Chains on the Ocean Circulation and Thermohaline Structure," in the book, Seamounts, Islands, and Atolls. It is an excellent primer on the subject and is highly recommend reading.

Seamounts are known to enhance boundary mixing in stratified flow, they generate wavetrains, eddies, cause flow intensification as well as flow deflection and give rise to Taylor columns (Roden 1987). Seamounts are like flat topped underwater mountains. They rise hundreds of meters, even kilometers, above the sea floor and can be kilometers in diameter. Even a small seamount could be 5 km across. Eddies and vortices have been modelled over seamount like obstructions in conditions where inertial forces were low, i.e. ocean current flow speeds were low (Verron & Provost 1985; Owens & Hogg 1979). Also, flow past seamounts has been measured and observed to increase in velocity on the flanks of seamounts by multiple times that of the far field velocity (Carter, Gregg and Merrifield 2006). Reported point velocity increases of up to 10 times (ibid) have been measured at seamounts.

Obstructions to Flow

Relating to seamounts, several papers have been published on quasigeostrophic flow over low seafloor structures. Quasigeostrophic flow is an ocean current where the Coriolis force and the pressure gradient is just prevented from balancing by small inertial forces. Under calm atmospheric conditions the period of minimal tidal currents around slack tide could be described as quasigeostrophic and an obstruction to quasigeostrophic flow can produce a Taylor column (Owens and Hogg 1979). Hide (1961) was the first to report a critical blocking parameter, $S_{critical}$, of order one for Taylor column formation. Hide is best known for his work on Jupiter's red spot, which was theorized by Hide to be a Taylor column. Taylor (1917) was the first to hypothesize the rigid column type dynamics in a rotational fluid with a relatively slow transverse current and he created one in a rotating tank with a coin which was moved as the obstruction.

Johnson (1982) studied flow effects of isolated elongated ellipses in the path of quasigeostrophic flows. In Johnson's model the fluid was not stratified. The models were in the f plane where the Coriolis parameter does not vary with latitude. Johnson's aim was to analytically describe the horizontal range of the effects caused by raised topography or bathymetry. His 1982 study concluded that the longest horizontal dimension of the ellipse determined the extent of the region affected by the topographical obstruction to flow

$$S = \frac{h_0}{(d \cdot R_0)}$$

Equation 3: Blocking Parameter. Height of an obstruction over the product of the water depth and the Rossby number (Hide 1961). When S is a critical value, $S_{critical}$ is O(1), closed streamlines over an obstacle may form in a homogenous flow.

For greatly elongated topography of variable cross section he concluded that the velocity parallel to the ridge was also proportional to the Coriolis parameter. Solutions to flow patterns were functions of the aspect ratio of the cylinder, long axis/ short axis, and the angle of the major axis of the ellipse to the impinging flow. In Johnson's model for an infinitely long ridge, flows increased their velocity on the left hand side looking down stream and decreased by the same amount on the right. For an elliptical cylinder in line with impinging flow Johnson concluded that this equaled the blocking parameter times the velocity of impinging flow. The increase in velocity decayed away from the obstruction on the scale L, length of long axis of the ellipse. The longest axis of the obstruction being referred to as the characteristic length. Johnson noted an unusual feature of very elongated ridges in line with flow, i.e. aligned parallel to streamlines. This feature was observed in his numerical models. The feature was that far upstream, the fluid to the right of the obstruction (looking downstream), turns to the left side to pass the obstruction on the left side before crossing back to the right. Furthermore, the larger (higher and/or longer) the obstruction, the greater the volume of fluid that crosses left, then back again. Similar findings were concluded by Carter, Gregg and Merifield (2006).
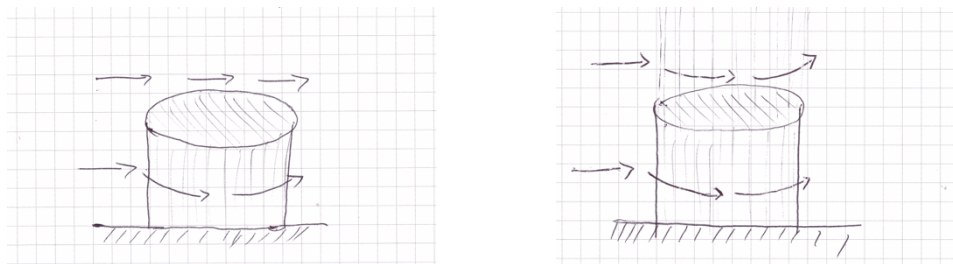
Figure 3 The concept of a Taylor column is displayed in the sketch on the right. On the left is a flow moving over an obstruction in an irrotational fluid. On the right, a Taylor column is present over the obstruction causing the water above it to behave as an extension of the obstruction. The Taylor column blocks the flow, causing it to divert as if there was a solid obstruction there.

McCartney (1975) showed that flow patterns, past an obstruction in a stratified quasigeostrophic flow, are dependent on their direction in relation to the greater rotating system in which they exist. The quasigeostrophic assumption removes surface gravity waves, inertial waves, Kelvin waves, and the topographical trapping of waves (Roden 1975). McCartney's model had 2 fluid layers. Under conditions like those in the northern hemisphere, a westward quasigeostrophic flow had patterns on either side of an obstruction that were similar, with a Taylor column over the obstruction, which McCartney described as a trapped anticyclonic eddy. In an eastward flow a large amplitude wake formed past the obstruction and the flow patterns on either side of the obstruction were not similar, the downstream pattern was much more turbulent.

Mason & Sykes (1981) modeled how water flows are affected by a bell shaped obstacle on the bottom boundary, in a rapidly rotating system. Their study was on a small scale with the obstruction having a length of 2cm. The tank had a thin, 3mm, plate on its surface and the depth of their fluid was 20cm. Their study concluded with affirmations of the Taylor Proudman theorem by defining a parameter $T_L = L/DR_0$ which was related to the character of the flow. For $T_L \gg 1$ the flows appeared two dimensional, in accordance with the Taylor Proudman theorem, and for $T_L \sim 1$ a Taylor column appeared over the obstacle. Mason and Sykes $T_L = L/DR$ is similar to Hide's blocking parameter. Hide uses the height of the obstacle where Mason and Sykes use the length. Johnson's solutions use the largest horizontal length as the characteristic length and also use the aspect ratio and angle of incident flow to determine streamline patterns. Solutions reviewed in the literature assume steady flow or work on long timescales which develop into a steady state (Verron and Provost 1985, James 1980). The period of the flow is an advective

timescale of L/U (ibid). An obstruction's effects on daily oscillating flows in a rotating fluid are not published at this time.

Taylor columns have been created in labs, in fluids under steady flow in a rotating system. They form very quickly and appear to form faster than the advective timescale L/U. Verron and Provost's (1985) comments on the timescales of eddies associated with Taylor columns imply that they will not likely form as a clear structure in a tidal environment. However no solid conclusions on their formation times were found. The literature was split on inviscid flows with a 2D approximation and dampening of the Taylor column above and away from the obstruction. Oceanographic measurements do support the existence of Taylor columns however the stratification in the sea, especially in the upper most layers, will suppress any Taylor Column effects (Roden 1987).   An MIT paper (Buckley 2004) uses the momentum equation to show that any density gradient eliminates the possibility of a Taylor column.

While fascinating studies on stratified flow and Taylor columns over seamounts exist, the most useful observation documented is that flow incident on a seamount will diverge around it rather than flow over it (Carter, Gregg and Merrifield 2006). A study in the Hawaiian Islands (ibid) observed that flow will go around a large obstruction or rather to the side of it instead of over it. This flow behavior is seen in numerical models (Johnson 1982, Verron & Provost 1985). Huppert (1973) easily explains this phenomenon as a consequence of the balance of buoyancy forces inhibiting vertical motions of fluid particles.

Modelling

Several numerical models have been published on the flow patterns around seamounts/bottom topography (Roden 1987; Verron & Provost 1985; Johnson 1982; James 1980) and some studies analyze field data and make comparisons to numerical models (Carter, Gregg and Merrifield 2006; Beckmann & Mohn 2002; Owens & Hoff 1979). The papers published on numerical models of flow obstruction (Verron & Provost 1985; Johnson 1982; James 1980) use steady state approximations and flow rates on the order of centimeters per second. A system harnessing tidal currents would experience such slow flow rates during a tidal change, at least two times a day so these studies give insight on what flow patterns could be expected.

For the purposes of modeling it was of importance to understand that the Taylor column has been produced in a rotating tank where a drop of silicone was the obstruction to flow (Bush, Stone and Tanzosh 1994).

The Coriolis force influences flow patterns in the ocean when the Rossby number is low, and the blocking parameter is $O(1)$. Published numerical studies on Taylor columns have been on quasigeostrophic flows at a steady state or developing into a steady state. This leaves a need in the research arena of rotational effects on oscillating flows in the sea. The literature review brought a good deal of information to consider in the design of a submerged array. The flow patterns around an obstruction on the sea floor are influenced by the velocity of the flow, and the dimensions of the obstruction. The literature shows the shape of the obstruction is not more influential than the length of the largest horizontal dimension. Large obstructions will divert greater volumes laterally around them. Small currents in a rotational fluid will diverge to the side of obstructions

rather than over them, and favor one side over another. The direction a flow in relation to the rotating system it is in will determine which side is favored. In a northern hemisphere environment, when looking down-stream, the left side is favored. And eastward flows past obstructions in the northern hemisphere vary in direction a great deal less than westward flows (Rossby waves).

Roden (1987) noted that for mathematical convenience numerical studies on seamounts use low amplitudes; the obstruction to flow has a vertical dimension that is a small fraction of the water depth. For submarine power systems this approximation is realistic and models using this simplification would be expected to produce results which an ocean engineer could apply. Prototype ocean tidal current turbines are a few meters high and even with supporting structures, they would not be expected to take up a significant fraction of the water column unless they were in a river or coastal channel. Even then, marine traffic might be an issue and deeper locations are likely to be more feasible.

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Problem

The Earth is rotating and as a result the oceans are a rotating fluid. Inertial forces from currents mask the rotational effects and rotational effects can be ignored without issue for a majority of engineering problems. However when currents are very weak, for example during the period of a slack tide, rotational effects can become more pronounced. On daily intervals, diurnal or semidiurnal, tidal currents will drop to low speeds where rotational forces will dominate. Sensitive equipment for tidal current energy extraction may be affected by rotational effects around the period of slack tide. It is not the concern that rotational effects will impact individual turbines but the array as a whole. It is the expected scale at which tidal turbine arrays are to be developed which brings rotational effects to the forefront. For a tidal turbine farm, at the scale of any modern day renewable energy project, rotational effects could be seen in flow patterns around a power production site.

The Taylor Proudman theorem, for flows which are slow relative to the rate of rotation, states that in a rotating fluid, the flows can be approximated as two dimensional for $\frac{L}{DR_0} \gg 1$ (Mason and Sykes 1981). Looking at an array 10,000m in length at a depth of $D=35m$, $L/D= 286$. For flows of 1m/s at Dania Beach the Rossby number is 1.56, making L/DR_0 over 183. For flows not fitting the 2-D approximation the Rossby number would need to be 183. Inertial flows could not approach those rates. The 2-D approximation fits in with the Taylor column phenomenon where flow patterns are independent of the coordinate parallel to the axis of rotation.

The flow patterns are unique to a transverse flow past and obstruction in a rotational fluid could affect the turbines and reduce their production times. These asymmetric flow patterns are documented around seamounts (Carter, Gregg and Merrifield 2006, Owens and Hogg 1979) and could also knock down production rates for half of a turbine array for periods when rotational forces dominate. The flow patterns may also help production times with a prudent design layout. Such a design was produced by this research and was presented in the results.

When rotational forces outweigh inertial forces, and a structure has a blocking parameter, S of O(1), a Taylor column may form. The column forms above/below an obstruction and incident flows are diverted around it. Observations over a seamount have confirmed a Taylor column for a blocking parameter as low as 0.7 (Owens and Hogg 1979). At a critical blocking parameter, $S_{critical}$ the flow behaves as though an obstruction is higher/taller/deeper, than it is. At the horizontal level of the obstruction a steady flow moves around it and this is duplicated on the adjacent horizontal planes, so that the flow is independent of the coordinate parallel to the axis of rotation. Speeds on the flanks of the obstruction increase markedly. And sometimes the flows become asymmetric, with one lateral side experiencing more significantly accelerated rates than the other. Could an underwater array of turbines in the ocean experience these flow accelerations from rotational effects for periods with low Rossby numbers? When the speed of the tidal current drops so that inertial forces are less than rotational forces, what is the ideal placement of turbines? Can flows moved by rotational forces be harnessed by a turbine placed perpendicularly incident to the flow? These are questions which are sought to be answered by this research.

The way rotational forces could be harnessed would be by a Taylor column forming and eliminating a cross-sectional area through which the flow would travel. This would increase flow rates around the Taylor column by conservation of energy. The array's physical, static presence, as an obstruction which takes up a fraction of the water column would cause a negative vorticity over the array because of the conservation of potential vorticity. A Taylor column would form perpendicular to Earth's axis, above the array.

Assumption made were that the strongest tide moves eastward, and the linear array was placed perpendicular to the strong tide, with turbines extending from the north to the south over a water depth that was a constant 35m deep. The turbines were taken to be 15m below the water line and were 10m high. The array was 10,000m in length and the Coriolis parameter was assumed to be constant at 26° N. These parameters would produce similar blocking parameters to those associated with observations of a Taylor column over a seamount in the ocean, see table 1 (ibid).

Putting a turbine array in almost any place will alter the flow patterns in that particular place. A turbine farm placed in mid to high latitudes will experience a larger Coriolis parameter, meaning that the Coriolis force will have a greater coefficient. The array will be assumed to be on the 'f' plane, where the Coriolis parameter does not vary. However a 'Beta' plane array, where the Coriolis parameter does change, because it extends over a range of latitudes is not outside of the scale of a feasible underwater array.

When the tide drops to slack tide the Earth is still spinning, producing unique patterns, which are not particularly intuitive. The problem is how to position the turbine array so that the flow patterns will be altered in the most beneficial way.

Owens & Hogg (1979) Observations above "bump" at 36 N, 55 W		Experimental Parameters	
structure height, h=	400 meters	structure height, h=	10 meters
Coriolis Parameter, f =	0.0000855 rad/sec	Coriolis Parameter, f =	0.0000618 rad/s
structure length, L=	25000 meters	structure length, L=	10000 meters
water depth, d=	5000 meters	water depth, D=	35 meters
U meters/sec	Blocking parameter	U meters/sec	Blocking parameter
2	0.086	2	0.088
1	0.171	1	0.177
0.75	0.228	0.75	0.235
0.5	0.342	0.5	0.353
0.25	0.684	0.25	0.706
0.2	0.855	0.2	0.883
0.1	1.710	0.1	1.766

Table 1 : Observations indicated a Taylor column at speeds of 0.25m/s in a study of a seamount in the Northern Atlantic by Owens and Hogg(1979). Highlighted in yellow are the current rate U and blocking parameters associated with these observations. The parameters for a proposed tidal array line up in a similar manner and are seen in the table on the right.

Governing Equations

For investigating flow patterns effecting tidal power systems during tidal changes in the presence of Coriolis force, the homogeneous, incompressible and inviscid fluid assumptions were made. The governing equations for Taylor columns may be derived as follows:

Equation 4 Governing Equations:

The continuity equation is given by:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

And the momentum equations are given by:

$$\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}\right] = -\frac{1}{\rho}\nabla p - g\hat{k} + \nu\nabla^2\boldsymbol{u} - (\boldsymbol{\Omega}x[\boldsymbol{\Omega} \times \boldsymbol{R}]) - (2\boldsymbol{\Omega} \times \boldsymbol{u})$$

The centrifugal acceleration, $\boldsymbol{\Omega}x[\boldsymbol{\Omega} \times \boldsymbol{R}]$ *where $\boldsymbol{R}$ is a position vector*, can be placed with the gravitational acceleration, $g\hat{k}$. Both centrifugal and gravitational acceleration are taken to be in the same direction. Writing gravity and centrifugal force terms in terms of potential fields, $g = \nabla\phi$, and $\boldsymbol{\Omega}x[\boldsymbol{\Omega} \times \boldsymbol{R}] = \nabla\left(\frac{1}{2}(\boldsymbol{\Omega} \times \boldsymbol{R})^2\right)$ respectively, we can express the momentum equation as:

$$\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}\right] = -\frac{1}{\rho}\nabla p + \nu\nabla^2\boldsymbol{u} - (2\boldsymbol{\Omega} \times \boldsymbol{u}) + \nabla\Phi$$

where $\nabla\Phi = \nabla(\phi + \frac{1}{2}(\boldsymbol{\Omega} \times \boldsymbol{R})^2)$. For the large length scale being considered, O(10,000m), the Reynolds number is large and the viscous term can be neglected to obtain:

$$\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}\right] = -\frac{1}{\rho}\nabla p - (2\boldsymbol{\Omega} \times \boldsymbol{u}) + \nabla\Phi$$

Also, during tidal shifts, for the long periods being considered, the Rossby numbers are small ($Ro \ll 1$) and the inertial terms can be neglected:

$$2\mathbf{\Omega} \times \mathbf{u} - \nabla\Phi = -\frac{1}{\rho}\nabla p$$

The vertical component of this equation is considered to be in "hydrostatic balance" and the horizontal components are considered to be in "geostrophic balance."

Taking the curl of both sides,

$$\nabla \times (2\mathbf{\Omega} \times \mathbf{u} - \nabla\Phi) = \nabla \times \frac{-1}{\rho}\nabla p$$

Assuming density to be constant and using the relation that the curl of a gradient vector is zero, we obtain for $\mathbf{\Omega} = $ constant:

$$2(\mathbf{\Omega}\nabla \cdot \mathbf{u} - (\mathbf{\Omega} \cdot \nabla)\mathbf{u}) = 0$$

Then using the continuity equation $\nabla \cdot \mathbf{u} = 0$, we obtain

$$\mathbf{\Omega} \cdot \nabla\mathbf{u} = 0$$

If we consider $\mathbf{\Omega} = \omega\hat{\mathbf{z}}$, that is, if we assume the horizontal (f - plane) components of the rate of rotation $\mathbf{\Omega}$ are negligible compared to the its vertical component, we obtain

$$\mathbf{\Omega} \cdot \nabla\mathbf{u} = \frac{\partial\mathbf{u}}{\partial z} = 0.$$

That is, the fluid velocity is independent of z, height or depth (Taylor-Proudman theorem) so that flow past an obstacle such as a cylinder of finite length in the region will extend above and below the cylinder past imaginary cylindrical extensions, in the form a Taylor column.

The Reynolds number Re for the flow is:

$$Re = \frac{uL}{v} = \frac{u10^4}{10^{-6}} = u10^{10} \gg 1$$

and the Rossby number Ro is:

$$Ro = \frac{u}{L2\Omega sin\phi} = \frac{u}{Lf} \quad , \qquad u \rightarrow 0 \therefore Ro \ll 1$$

Conservation of Potential Vorticity

The conservation of potential vorticity, where D is the depth of the water column and $\zeta + f$ is the absolute vorticity, explains why rotation rates change when the depth D changes. When there is increase/decrease in the column depth, vortex compression/expansion occurs in view of conservation of mass. This in turn results in increase/decrease in the absolute vertical vorticity $\zeta + f$. The direction of a negative change in vorticity is anticyclonic (northern hemisphere). Thus when the depth, D, decreases because of an obstruction in the water column, and the Coriolis parameter f stays constant, then the vorticity ζ must decrease. So a parcel of water moving over the array will acquire negative vorticity

$$Q = \frac{\zeta + f}{D},$$

Equation 5 :Conservation of potential vorticity.

Goal and Objectives

The goal of this study was to assess quasi-geostrophic flows as resources for tidal current energy. In support of this goal, the study was undertaken with the following objectives:

1. The initial objective was to model the formation of a Taylor column in a rotating flow with a current, where the Rossby number was low. This would then be applied to the study of rotational flows in the vicinity of a linear array which was 10,000m long, and composed of a single line of turbines. A single line of turbines was chosen to eliminate wake effects. Analysis was then to be applied to the layout design of an array to place turbines in positions which would receive currents for the longest periods, with the aim of increasing their production times. The research in Fluent was to study the spatial development of a Taylor column, and to see if streamlines accelerated asymmetrically around a linear obstruction before and after a Taylor column had formed.

2. The second objective was to use Johnson's (1982) solutions to characterize the flow and streamlines around a linear array. Johnson's solutions illustrated occurrence of flows with closed streamlines or a Taylor column, which is dependent on the blocking parameter, S, current speed, U, and the physical dimensions of the array. Consistent with the Taylor column, these solutions were for steady flow. However, considering slow temporal variations associated with tides, the flow features can be regarded as being representative over the period of slack tide for different S and U, in support of estimating the characteristics of rotational flows around a linear array. The objective was to model an obstruction with flow rates seen during slack tide to see if rotational flows associated with the

Coriolis force would manifest. The model obstruction to flow would be similar in size to that of an underwater power generating structure. The structure, composed of turbines and other equipment was regarded as a single object. Flows for conditions with current speed of <1m/s were considered. Consideration of an inviscid solution for the overall Taylor column flow was appropriate on the basis that the role of viscosity is secondary, leading to slow dissipation of the flow.

It was investigated weather a Taylor column could be forced over an array through increasing the height of a fraction of the array in order to raise its blocking parameter. If a Taylor column could be forced, then its location could be manipulated and the accelerated flows around it could be harnessed.

Methods

For simulations in ANSYS Fluent, a variety of conditions were attempted in an effort to visualize a Taylor column. In ANSYS Fluent the pressure-based solver was selected. The solver used the momentum equations to obtain the velocity field. The solver used control volumes to solve the integral equations for the conservation of mass, conservation of momentum, and conservation of energy. A projection method algorithm used the constraint of mass conservation to solve a pressure correction equation. This equation was hidden in the algorithm and the pressure correction equation was not included in the ANSYS Fluent program help files or associated manuals. For modeling the rotational fluid, the pressure based coupled method was selected. This method couples the momentum and pressure equations which are discretized in the segregated algorithm. While the coupled method has twice the memory requirement as the segregated method, the remote access for FAU's College of Engineering and the university's KOKO High Performance Computing (HPC) systems were well equipped for the load.

The model was a moving puck to produce a relative current in a rotating frame and to achieve this the solver added additional acceleration terms when the motion of the reference frame was defined as rotational. The "Sliding Mesh" approach was employed to capture the interaction between the rotating fluid and the moving obstruction, which was set to move in the majority of experimental trials to create a relative current. Notes on trials are included in the appendix. In the calculations Fluent determined the pressure

from the mass conservation and momentum equations in a coupled manner which was also corrected by the pressure. Iterations of the coupled, nonlinear equations were repeated by the solver until a solution converged.

Numerical Solutions for Quasigeostrophic Flow

Johnson's (1982) numerical solutions for quasigeostrophic flow over elongated topography were used to visualize streamlines and analyze the spatial development of closed streamlines which are associated with a Taylor column. Matlab was used to plot closed streamlines over an elliptical obstruction to flow.   The code is found in Appendix A. The array is taken to be linear; a single line of turbines which fit the description of elongated topography. Johnson's solutions vary with the angle of incidence and with the aspect ratio. Since the ideal angle of incidence was determined to be $\pi/2$, the value of aspect ratio is the primary factor in determining streamline patterns for this angle of incidence.

The entire array was modeled as a single obstruction for flows <1m/s. The changing flow speeds during a tidal change were used as the driver of changing blocking parameters. Flow patterns were produced for different blocking parameters. The obstruction to flow was assumed to produce effects on flow in a range equal to its largest dimension, L. Therefore, within this distance L from the array, there were no other modeled obstructions.

Blocking parameters for a small, raised section of the 10,000m long array was analyzed to see if a Taylor column could be forced and its location manipulated for the benefit of the greater remaining fraction of the array.

Results

Regarding the first objective, it was unfortunate that numerical models in ANSYS Fluent, failed to produce a Taylor column or closed streamlines, or stagnation points above an obstruction to flow. The first attempts were re-creations of the Mason and Sykes (1981) work and included the rotational effects as an additional body force added as a user defined function (UDF). The first model had a stationary obstruction, on the base of a rectangular tank. The Coriolis force was added as a body force with a user defined function which is available in Appendix 2. A pressure difference was used to induce a small velocity current. This did not produce a Taylor column. Disorganized streamlines, varying significantly in the vertical direction were produced, Figure 2.

Then an undergraduate lab (Buckley 2004) which produced a Taylor column in a rotating tank of water was attempted to be reproduced in Fluent. The lab was from the Massachusetts Institute of Technology (MIT), published in free online course material for a meteorology course. MIT as well as Johns Hopkins University had videos on youtube where the formation of a Taylor column in a tank can be seen in real time. To confirm the accuracy of the modeling in Fluent these examples of Taylor columns formed in labs with rotating tanks were attempted with computational fluid dynamics (CFD) in Fluent. The attempts in Fluent were at the scale of published experiments, as well as smaller and larger scales.

For all the following models, a rotating cylindrical tank with a puck moving along the bottom of the tank was set up in ANSYS Fluent. Two different motion paths for the puck were attempted: 1) moving the disk along the tank's radius; 2) moving the disk in a circle on the base of the tank, maintaining a constant distance from the center, both clockwise and counter-clockwise motions were attempted. These experiments were set up as transient with sliding mesh to move the disk. The tank was set as the absolute rotating reference frame. Trials were also run with the tank rotating, i.e. the tank's walls were moving mesh. With CFD there was no success in producing a Taylor column in a rotating fluid with a slow relative current around an obstruction.
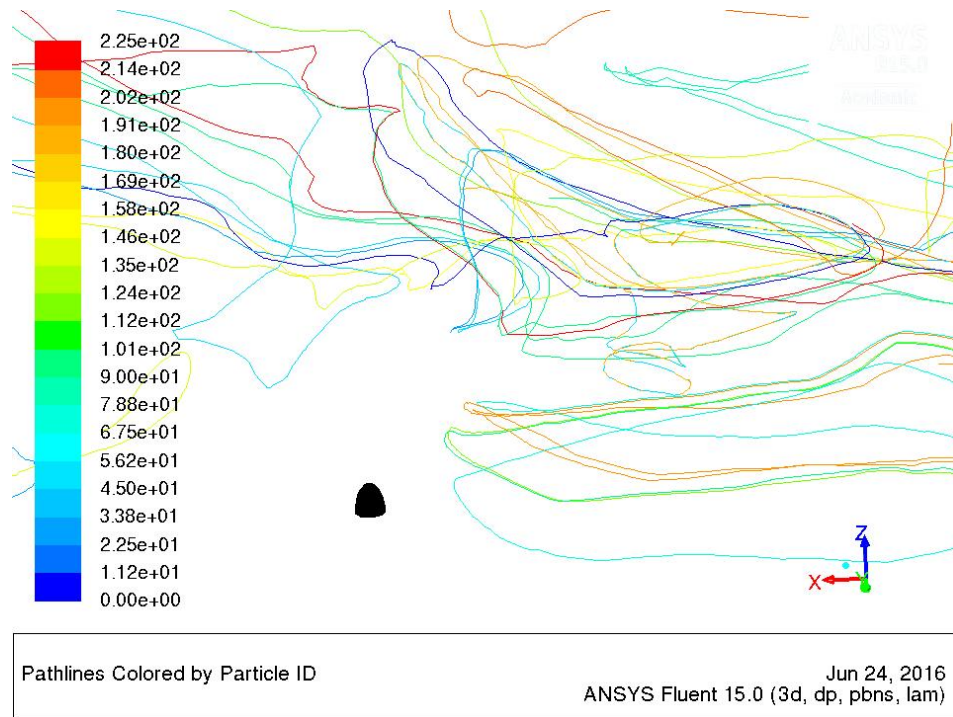

Figure 4 Fluent modeling with a pressure difference to produce a small current and, using user defined functions to add the Coriolis force as a body force produced inaccurate streamlines. The blocking parameter for this snapshot of streamlines was 26.

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Coarse mesh was initially attempted however errors always prevented the model from running. Most of the numerical models in the literature review were for very coarse models from the 1970s and 1980s, so it was expected that coarse mesh would model appropriately. Unfortunately, this was not the case. Fine mesh produced the least errors and transient models typically always ran with fine mesh. Fluent's 6-DOF modeling method was attempted for the puck in a rotating fluid, however the 6-DOF modeling method is only recommended for when a fluid moves the mesh, such as when an object is dropped into a fluid. The recommended criteria for the 6-DOF model not fitting the research aim and the high complexity of 6-DOF modeling caused this method to be abandoned.

The trickiest part of the ANSYS Fluent modeling was to define the boundary of the puck. The puck had to be produced as a void during the meshing of the model for the boundary to model properly in the fluid, Figure 3. The model was run numerous times at different timesteps ranging from 0.01s to 2s, for periods of 20s to several minutes. Laboratory experiments on a similar scale as the models showed Taylor columns form almost instantly once the puck was moved in a rotational fluid. With the CFD models flows always passed directly over the obstruction which was not physically accurate. The formation of a Taylor column could not be produced in Fluent.

A transient fluids model using ANSYS Fluent was unable to produce a Taylor column. A great effort was made with Fluent and the High Performance Computing (HPC) resource at FAU to run and troubleshoot models. Models were able to be set up in a period of an hour, with run times spanning several hours, which were often ran over night. With no results the Fluent modeling was abandoned.

Replicating laboratory conditions which produced a Taylor column were disappointingly unsuccessful. Only at the depth of the obstruction was flow acceleration around the obstruction observed in Fluent. The acceleration did not appear asymmetric as would be expected in a rotating fluid with a slow relative current. Flows moved with no indication of a Taylor column, at all depths above the obstruction. The phenomenon of Taylor columns in a rotating flow is not well understood and does not appear in off the shelf physical modeling software ANSYS Fluent.

Johnson's solutions for quasigeostrophic flow were used to produce streamlines over a linear array, sitting on the bottom of the sea, which obstructs a quasigeostrophic current. The aspect ratio of 1:1000 was used to fit an array of width 10m and length
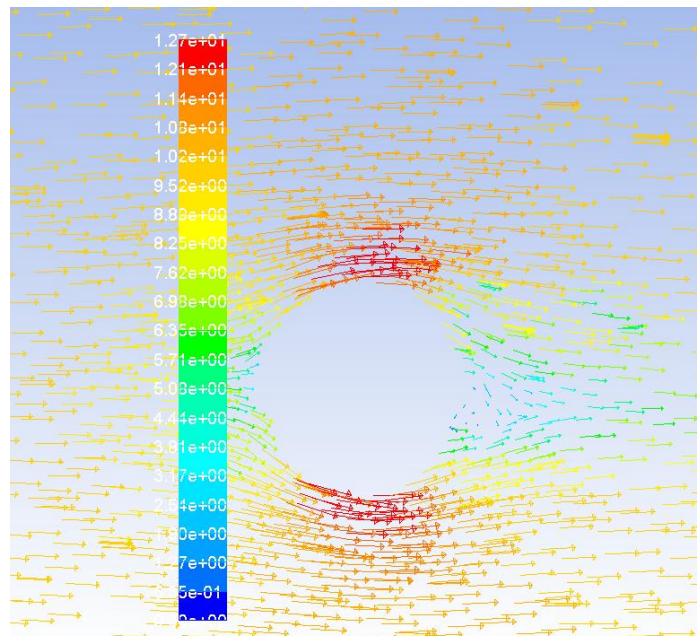
Figure 5 This image shows vectors for tangential velocity in meters per second (m/s) of fluid particles in a rapidly rotating tank. The circular void in the center is the location of a puck which obstructs flow. The puck moves at a fraction of the rate of the rotating tank to maintain a low ratio of inertial to rotational forces. When rotational forces dominate, a column of stagnant fluid is expected above the obstructing puck. Such conditions have a blocking parameter O(1). In Fluent the model only showed the obstruction to flow at the heights where the puck was present.

10,000m. Flow patterns for an array with an aspect ratio of 25 were compared with an array with an aspect ratio of 1,000 and the variation was not remarkable. The streamlines around an obstruction did not vary significantly with a much smaller aspect ratio, see Figures 6 and 7.

Matlab was used to plot the streamlines expected at conditions around tidal changes, i.e. for slow flow rates which produced critical blocking parameters. The largest blocking parameters were associated with flows speeds <0.1m/s. The streamline plots in the immediate vicinity of the obstruction were asymmetric. Slow current perpendicularly incident on a linear array formed closed streamlines above the southern end of the array. As the value of S increased for decreasing flow velocities, first the spacing between streamlines widened at the southern end of the obstruction, indicating slower velocities. Simultaneously the spacing narrowed at the northern end, indicating higher velocities. Very close streamlines still appeared at the northern end of the obstruction even as the blocking parameter approached 4. At this high value of the blocking parameter closed streamlines on the southern end indicated stagnant flow while the narrowly spaced streamlines at the northern end, just outside the closed streamlines, indicated rapid flow rates.   As the blocking parameter increased, the closed streamlines enclosed the entire area of the array. Color velocity plots for two critical blocking parameters illustrate the asymmetric acceleration around an obstruction in Figure 9.

When the height of the obstruction was increased, then a higher current speed was able to raise the blocking parameter to the critical value, O(1). Considering a tidal cycle, when the critical blocking parameter is reached at a higher speed, then the longer the period in a tidal cycle where the blocking parameter is critical. For a water depth of 35m,

by increasing the value of the height of the obstruction, a critical blocking parameter

could be reached when the local current was 0.5m/s, Figure 7.

Figure 6 Streamlines around the obstruction with an aspect ratio of 25 are shown
for different blocking parameters, S. The streamlines do not vary remarkably from
an obstruction with an aspect ratio of 1,000, as seen in the previous figure, Figure
5. The coordinate system is normalized using l, the minor axis of the elliptical
bodyaxis, l= 10m. The coordinate system is normalized using l, the minor axis of
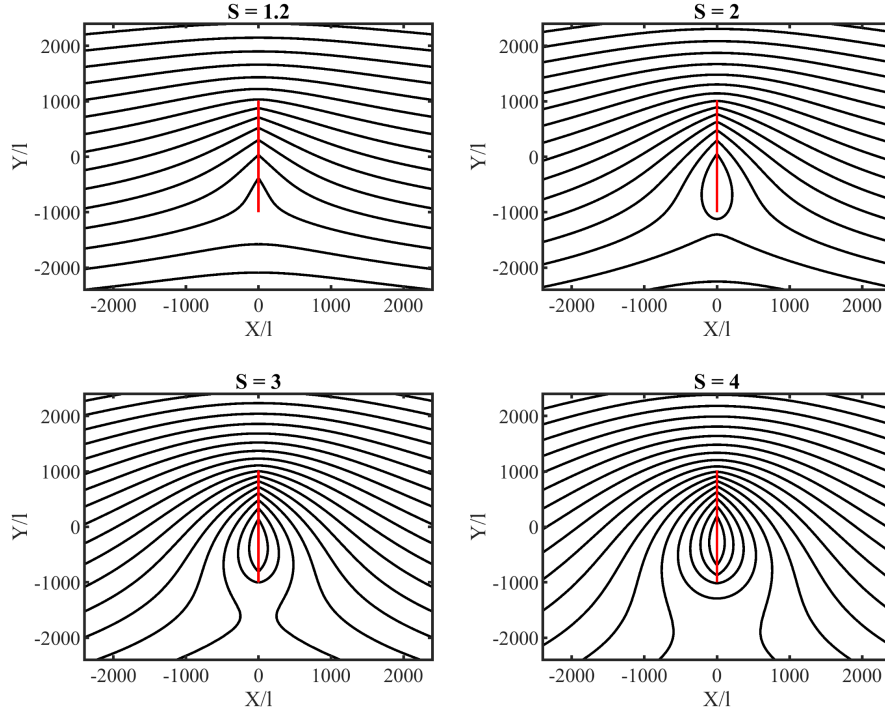the elliptical body.
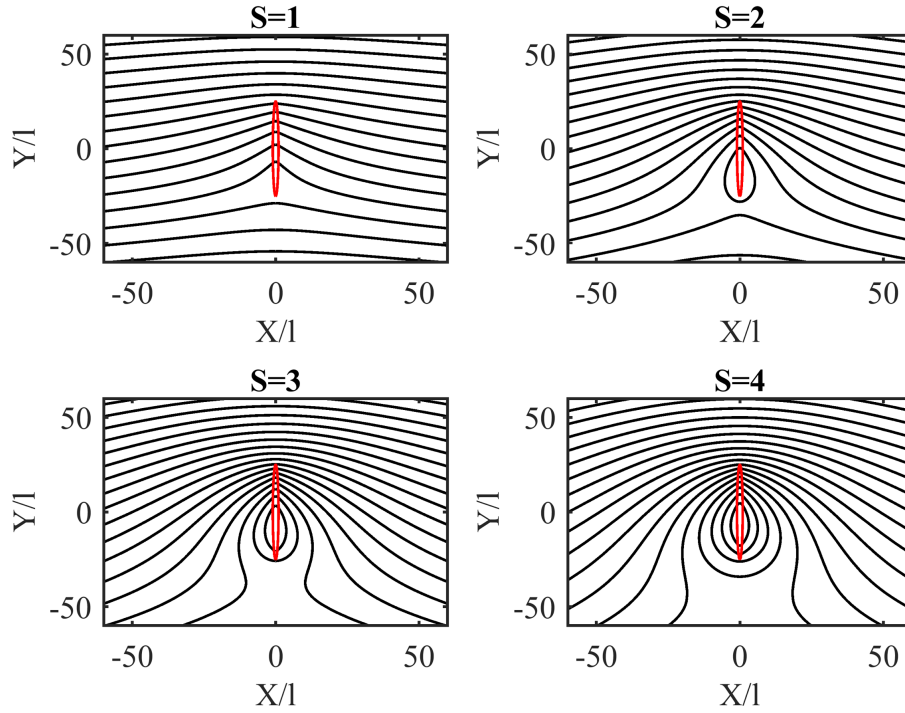
Figure 7 Streamlines around the obstruction with an aspect ratio of 25 are shown for different blocking parameters, S.   The streamlines do not vary remarkably from an obstruction with an aspect ratio of 1,000, as seen in the previous figure, Figure 5. The coordinate system is normalized using l, the minor axis of the elliptical body.

A trial was run for an array which was parallel to the far field streamlines and this orientation for the structure showed greater engulfment by closed streamlines. These results indicate that an obstruction parallel to quasigeostrophic flows may experience more rotational effects at lower values of the blocking parameter, Figure 6. Other than a greater area of a linear array being affected during periods of a critical blocking parameter, velocity plots were produced to better visualize what occurs to a structure parallel to incident flow in a quasigeostrophic flow. These plots were produced for two blocking parameter values, S=1 and S=2.   The plots showed that on the north side velocity was accelerated over the far field and on the south side there was a great

decrease from far field velocity. Having adjacent regions, separated by a single line of turbines, with such great differences in velocities is not realistic and indicates that high levels of turbulence could be expected directly on the line of turbines in such conditions. This configurating would not be recommended for an array because of the expected mechanical stresses placed on the array.
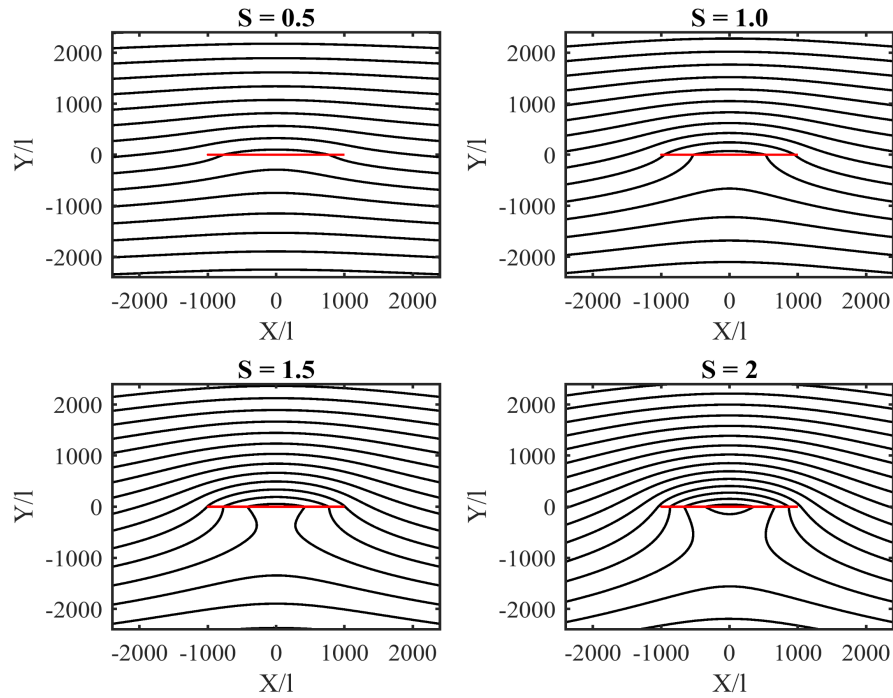
Figure 8 Orienting the obstruction to be parallel with the flow caused greater engulfment by a region of slower or stagnant flow, indicated by wide streamlines or closed streamlines. These results indicate that an obstruction parallel to quasigeostrophic flows will experience more rotational effects at lower values of the blocking parameter.
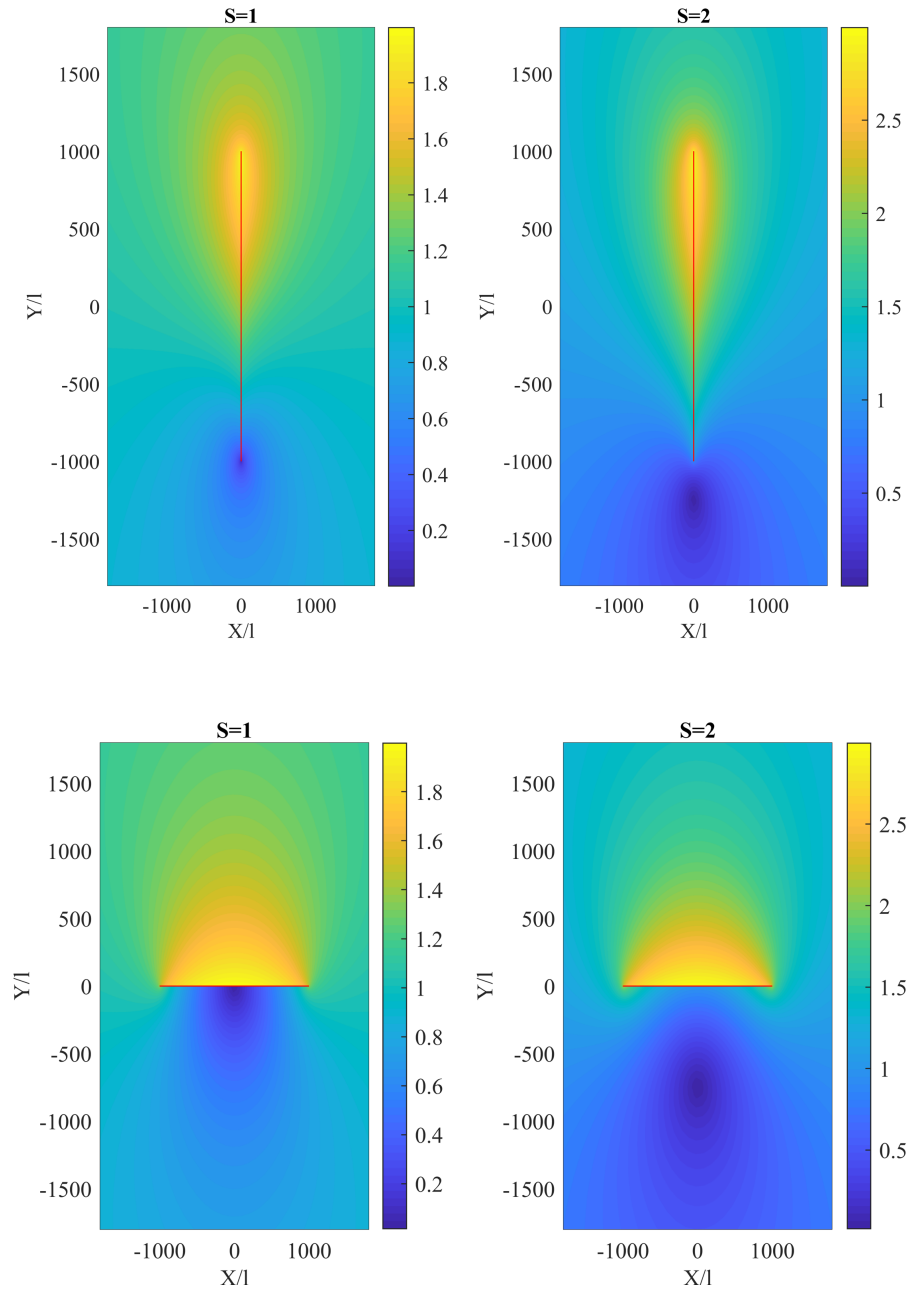
Figure 9 Velocity plots for two critical blocking parameters, for perpendicular and parallel incident flow illustrate asymmetric flow patterns when the Coriolis force dominates inertial forces. Velocity scale is u/U_0, where U_0 is ambient velocity away from structure. The coordinate system is normalized using l, the minor axis of the elliptical body. For the parallel structure (lower plots) the high variation over a small spatial scale are not realistic and indicate that a high level of turbulence may be expected over the obstruction at critical blocking parameters.

For a 10m high structure, 10,000m in length the blocking parameter reached a critical value at low speeds, see Figure 7. The blocking parameter grew very quickly with velocities approaching zero. For the ocean observations of a Taylor column the conditions were: flows of 0.25m/s had a blocking parameter of 0.7, see Table 1 (Owens and Hogg 1979). A similar blocking parameter would occur at 0.5m/s for the proposed structure at Dania Beach. At 0.1m/s the blocking parameter was 1.77. It is during the period when flows are 0.5m/s and below that a possible Taylor column would form and caused increase flow rates on the opposite end. If the rate increased to a level where turbines would spin, then the turbines would be harnessing the Coriolis force.

Since the height of the structure is directly proportional to the blocking parameter, a structure which would take up a greater percentage of the water's depth may attain a critical blocking parameter at higher current velocities, see Figure 11.

Blocking Parameter for for Dania Beach, L=10km.
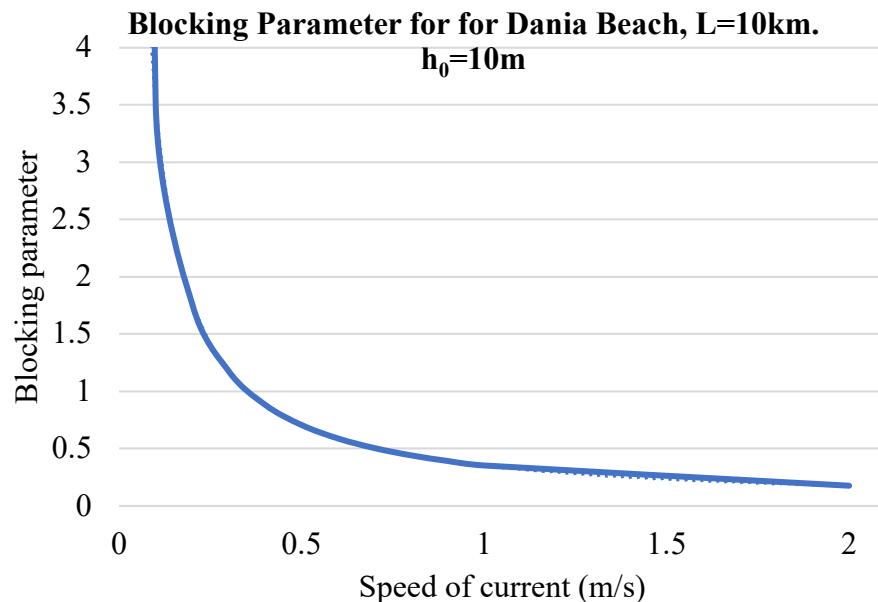**h_0=10m**

Figure 10 The graph illustrates how the blocking parameter changes with the speed of the current for a structure 10,000m long, 10m high at the latitude of 26ºN where water depth is 35m.

Blocking Parameter Increases with Decreasing Speed
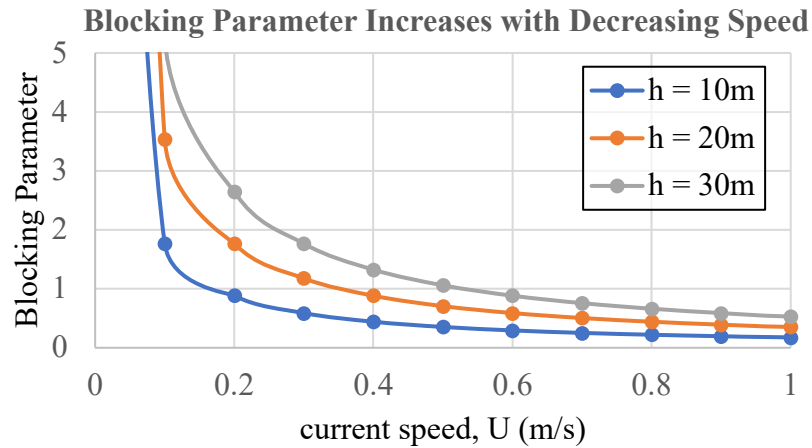
Figure 11 The greater the obstruction height, the higher the current speed is required to reach a critical blocking parameter of O(1). When a higher speed leads to the critical value, the longer the period where the blocking parameter is critical. The graph is for an obstruction 10,000m long, in 35m of water offshore of Dania Beach, Florida and three possible heights are plotted: 10m, 20m, 30m

A pair of linear obstructions, one on top of another were investigated. A small section of a large array was raised with the aim of forcing a Taylor column over that fraction of the array. The raised fraction had a smaller length scale than the array and a greater height. The raised section did not have a greater blocking parameter. In fact, its blocking parameter was less than the entire 'base' array as a whole. It was found that a Taylor column could not be forced by increasing the height of a fraction of the array.

While Taylor columns are not expected to form distinctively in the short slack tides in a diurnal zone, the results indicated rotational flows form on the southern end of an obstruction oriented N-S with an eastward flow. Viewing a turbine array as the obstruction, the south end would experience the drops of velocity with the changing tide first. For a reversed westward flow, the north end would experience the slowest current

velocities first. If the tidal current deviated from the ideal perpendicular angle of incidence on the array, then the location first experiencing the slowest flows would move closer to the center of the array. Turbines attempting to harness the current acceleration around a Taylor column would physically need to be placed opposite from the side of the array experiencing low Rossby numbers first. Such turbines would need to be sensitive to currents <1m/s. So as to place turbines in the most dynamic arrangement, the site specific dominant tidal direction needs to be determined.

Blocking Parameter

The blocking parameter is the value used to predict the formation of a Taylor column. There is no consensus on the blocking parameter. In 1917 Taylor linked the formation of a Taylor column to the Rossby number dropping below the value of $1/\pi$. Effectively Taylor's critical blocking parameter was when the Rossby number was $1/\pi$ or smaller. Hide's work in 1961 on rotational fluids added a coefficient to the inverse of the Rossby number, and this produced the blocking parameter, S, used in this study. The coefficient Hide added was the ratio of the height of the obstruction to the depth, d. In 1981 Mason and Sykes used the ratio of the greatest horizontal length of the obstruction to the water depth as the coefficient to the inverse of the Rossby number. The similar thread in all these blocking parameters is the Rossby number.

The Rossby number is the ratio of inertial forces to rotational forces. The rotational forces are represented in the Rossby number by the terms fL. Where f is the Coriolis parameter and L is the characteristic length, the longest horizonal dimension of the object. The Coriolis parameter is larger for higher latitudes. The Rossby number will

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decrease for higher latitudes because at these locations rotational forces are stronger. The Rossby number is inversely proportional to the Coriolis parameter

$$R_0 \propto 1/f$$

And the blocking parameter is proportional to the Coriolis parameter

$$S \propto f$$

Consequently, a location at a higher latitude will experience higher blocking parameter; with velocity and structure size and water depth all remaining constant.

$$S = h/(d\ R_0)$$

$$S = fhL/dU$$

The Rossby number includes L, the characteristic length of an object. So, Hide's blocking parameter is the most complete because it not only includes the characteristic length of an object, but it also includes its height. Hide's blocking parameter is used in this study. The Mason and Sykes blocking parameter is redundant in that it uses the length of the obstruction in its coefficient and the length also appears in the Rossby number. So effectively the Mason and Sykes blocking parameter uses the square of the characteristic length.

Returning to the perspective of a tidal current, the velocity changes and the blocking parameter will change along with it. Recall that the blocking parameter is inversely proportional to the velocity

$$S \propto 1/U$$

At sites where tidal turbine arrays will be deployed the assumption is that the Coriolis parameter will not change. This is an f plane approximation. If the array was to extend over several degrees of latitude the Coriolis parameter would change and a structure so great in size would need to be analyzed in the β-plane. For the f plane approximation, the absolute velocity of the tidal current will be the parameter that changes the blocking parameter. Looking at the critical blocking parameter for various latitudes, faster speeds can bring about a critical blocking parameter at higher latitudes. This means that the period of the tidal cycle where the blocking parameter is critical will be longer at higher latitudes.

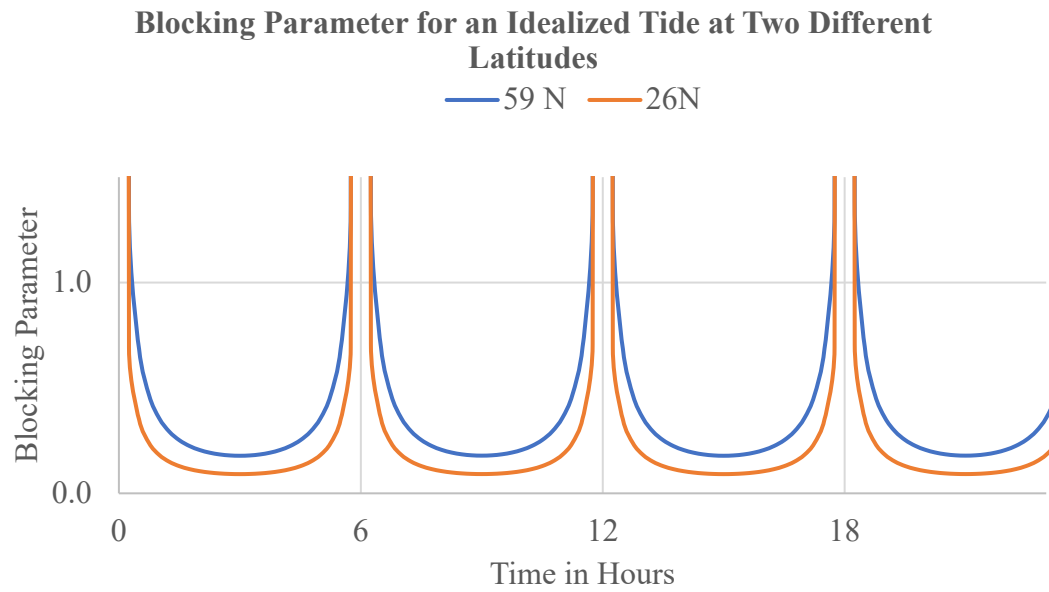Blocking Parameter for an Idealized Tide at Two Different Latitudes

Figure 12 Blocking parameter over an idealized tidal cycle. The blocking parameter is an absolute value which peaks with low speeds which characterize a changing tide. The tidal cycle for which the above graph corresponds to is Figure 13 below

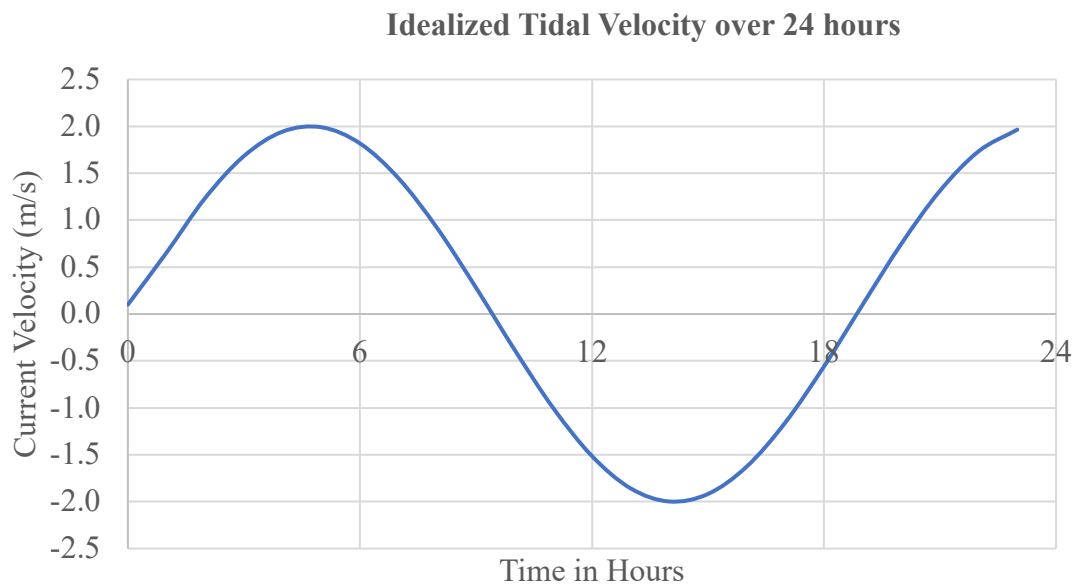Idealized Tidal Velocity over 24 hours

Figure 13 Graph of velocity over time for an idealized diurnal tide

An unfortunate consequence of the ANSYS Fluent model not producing accurate results was that the critical blocking parameter could not be scrutinized. Observations by Owens and Hogg indicated that a critical blocking parameter was 0.7. For the seamount parameters and the current rate of 0.25m/s a Taylor column was observed at 35 o N in the Atlantic, see Table 1. Numerical studies all indicate a blocking parameter of 1 or greater. While 0.7 could be argued to be O(1) a more precise value for the critical blocking parameter is desired for engineering applications. It is likely that the blocking parameter would be affected by local bathymetry and other local environmental factors and would need site specific evaluation.

U meters/sec	Blocking Parameter				
	59° N European Marine Energy Center	48° N Puget Sound	41° S Cook Strait	35 ° N Cape Hatteras	26° N Dania Beach
2	0.179	0.155	0.137	0.120	0.088
1	0.357	0.310	0.273	0.239	0.177
0.75	0.476	0.413	0.364	0.319	0.235
0.5	0.714	0.619	0.547	0.478	0.353
0.25	1.429	1.239	1.093	0.956	0.706
0.2	1.786	1.548	1.367	1.195	0.883
0.1	3.572	3.097	2.734	2.390	1.766

Table 2 The blocking parameter is given for selected latitudes at different current speeds, U (m/s). Blocking parameters were a Taylor column is likely to be observed are highlighted in yellow.

The blocking parameter is greater at higher latitudes and to get an idea of how that would translate into power, the blocking parameter was used to plot how the two ends of the structure experience the tidal current.

Blocking Parameter of a Varying Current at Different Latitudes
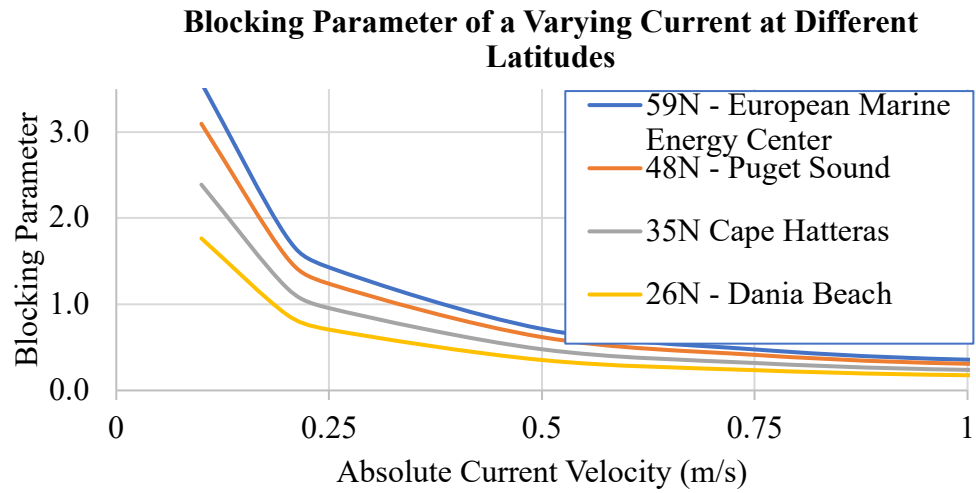
Figure 14 Values from Table 2 are graphed to show how the blocking parameter changes with latitude.

Figure 9 assigns a color to the multiplier of the current velocity. The graph below shows the velocity felt at each end of the array.

Velocity at the Top of the Array and the Bottom, with Idealized Tidal Velocity over 24 hours at 59N
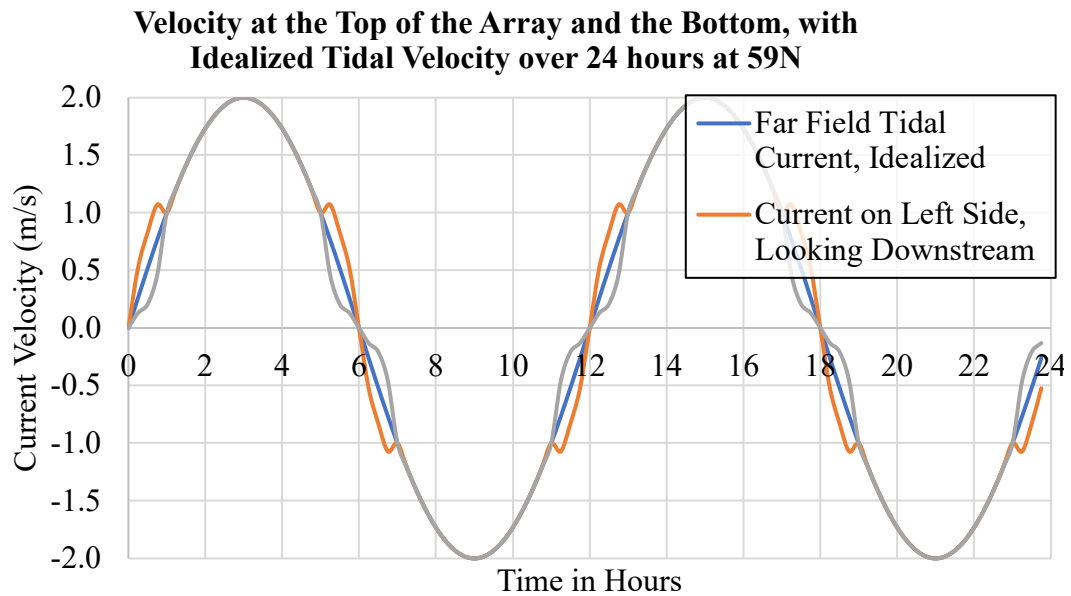
Figure 15 Velocity felt at the two ends of the structure, calculated with blocking parameters for a 10,000m structure, 10m high at a latitude of 59N, in 35m of water.

Periods when the tidal current is <0.5m/s are assumed as a period when energy in the ocean/tidal current is too low to be extracted. From figure 16 it can be seen that for periods when the absolute speed is between 1m/s and 0.5m/s, the area under the curve increases more for the left side of the array than, the area under the curve decreases for the right side of the array of the array. Again, right and left are from the perspective of looking downstream This difference shows that the net effect on power from rotational effects is expected to be positive if energy from the Coriolis force is harnessed. However weather or not the mechanical system is able to harness this boost of speed is dependent on factors outside the scope of this research.

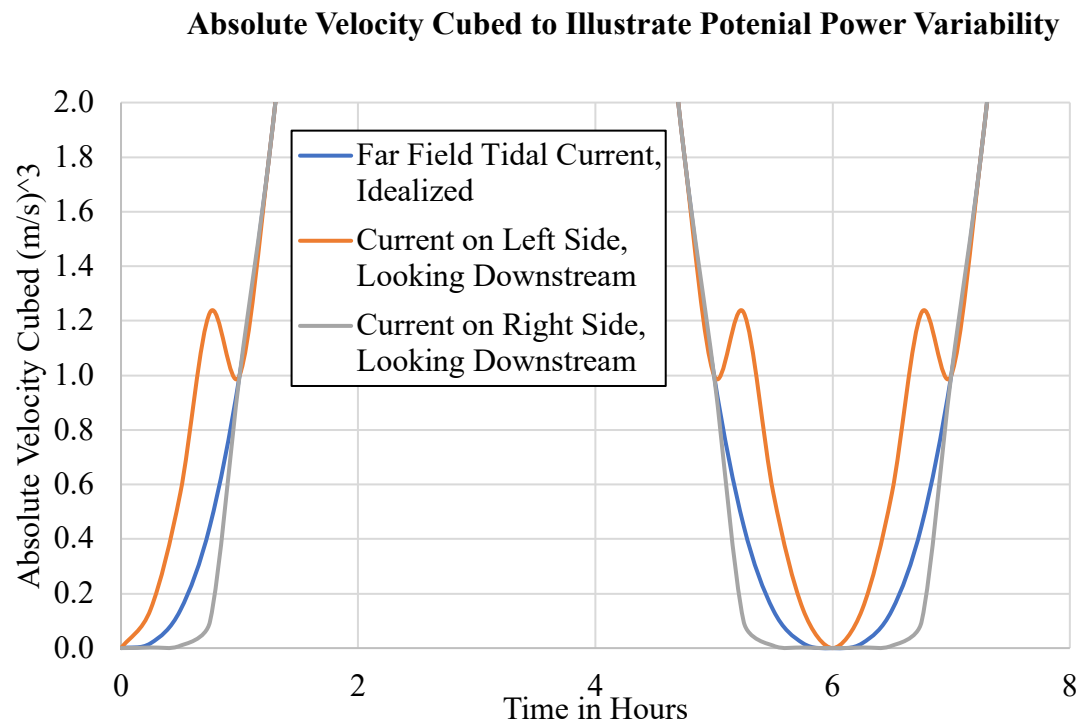**Absolute Velocity Cubed to Illustrate Potenial Power Variability**

Figure 16 Absolute velocity cubed to illustrate power potential at the ends of the array. The array is 10,000m long, 35m underwater, 10m high and composed of a single line of turbines at 59N latitude.

Proposed Array Design

Consideration rotational effects a proposed design for a tidal turbine array was developed. The array has three components: the major field, the minor field and the supporting equipment between the two, Figure 15. The array is a single turbine high, and the design is for a diurnal tidal zone. A single line of turbines is recommended for each field so that all turbines harness the direct tidal current with no wake effects. Furthermore, all turbines receive the full energy from the incident current because there is no turbine in front of them extracting energy from the current. Looking downstream, the major field is located on the left side of the array. In the northern hemisphere flows move past this side of an obstruction with greater speed (2006 Carter, Gregg, Merifield) for low rates, when the Coriolis force dominates over the inertial forces. Therefore this side would be where the most energetic flows could be expected. The major filed is larger to harness a greater amount of energy. Weather it is able to increase its period of production or weather it has turbines sensitive enough to harness this boost of speed by rotational effects is dependent on factors outside the scope of this paper.

Major field	Supporting equipment	Minor field

Figure 17 Plan view of an array design for the enhancement of tidal power with the Coriolis force. The current goes 'up' through this schematic for the dominate ebb tidal flow. The design is 2-D because flows in an incompressible, rotating fluid can be described as 2-D with no vertical velocity gradients according to the Taylor Proudman theorem

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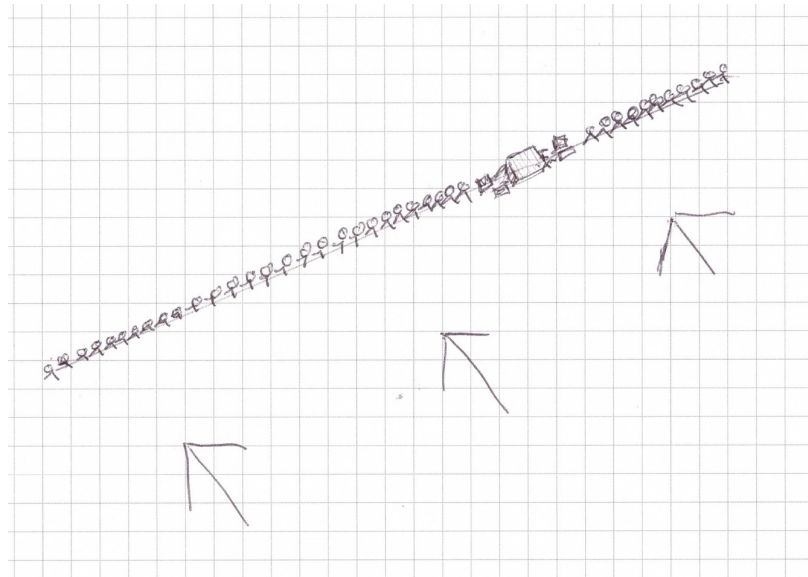
Figure 18 Array composed of bottom seated turbines, with supporting equipment between the minor and major fields.

When the tide reverses, the flow direction reverses and the opposite side of the array, the right side, could be directly in the most energetic current following tidal change. For the flood tide the opposite side of the array would be expected to be the location of the more energetic flows, and this is the location of the minor field, see Figure 15. If only the ebb tide was harnessed for energy, then the supporting equipment could be slid down to the right side of the array and there would be no need for a minor array. All the turbines would be in a single field on the left side of the array (looking downstream). It is important to remember that monitoring equipment for the environment as well as for monitoring the turbines, and supporting electrical equipment for the turbines producing electricity would be part of any array.

The greater the fraction of the water column which the array takes up, the larger the blocking parameter. The blocking parameter becomes important at low current speeds. For example, if an array has a low profile, its blocking parameter will not rise to critical levels until speeds are very low. Having a low profile will also reduce the period for which the array has a critical blocking parameter. If the array is taller and takes up a larger fraction of the water column then it will have a critical blocking parameter at higher speeds. The period where it has a critical blocking parameter will be greater. Meaning that a taller array can experience greater affects from the Coriolis force.

To reduce the complexity of the flow at low inertial periods, and to minimize variation locally in the vicinity of the array, construct the array in such a manner which limits its blocking parameter. To limit an array's blocking parameter do not expand the array vertically and limit its length.

Conclusion and Discussion

At 0.1m/s flows the blocking parameter was 1.77 at the Dania Beach location which corresponds to a latitude of 26 degrees. This high value of the blocking parameter is a critical value for the Johnson model and indicates the presence of closed streamlines or a Taylor column. The parameters for Dania Beach had a striking comparison with the Owen's and Hogg data for which a Taylor column was observed for a blocking parameter of 0.68. Taking the blocking parameter of 0.68, a Taylor column could be expected for a velocity of 0.25m/s, at Dania Beach for a 10,000m array in 35m of water, and the array being 10m high.

For a steady, slow, eastward flow it could be likely for a Taylor column to form over the southern section of the linear array oriented N-S., It would be expected that flows over the northern section would increase in speed due to the blocking nature of the Taylor column. However in an oscillating tidal flow the time required for this flow pattern to develop at this scale is unknown. In laboratory tank experiments a Taylor column forms almost instantly before your eyes, but on the scale of kilometers the time could be on the advective timescale of L/U. This is a large period and would be a limiting factor for Taylor column formation. Such a long period would not be possible with the tidal current's oscillation. What is however likely is that the southern end of the array, with an eastward current, is the first to experience the drop off of energy production when the tide changes. For the converse flow the northern end of the array would first experience stagnant flows during a tidal change. The spatial variance in the

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tidal current velocity will be important to operators of a tidal current power array. The special difference in velocity changes will dictate how the resources within the operation are used. Maintenance practices will be affected by spatial differences in the tidal current velocity. Also, electrical equipment "refining" the electricity before it is sent to the grid will have to take into account the periodic asymmetric flow patterns because they will affect the production of the array.

Even if a Taylor column did not form, the appearance of asymmetric velocity increases around an array during low inertial periods of the tide could be expected. The presence of Taylor columns is one resultant of an obstruction to flow in a rotating fluid. Asymmetric acceleration of a tidal current (M2) without a Taylor column have been observed around a seamount (2006 Carter, Gregg, Merrifield). In the northern hemisphere the volume of water passing the seamount was greater on its left flank than its right (looking downstream). And when data collected was plotted as streamlines it could be seen that as the current approached the seamount, streamlines on the right side of the seamount crossed to the left. Once the streamlines passed the seamount, about the length of the seamount downstream, the streamlines crossed back to the right.

The knowledge and understanding of how Earth's rotation affect flows around obstructions in the sea has yet to be fully developed. Asymmetric flow patterns around obstructions when rotational forces dominate is a firm conclusion. The exact conditions for Taylor column formation in the sea have yet to be nailed down. Furthermore, the dynamics inside the Taylor column are dim in the understanding of science and engineering. This study found that a Taylor column could not be forced, or its location manipulated by raising a fraction of the array. It was found that a fractional raised

section of the array has a smaller blocking parameter because of its significant decrease in length. The formation of a Taylor column was viewed as beneficial because of the increase in speed it produces in flows passing around it.

For the proposed design of an array, the design assumed no vertical variation in the flow as predicted by Taylor-Proudman theorem. The design was 2-D. The proposed design for an array placed the majority of turbines in the 'major field' on the left side of an array in the northern hemisphere. The ebb tide was assumed to be stronger than the flood tide and it flowed "up", perpendicularly incident through the array. Since local site characteristics like bathymetry can influence the currents with the tidal changes, specific details like the duration and strength of ebb and flood currents would dictate if it was appropriate to place more turbines, or turbines with higher production ratings, at one end of an array.

A valid design decision could be to put supporting equipment for the array on the end of the array. This way for ebb/flood the supporting equipment would first experience stagnant flows, allowing the moving streamlines to stay over turbines for longer periods. A possible downside could be, that if one end had a higher production capacity for a tidal direction, on the opposite tide, the end with supporting equipment and not turbines would have the more productive streamlines. This is why a "minor filed" was placed on the right side of the array with the supporting equipment off-centered, in the middle.

Future Considerations

The Fluent modeling of a Taylor column was unsuccessful because the inside of the Taylor column may have properties which the Reynolds Averaged Navier Stokes (RANS) CFD modeling cannot properly illustrate. With the associated vortex compression at an obstruction and the condition of "sudden" stagnant flow in the Taylor column, implied by closed streamlines, it is likely a RANS CFD, like Fluent, cannot determine how the flow separates into the column above (parallel to the axis of rotation) the obstruction. The program must be able to insert the boundary of the Taylor column into the flow on its own. This is a problem rooted in the Reynolds closure problem of not having enough equations to solve for all the variables in the Navier Stokes equations. Furthermore, it is speculated that viscous forces dominate the flow inside the Taylor column. The homogeneous, inviscid, incompressible assumptions for Taylor Proudman theorem would not be valid inside the Taylor column and could prevent an off the self RANS CFD, like Fluent, from accurately modeling the phenomenon.

Without explicitly defining under what conditions flow separates into this behavior characterized by Taylor columns, it may not be properly modeled with RANS CFD until the Reynolds closure problem is solved.

To better understand rotational flow patterns around structures, field measurements and experiments in the ocean would be the best methods. Since rotational effects are greater at higher latitudes it would be prudent to conduct such experiments at these locations. Furthermore, conducting experiments well below the surface, >10m,

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would minimize Ekman effects. Also, locations with a uniform bathymetry and good data on local currents would be ideal so that Coriolis effects could be filtered out from field data more easily.

Appendix

Matlab code

Appendix 1 – Matlab code

```matlab
%Ploting streamlines for quasigeostrophic flow over an isolated elliptical

%topography (Johnson, 1982, Deep-sea Research, vol 29, pp 1085-1097

% Length of semi-major axis of ellipse = L; semi-minor axis = l;

%gamma = L/l; height of cylinder h = l;

%S = h0/(d*R0), where R0 = U/(f*l) is Rossby number; h0 is height of

%cylinder and d is water depth.

M=50;N=100;% number of M x N grid points in (μ, theta) coordinates

al=[pi/2];%al is angle of orientation of ellipse to y-axis

S=[1.2, 2, 3, 4];% S will increase as U decreases;

gamma=1000; %Choosing parameters S, gamma

mu0=atan(1/gamma);

a=2*sqrt(gamma^2-1);% determine μ0 and a from (3.5)

mu=[linspace(0,mu0,5) linspace(mu0,2,M)];

theta=linspace(-pi,pi,N); %Set up μ (μ0<μ<2

%and -?<theta<?)for flow outside of the elliptical cylinder

[MU,TH]=meshgrid(mu,theta);%define grid matrices in (μ,theta) coordinates

tsy=1/32*a^2*sinh(2*mu0).*(exp(-2*MU).*cos(2*TH)+2*(MU-mu0));% Eqn (3.3b)

for μ>μ0

tsy(:,1:5)=1/32*a^2*((1-exp(-

2*mu0)*cosh(2*MU(:,1:5))).*cos(2*TH(:,1:5))+...

cosh(2*MU(:,1:5))-cosh(2*mu0));% Eqn (3.3a) for μ<μ0

Zeta=0.5*a*cosh(MU).*cos(TH);

Eta=0.5*a*sinh(MU).*sin(TH);%determine (zeta, eta)

%see paragraph before equation (3.1)

% Calculate derivatives needed to evaluate u = -?p/?y and v = ?p/?x

dtsybydmu = 1/32*a^2*sinh(2*mu0).*(-2*exp(-2*MU).*cos(2*TH)+2);% ?tsy/?μ

dtsybydtheta = -1/32*a^2*sinh(2*mu0).*2*exp(-2*MU).*sin(2*TH);% ?tsy/?theta

%for μ<μ0:

dtsybydmu(:,1:4)=1/16*a^2*(-exp(-2*mu0)*sinh(2*MU(:,1:4))....
```

```matlab
.*cos(2*TH(:,1:4))+sinh(2*MU(:,1:4)));
dtsybydtheta(:,1:4)=-1/16*a^2*((1-exp(-2*mu0)*cosh(2*MU(:,1:4)))....
.*sin(2*TH(:,1:4)));
dmubydz=2/a*sinh(MU).*cos(TH)./(cosh(MU).^2-cos(TH).^2);% ?μ/?zeta
dthbydz=-2/a*cosh(MU).*sin(TH)./(cosh(MU).^2-cos(TH).^2);% ?theta/?zeta
dmubydeta=2/a*cosh(MU).*sin(TH)./(sinh(MU).^2+sin(TH).^2);% ?μ/?eta
dthbydeta=2/a*sinh(MU).*cos(TH)./(sinh(MU).^2+sin(TH).^2);% ?theta/?eta
for j=1:4
X=Zeta*cos(al)-Eta*sin(al);%Determine corresponding (X,Y) coordinates
Y=Zeta*sin(al)+Eta*cos(al);
dzbydy=sin(al);detabydy=cos(al);% ?zeta/?y; % ?eta/?y
dzbydx=cos(al);detabydx=-sin(al);% ?zeta/?x; % ?eta/?x
p=-Y-S(j)*tsy; %using equation preceding (2.4a)
dpbydy=-1-S(j)*((dtsybydmu.*dmubydz+dtsybydtheta.*dthbydz).*dzbydy+....
(dtsybydmu.*dmubydeta+dtsybydtheta.*dthbydeta).*detabydy);% ?p/?y
dpbydx=-S(j)*((dtsybydmu.*dmubydz+dtsybydtheta.*dthbydz).*dzbydx+....
(dtsybydmu.*dmubydeta+dtsybydtheta.*dthbydeta).*detabydx);% ?p/?x
U=-dpbydy;% x component of velocity
V=dpbydx;% y component of velocity
%Plot streamlines
figure(1);subplot(2,2,j)
contour(X,Y,p,22,'k'); %plot streamlines as constant contours of p
hold on; plot(X(:,6),Y(:,6),'r');% plot ellipse
set(gca,'FontSize', 8, 'FontName', 'Times New Roman')
axis([-2.4*gamma,2.4*gamma,-2.4*gamma,2.4*gamma]); %choose suitable domain
of the plot
xlabel('X/l');ylabel('Y/l');
if j==1; title('S = 1.2')
elseif j==2; title('S = 2')
elseif j==3; title('S = 3')
elseif j==4; title('S = 4')
```

```matlab
end
figure(2);subplot(2,2,j)
contour(X,Y,p,30,'k'); %plot streamlines as constant contours of p
hold on; plot(X(:,6),Y(:,6),'r');% plot ellipse
axis([-2.4*gamma,2.4*gamma,-2.4*gamma,2.4*gamma]); %choose suitable domain
of the plot
xlabel('X/l');ylabel('Y/l');
if j==1; title('S = 1.21')
elseif j==2; title('S = 2')
elseif j==3; title('S = 3')
elseif j==4; title('S = 4')
end
% Plot Velocity vectors
%{
figure(2);subplot(2,2,j)
quiver(X(1:2:end,1:2:end),Y(1:2:end,1:2:end),U(1:2:end,1:2:end),....
V(1:2:end,1:2:end)); %plot velocity vectors
hold on; plot(X(:,6),Y(:,6),'r');% plot ellipse
axis([-2.4*gamma,2.4*gamma,-2.4*gamma,2.4*gamma]); %choose suitable domain
xlabel('X/l');ylabel('Y/l');
if j==1; title('\alpha = 0')
elseif j==2; title('\alpha = \pi/4')
elseif j==3; title('\alpha = \pi/2')
elseif j==4; title('\alpha = 3\pi/4')
end
figure(3);subplot(2,2,j)
pcolor(X,Y,sqrt(U.^2+V.^2));shading interp; %plot absoloute speed
hold on; plot(X(:,6),Y(:,6),'r');% plot ellipse
axis([-2.4*gamma,2.4*gamma,-2.4*gamma,2.4*gamma]); %choose suitable domain
%size
xlabel('X/l');ylabel('Y/l');axis equal;colorbar
```

```matlab
if j==1; title('\alpha = 1.04')

elseif j==2; title('\alpha = \pi/4')

elseif j==3; title('\alpha = \pi/2')

elseif j==4; title('\alpha = 3\pi/4')

end

%}

end
```

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