OES Task 10 WEC heaving sphere performance modelling verification

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ABSTRACT: OES Task 10 Modelling, Verification and Validation of Wave Energy Converters (WECs) is a task under the IEA Technology Collaboration Program for Ocean Energy Systems (OES). The long-term goals are to:

1. Assess the accuracy of, and establish confidence in, the use of numerical WEC models
2. Determine a range of validity of existing computational modelling tools
3. Identify uncertainty related to simulation methodologies
4. Define future research.

To some extent, this project builds on the experience from a similar effort carried out to verify modelling of wind turbines as part of the IEA Wind Task 30 on wind OC3-OC5.

1 INTRODUCTION

1.1 *OES Task 10.1*

The first joint effort within IEA OES Task 10 focused on a heaving sphere as shown in Figure 1 and compared results of different "numerical experiments," such as heave decay test, regular wave cases of varying steepness and periods, and irregular wave cases, with and without external damping. The numerical models included Cummins' equation-based linear and slightly nonlinear models, fully nonlinear time-domain boundary element method (BEM) as well as computational fluid dynamics (CFD) simulations. The initial work is described in the joint reference paper (Wendt et al., 2017)

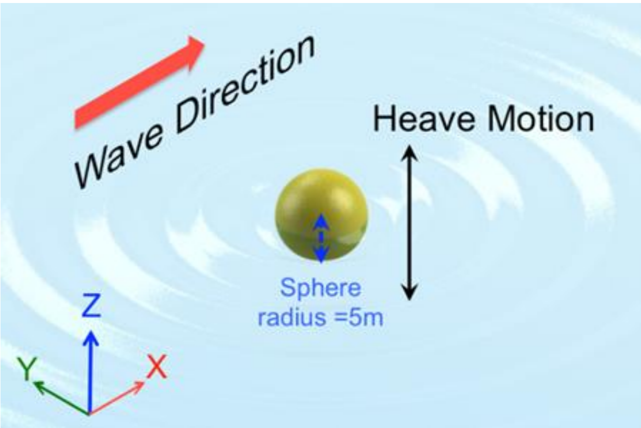

Figure 1. A floating sphere WEC system has been modeled for the IEA OES Task 10.1 Project.

1.2 *OES Task 10.1.2*

This paper summarizes the theory behind the results of Task 10.1 and presents the results of Task 10.1.2, which include comparisons of power generation calculations in six irregular sea states with optimal linear damping and the same calculations with negative springs. The annual energy production was calculated following the simplified methodology of Nielsen & Pontes (2010), using the six sea states and the corresponding weighting factors to estimate the averaged power output at a selected generic location representing the North Sea.

1.3 *Definition of float geometry*

The general properties for the floating sphere system is defined in Table 1.

Table 1. The general properties for the floating sphere system.

Parameters	Value
Radius of the sphere	5.0 m
Origin of the sphere in global coordinates	0.0, 0.0, 0.0 (m)
Center of gravity	0.0, 0.0, -2.0 (m)
Mass of the sphere	261.8×10^3 kg
Moment of inertia	TBD
Water depth	Infinite
Water density	1000.0 kg/m^3

The resonance frequency for a heaving sphere is given by the approximate solution Falnes (2002):

$$\omega_0 \approx 1.025\sqrt{g/a} \tag{1}$$

where, g is the acceleration of gravity and a is the sphere radius. For $a = 5.0\ m$ radius sphere, the resonant period is $T_0 = 2\pi/\omega_0 \approx 4.4\ s$.

2 HYDROSTATIC TEST AND THE HEAVE DECAY TEST

The initial test specified for the group of IEA OES Task 10 participants was the hydrostatic and heave decay test of the sphere. The hydrostatic test ensures the float floats at the prescribed waterline, in this case specified on equator. This means the float mass M_f must equal the volume of displaced water or $M_f = \rho 2\pi a^3/3$. With radius $a = 5m$ and $\rho = 1000\ kg/m^3$, the displaced mass of water is 261799.38 kg. The specified mass of the float in Table 1 is 261800 kg which is 0.62 kg more.

At the equator, the waterplane area is $S_w = \pi a^2$. If the float is displaced a vertical distance s_3 from equilibrium, it will experience a hydrostatic force that is proportional to the displaced volume. If the displacement is small, the hydrostatic force F_b becomes $F_b \approx s_3\rho g S_w = s_3 S_b$ where S_b is called the hydrostatic stiffness. If we take the shape of the sphere into consideration, the hydrostatic force will follow the formulation $F_b = s_3\rho g S_w(1-s_3^2/3a^2)$. The hydrostatic force as a function of the vertical displacement is shown in Figure 2. From the figure, it appears that displacements with amplitudes less than two meters the linear approximation seems reasonable.

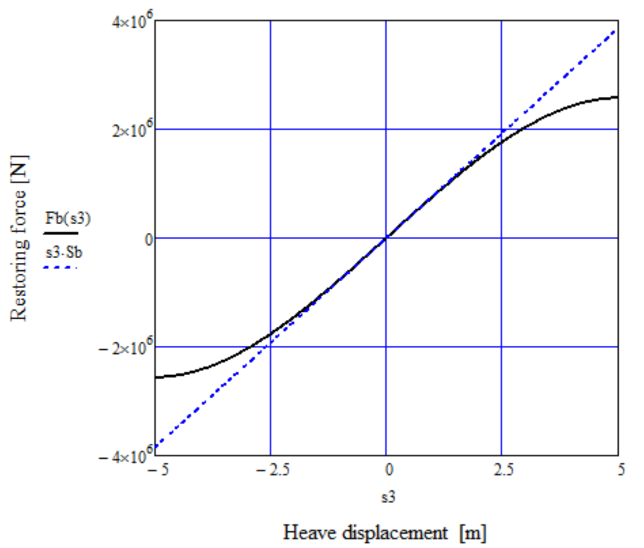

Figure 2. Hydrostatic force on the floating sphere WEC system modeled for the IEA OES Task 10.1.

2.1 *Decay test*

The second task was to simulate how the float moves up and down in the water if released from a given initial elevation. This experiment can be simplified to the analogue of a free oscillator with a mass m (the float mass + added mass), a damping coefficient R, a spring coefficient S displaced to an initial displacement x_0. The governing equation for the free, damped oscillation is specified below:

$$M\ddot{x} + R\dot{x} + Sx = 0 \tag{2}$$

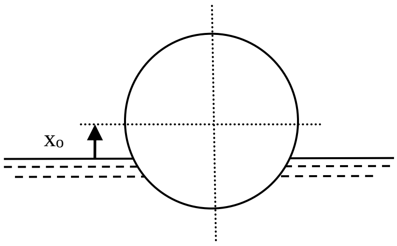

Figure 3. Initial displacement of the floating sphere WEC system for the decay test modeled for the IEA OES Task 10.1.

The theoretical solution to this experiment is given by Equation 3:

$$x = (C_1 \cos\omega_d t + C_2 \sin\omega_d t)e^{-\delta\cdot t} \tag{3}$$

where $\delta = R/2M$, $\omega_o = (S/M)^{0.5}$, $\omega_d = (\omega_o^2 - \delta^2)^{0.5}$ and the integration constants C_1 and C_2 can be determined from the initial displacement x_o and velocity u_o as $C_1 = x_0$ and $C_2 = (u_o + x_o\delta)/\omega_d$. The floating sphere is released from a position x_0 above its equilibrium in still water, and the float will move up and down, gradually converting the initial potential energy to waves traveling outwards from the float. The similarity with the damped oscillator is explained as the spring term S is comparable to the hydrostatic stiffness S_b and the damping force proportional to the hydrodynamic damping coefficient $B_{33}(\omega)$ and the inertial part proportional with the acceleration is expressed as the sum of the float mass M_f plus added mass $A_{33}(\omega)$.

Both the added mass and damping are frequency-dependent hydrodynamic properties, and in the case of the sphere, it is possible to derive an analytical solution to these properties. Such a solution and results were first presented by Havelock (1955) and Hulme (1982). A comparison of the early analytical values to calculations of values carried out using the modern computer program, WAMIT Lee & Newman (2015) is shown in Figure 4. To compare WAMIT values with the theoretical values derived by Hulme (1982), the WAMIT coefficients have been nondimensionalized accordingly. Researchers at the National Renewable Energy Laboratory (NREL) generated the coefficients to be used by all IEA OES Task 10 participants. Table 2 shows examples of selected values of the calculated coefficients using WAMIT and computed at the Center of Gravity (CG) as well as at the mean water surface. The hydrodynamic coefficients include the added mass, radiation damping and wave-exciting forces, including both diffraction and linear Froude–Krylov forces.

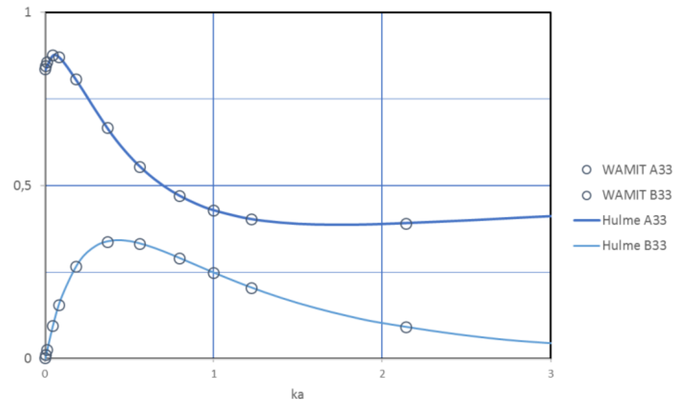

Figure 4. Comparison of analytical coefficients (Hulme, 1982) and coefficients derived using WAMIT.

Accordingly, the free decay test can be formulated as:

$$(M_f + A_{33}(\omega))\ddot{s}_3 + B_{33}(\omega)\dot{s}_3 + S_b s_3 = 0 \qquad (4)$$

The theoretical solution for Equation 2 is found using $\omega_d = (\omega_o - B_{33}(\omega_d)/(M_f + A_{33}(\omega_d))^{0.5}$. The values used in the equation are given in Table 1 and Table 2 and the solution of Equation 3 is presented in Figure 5.

Table 2. The hydrodynamic coefficients B_{33} and A_{33} for selected values of T in deep water.

T_d	ω_d	$A_{33}(\omega_d)$	$B_{33}(\omega_d)$	$\delta(\omega_d)$	C1	C2
sec	sec^{-1}	kg	kg sec^{-1}	sec^{-1}	m	m
4.384	1.433	1.106×10^5	8.962×10^5	0.12	1.0	0.084

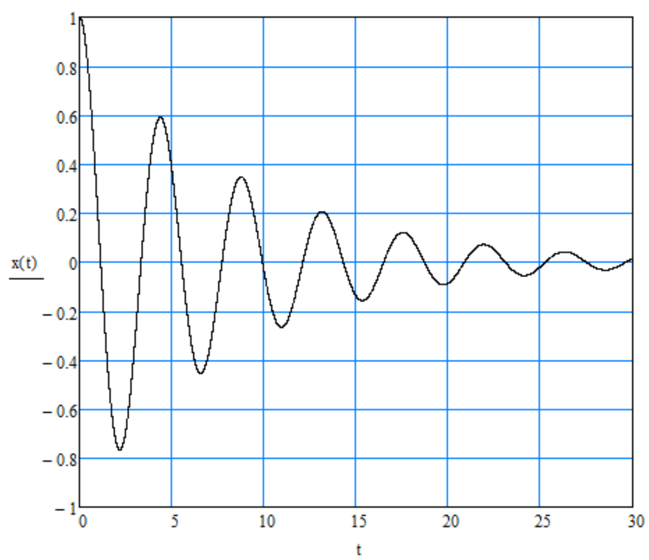

Figure 5. Plot of free decay experiment in theory Equation 3.

The free decay test for the three initial displacements as calculated by the participants using each their different numerical tools and codes was presented in 2017 at the European Wave and Tidal Energy Conference (EWTEC) Wendt et al. (2017).

3 FORCED OSCILATION

Considering the case in which the sphere is restricted to vertical forced motion, the equation of motion reads:

$$M\ddot{x} + R\dot{x} + Sx = F = F(t) \qquad (5)$$

When the sphere is placed in water with incoming sinusoidal waves, a force will act on the float and it could have the general form:

$$F(t) = F_0 \cos(\omega t + \varphi_F) \qquad (6)$$

where F_o is the force amplitude and φ_F a phase constant for the force. The solution to Equation 5 has a similar form for which the position can be expressed as:

$$x(t) = x_0 \cos(\omega t + \varphi_x) \qquad (7)$$

and the velocity

$$\dot{x}(t) = u_0 \cos(\omega t + \varphi_u) \qquad (8)$$

The velocity amplitude u_0 is related to the amplitude of the motion x_0 by $u_0 = \omega x_0$ and the phase constants by $\varphi_u - \varphi_x = \pi/2$. To be a solution to Equation 5 the excursion amplitude is given as:

$$x_0 = \frac{u_0}{\omega} = \frac{F_0}{|Z|\omega} \qquad (9)$$

The phase difference between the force and the velocity is:

$$\varphi = \varphi_F - \varphi_u = \varphi_F - \varphi_x - \pi/2 \qquad (10)$$

The phase angle satisfies the equation:

$$\tan\varphi = (\omega m - S/\omega)/R \qquad (11)$$

$|Z|$ is the absolute value of the complex impedance, which can be expressed as:

$$|Z| = \left\{R^2 + (\omega m - S/\omega)^2\right\}^{1/2} \qquad (12)$$

Extending this theory of forced oscillations into the problem of the heaving sphere, we can compute the response amplitude for the sphere to incoming sinusoidal waves of amplitude $A=1m$:

$$\frac{x_0}{A} = \frac{X_3}{\omega\left[B_{33}{}^2 + (\omega(M_f + A_{33}) - S_b/\omega)^2\right]^{1/2}} \qquad (13)$$

The phase angle between force and velocity is found as:

$$\varphi = \arctan((\omega(M_f + A_{33}) - S_b/\omega)/B_{33}) \quad (14)$$

Using the frequency-dependent exciting force amplitude X_3 and hydrodynamic coefficients A_{33} and B_{33} calculated using WAMIT inserted into Equation 13, we get the response curve as indicated in Figure 6.

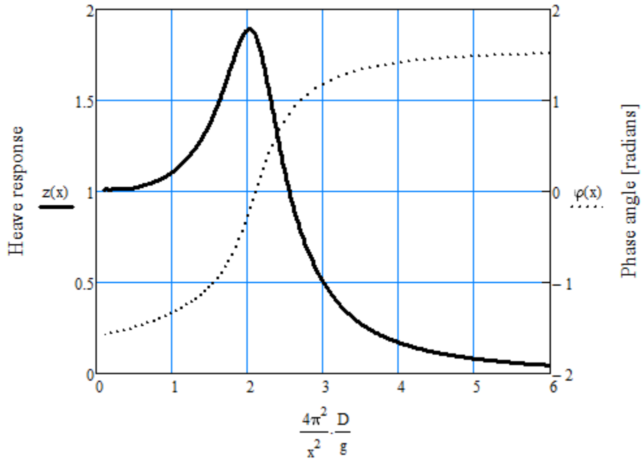

Figure 6. Plot of the response of the sphere to incoming waves.

The response of the sphere with no external Power Take-Off System (PTO) damping was calculated by the participants for three steepnesses of regular waves. Participants with non-linear codes obtained noticeable lower Response Amplitude Operators (RAOs) at resonance compared to linear simulations.

4 ENERGY EXTRACTION AND OPTIMAL DAMPING

To extract energy from the floating sphere, one must imagine a damper fixed between the float and an external reference frame as illustrated in Figure 7. In practice, the damper could be a hydraulic circuit including a hydraulic motor and generator to convert the absorbed power to electricity or it could be a linear electric magnetic generator. In linear theory, the damper is expressed as a coefficient multiplied with the velocity of the float.

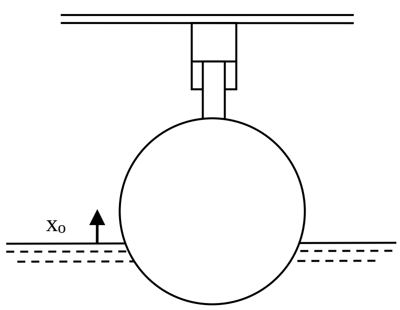

Figure 7. Float connected to external damper.

The OES Task 10 group was tasked with calculating the absorbed power in the regular waves of three different steepnesses.
The steepness was defined as $s = H/(gT^2)$, and the steepness values of 0.0005, 0.002 and 0.01 were chosen to see whether this would impact the solutions when models other than linear theory were applied.

Table 3. The wave parameter for selected values of T in deep water.

T	λ	B_{opt}(T)	B_{33} (T)	A_{33} (T)	X_3
sec	m	kg sec^{-1}	kg sec^{-1}	kg	kg m^{-2}sec^{-2}
3.0	14.1	3.99×10^5	4.70×10^4	1.03×10^5	9.78×10^4
4.0	25.0	1.19×10^5	8.24×10^4	1.05×10^5	2.00×10^5
4.4	30.2	9.01×10^4	8.98×10^4	1.11×10^5	2.41×10^5
5.0	39.0	1.62×10^5	9.46×10^4	1.22×10^5	3.00×10^5
6.0	56.2	3.23×10^5	9.17×10^4	1.45×10^5	3.87×10^5
7.0	76.5	4.80×10^5	8.06×10^4	1.67×10^5	4.58×10^5
8.0	99.9	6.34×10^5	6.87×10^4	1.84×10^5	5.14×10^5
9.0	126.4	7.85×10^5	5.68×10^4	1.97×10^5	5.57×10^5
10.0	156.1	9.32×10^5	4.65×10^4	2.07×10^5	5.92×10^5
11.0	188.9	1.08×10^6	3.89×10^4	2.16×10^5	6.23×10^5

The float amplitude of the damped motion applying external linear damping can be calculated as:

$$(M_f + A_{33}(T))\frac{dz}{dt^2} + (B_{33}(T) + fr)\frac{dz}{dt} + S_s \cdot z = AX_3(T)\cos(t) \quad (15)$$

In this equation, the damping ratio *fr* is expressed as a damping factor *rd* multiplied with $(M_f\ S_b)^{0.5}$:

$$fr = rd\sqrt{M_f \cdot S_b} \quad (16)$$

Rearranging the variables in Equation 15, we can express the equation as:

$$\frac{dz}{dt^2} + 2\frac{(B_{33}(T) + fr)}{(M_f + A_{33}(T))}\frac{1}{2}\frac{dz}{dt} + \frac{S_b}{(M_f + A_{33}(T))}z = \frac{AX_3(T)\cos(t)}{(M_f + A_{33}(T))} \quad (17)$$

Defining the following variables:

$$\gamma(T, rd) = \frac{\left(B_{33}(T) + rd\sqrt{M_f S_b}\right)}{(M_f + A_{33}(T))}\frac{1}{2} \quad (18)$$

$$\omega_0 = \frac{S_b}{(M_f + A_{33}(T))} \quad (19)$$

$$\omega(T) = \frac{2\pi}{T} \quad (20)$$

The resulting equation becomes:

$$\frac{dz}{dt^2} + 2\gamma(T, rd)\frac{dz}{dt} + \omega_o(T)z = \frac{AX_3(T)\cos(t)}{(M_f + A_{33}(T))} \quad (21)$$

The solution to this equation of motion is a harmonic oscillation with the same frequency of oscillation as the applied force. The amplitude of the oscillation can be calculated as:

$$z_d(T,rd) = \frac{AX_3(T)}{(M_f + A_{33}(T))} \frac{1}{\sqrt{\left(\omega(T)^2 - \omega_o(T)^2\right)^2 + 4\left(\gamma(T,rd)^2\right) + \omega(T)^2}} \quad (22)$$

The phase between the velocity of the sphere and the applied force is:

$$\alpha(T,rd) = a\tan\left(\frac{\left(\omega(T)^2 - \omega_o(T)^2\right)}{2\gamma(T,rd)\omega(T)}\right) \quad (23)$$

Using this information, the float position in relation to the wave elevation and the position of the center of the float can be presented as a function of time with its phase lag compared to the wave elevation at the center of the float *(φF(T)* is the phase lag between the wave and the exciting force):

$$z_f(t,T,rd) = z_d(T,rd)\cos\left(\omega(T)t + \varphi F(T) + \alpha(T,rd) - \frac{\pi}{2}\right) \quad (24)$$

The average power generated by the float can be expressed as:

$$P_{abs}(T,rd) = \frac{1}{2}\left(rd\sqrt{M_f S_b}\right)\cdot \omega(T)^2 z_d(T,rd)^2 \quad (25)$$

The maximum absorbed power for a given wave period T can be found by optimizing damping coefficient *rd* that provides the maximum power output in each wave period Falnes (2002):

$$B_{opt}(T) = B_{33}(T)\sqrt{1 + \left(\frac{S_b - \omega(T)^2\left(M_f + A_{33}(T)\right)}{\omega(T)B_{33}(T)}\right)^2} \quad (26)$$

The optimal damping B_{opt} can be expressed as a damping factor

$$rd_{opt}(T) = \frac{B_{opt}(T)}{\sqrt{M_f S_b}} \quad (27)$$

The incoming power in the wave is:

$$P_w(T) = \frac{\rho g^2}{32\pi}(2A)^2 T \quad (28)$$

The capture width ratio (CWR) expresses how much power the float absorbs compared to the incoming wave power over its diameter D:

$$CWR(T) = \frac{P_{abs}(T,rd)}{DP_w(T)} \quad (29)$$

For point absorbers, Falnes (2002) derived the theoretical maximum for resonance absorption as:

$$PAth(T) = \frac{\lambda(T)}{2\pi\cdot D} = \frac{gT^2}{4\pi^2\cdot D} \quad (30)$$

From Figure 8, one sees that heaving sphere reaches its theoretical maximum CWR at its resonance period, which when inserted in Equation 30 gives *PAth(4.4 sec)= 0.48* .

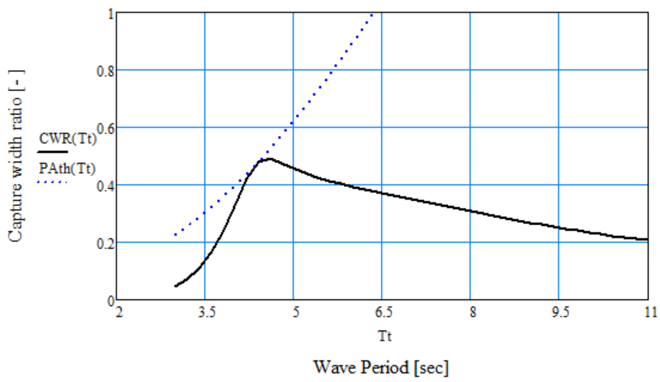

Figure 8. The CWR as a function of the wave period for the sphere using the specified optimal damping. Also, the theoretical upper limit *PAth* is shown for the heaving point absorber.

5 POWER EXTRACTION IN IREGULAR WAVES

The second task involved predicting the annual energy production as calculated following the simplified methodology provided by Nielsen & Pontes (2010). This methodology indicates a linear relationship between *Hs* and *Tp* for the location used in this example. The relation is:

$$T_p(H_s) = 0.876\cdot H_s + 5.75 \quad (31)$$

Based on the distribution of six sea states as specified in Table 4 given by the weight in percent (%), an estimate of the annual energy production can be made using the six sea states. The weighting factor indicates how much the sea state contributes to the annual energy production. The chosen distribution and combination provide an estimate of the averaged power representing the North Sea with an average wave power resource of 20 kW/m.
The irregular sea is described using a Bretschneider spectrum with the significant wave height H_s and peak period T_p as input:

$$S(t) = A_s\cdot t^5 \exp\left(-B_s\cdot t^4\right) \quad (32)$$

where:

$$A_s = \frac{5}{16}H_s^2\left(\frac{1}{T_p}\right)^4 \text{ and } B_s = \frac{5}{4}\left(\frac{1}{T_p}\right)^4 \quad (33)$$

The damping parameter is constant in each irregular sea state, and larger sea states with longer wave periods require more damping to extract the most energy. The optimal damping is chosen using Equation 26 with the period $T=T_p$, the peak period of the spectrum.
The float response amplitude in these irregular wave cases, is noted to be less than 4 meters, which is realistic. The annual average absorbed power based on these simulations from the OES Task 10 team is presented in Table 6.

Table 4. The selected six irregular sea states weight and selected damping parameters for each case. The average absorbed power in each sea state is indicated as reported by one of the participants (Wave Venture). Further, the annual Average Absorbed Power is indicated as AAP.

Hs	Tp	Weight	B_{opt}	Pabs	ΔPabs
m	sec	%	kg sec^{-1}	kW	kW
1	6.6	36.95	4.24×10^5	7.5	2.7
2	7.5	31.43	5.58×10^5	31.4	9.9
3	8.4	16.96	6.90×10^5	74.9	12.7
4	9.2	7.23	8.19×10^5	140.1	10.1
5	10.1	2.91	9.47×10^5	226.6	6.6
6.1	11.1	1.41	1.09×10^6	344.4	4.9
			Average Absorbed Power (AAP):		46.9

5.1 *Negative spring*

As an initial introduction to controlled motion of WEC it was agreed by the team to try calculating the effects of negative springs. Equation 26 is modified to describe the optimum PTO damping of the system including a negative spring *Ns*:

$$B_{opt}(T) = B_{33}(T)\sqrt{1+\left(\frac{(S_b - Ns) - \omega(T)^2\left(M + A_{33}(T)\right)}{\omega(T)B_{33}(T)}\right)^2} \quad (34)$$

The damped natural frequency of the heaving sphere at the optimum PTO damping can be calculated using the equation:

$$\omega(T) = \sqrt{\frac{S_b + N_s}{M_f + A33(T)}}\sqrt{1+\left(\frac{(B_{opt}(T) + B33(T))^2}{4(M_f + A33(T))(S_b - N_s)}\right)} \quad (35)$$

The additional negative spring stiffness that creates resonance at the longer peak period of the spectrum can be computed using iteration. The resulting spring stiffness and damping values are summarized in Table 5. Note that the spring stiffness N_s values in Table 5 do not include the hydrostatic stiffness term. Further, the average absorbed power in each sea state is indicated as reported by one of the participants (Wave Venture).

Table 5. The selected six irregular sea states are given in terms of H_s, T_p and weight and negative spring, damping parameters for each case.

Hs	Tp	Weight	Ns	$B_{opt\ (NS)}$	Pabs
m	sec	%	kg sec^{-2}	kg sec^{-1}	kW
1	6.6	36.95	-3.76×10^5	8.65×10^4	15
2	7.5	31.43	-4.50×10^5	7.54×10^4	84
3	8.4	16.96	-5.07×10^5	6.42×10^4	271
4	9.2	7.23	-5.50×10^5	5.44×10^4	666
5	10.1	2.91	-5.84×10^5	4.58×10^4	1355
6.1	11.1	1.41	-6.15×10^5	3.74×10^4	2502

The negative spring simulations presented in Table 5 were conducted with unrestricted float heave motion. These negative spring simulations predicted very large heave motion amplitudes, even in sea states Hs = 2m large values over 5m have been observed (i.e. the buoy radius). In the larger wave cases the motions become unrealistic with heave amplitudes of more than 10m. This is a result of trying to force resonance in long waves where the linear damping is so small that it cannot possibly constrain the motions to realistic values. In order consider negative springs in a realistic way, future work will focus on tuning the damping in each case to ensure that the motion amplitudes are held within 5m.

6 SUMMARY OF THE RESULTS FROM PARTICIPATING PARTNERS

The APP values reported by the participants are presented in Table 6. Overall, the results show good agreement and the difference in results are in most cases due to truncation of the time series.
During the initial task, a difference between linear and non-linear codes was visible around the resonance frequency of the heaving sphere.

Table 6. The annual average absorbed power (AAP) reported by the participants in Task 10.1.2 for the case with optimal damping and without negative spring.

OES Task 10 Participant	AAP [kW]
#1 NREL/SANDIA	47.7
#2 Tecnalia	59.7
#3 KRISO	48.7
#4 Wave EC	46.4
#5 University College Cork	47.1
#6 Aalborg University	47.0
#7 Innosea	47.7
#8 EC Nantes	46.4
#9 Wave Venture	46.9
#10 Dynamic Systems Analysis	49.3
#11 Technical University of Denmark	49.3

7 CONCLUSION

The results provided by the OES Task 10 participants in calculation the annual energy production shows good agreement. Wave power and wave energy capture can however be quite dependent on site and WEC. Therefore, the influence of sea state input from the target location site will be considered in future work. Comparing the results in Table 4 and Table 5, one can see that the theoretical power absorbed in each sea-state can be increased almost by a factor of 5 by tuning the resonance period of the sphere to the incoming waves. However, these results are theoretical and represent only an initial aspiration to conduct further numerical simulation of controlled WEC systems which will consider realistic control strategies and equipment.

8 ACKNOWLEDGEMENT

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