

# Advancements in High-Flow Hydraulic Power Take-Off Systems for Wave Energy Converters

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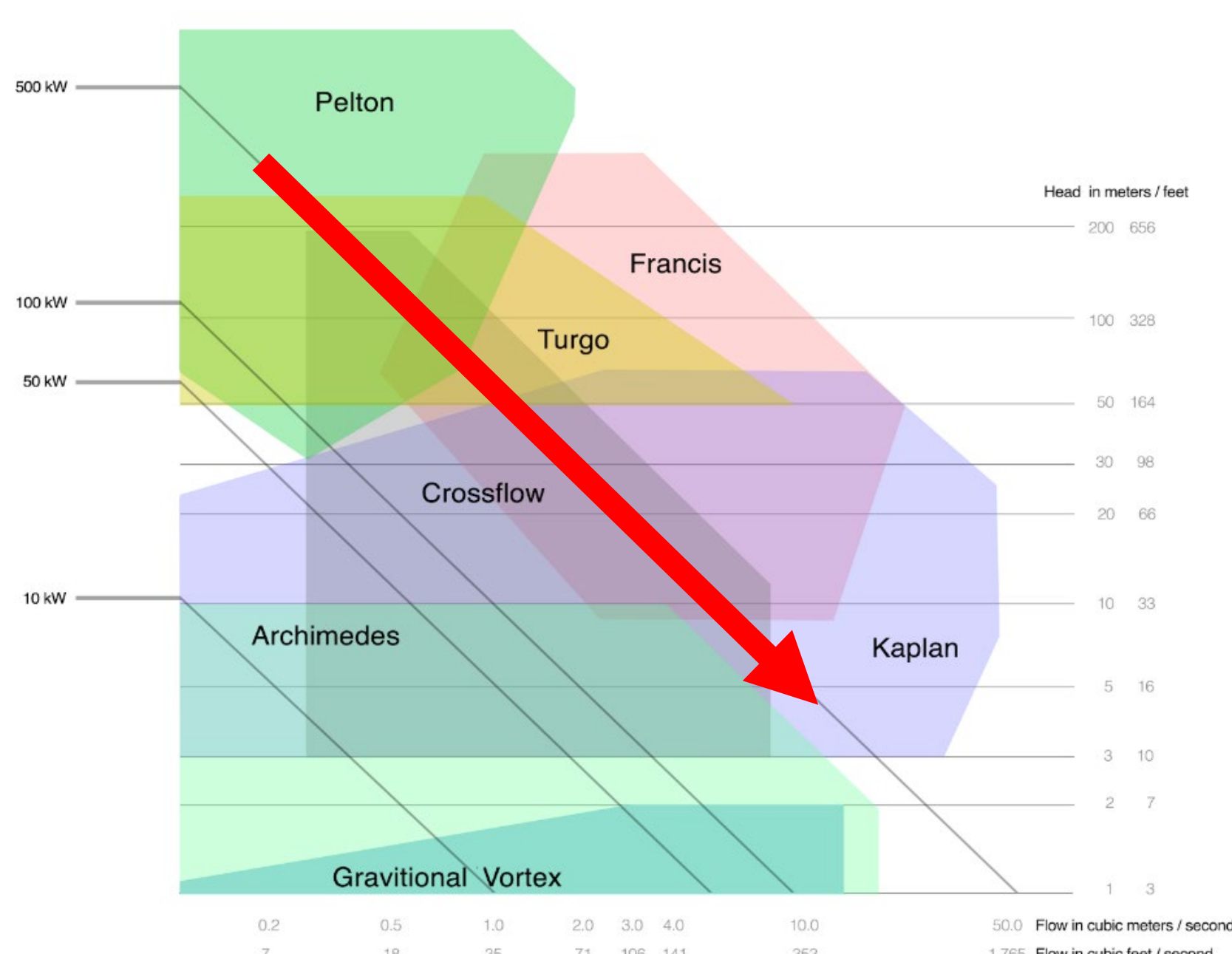
A coupled slider-crank and hydraulic modeling framework has been developed and verified for a high-flow positive-displacement pump intended to impedance-match with an oscillating surge wave energy converter. While perfect conjugate matching has not yet been achieved, this validated platform establishes the foundation for future optimization to improve energy transfer efficiency across the operating band.



## Background

Oscillating surge wave energy converter (OSWEC) designs employing hydraulic power take-off (PTO) systems often rely on high-head, low-flow systems driven by positive-displacement pumps, which can suffer from significant pressure losses and the need for complex control solutions to manage power variability. When paired with a high-efficiency hydro turbine for electro-mechanical power conversion, turbine selection is often bound to Pelton or Turgo types.

This project explores the design space of positive-displacement pumps for low-head, high-flow applications, thereby broadening opportunities for turbine selection. Paired with an OSWEC, a high-flow PTO will open opportunities in pumped storage solutions.



Impedance matching co-design has become the focal point of PTO research and development in recent years. In theory, impedance matching will provide maximum power transfer from the waves to the PTO damper when the PTO impedance equals the complex conjugate of the intrinsic impedance ( $Z_{PTO} = Z_i^*$ ). Adopting this principle, our design methodology focus on identifying positive-displacement pump characteristics which achieve this goal.



## Methodology

### 1. Analytic Model Development

A linear positive-displacement pump is coupled to the pitch motion of an OSWEC using a slider-crank mechanism. An analytic model capturing the nonlinear kinematics and force transmission through a slider-crank geometry is developed using the concept of virtual work. Torque at the axis of rotation is expressed in terms of the crank inertia and the slider force, coupling the change in linear motion with change in angular motion.

$$\tau_{external} = I\ddot{\theta} + F_{slider} \frac{dx}{d\theta}$$

### 2. Linearization and Frequency-Domain Transformation

The analytic model is linearized around a nominal operating point,  $\theta_0$ , to foster frequency-domain impedance analysis. To obtain a linear equation, the Taylor series expansion of the torque is taken to the first order, such that

$$\tau_{external} \approx \tau(\theta_0) + \Delta \tau \frac{d\tau}{d\theta} \bigg|_{\theta_0}$$

### 3. Hydrodynamic Target from Prior WEC Study

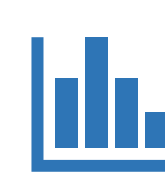
The flap-type wave energy converter (WEC) is characterized using frequency-domain hydrodynamic coefficients derived from Capytaine in the previous HAWSEC (Hawaiian Wave Surge Energy Converter) study. These coefficients define the flap's impedance:

$$Z_i = B(\omega) + i \left[ \omega(m + A(\omega)) - \frac{\rho g K_{hs}}{\omega} \right]$$

### 4. Impedance Matching Strategy

The PTO model is governed by a linear time invariant second order differential equation. The form of the impedance matches that of the hydrodynamic model above. Parameters defining the slider-crank geometry and hydraulic properties are treated as free variables in an optimization routine and selected to minimize the impedance matching error:

$$E = \int_b^a \sigma \left( \Re(Z_i^* - Z_{PTO}) \right)^2 + (1 - \sigma) \left( \Im(Z_i^* - Z_{PTO}) \right)^2 d\omega$$

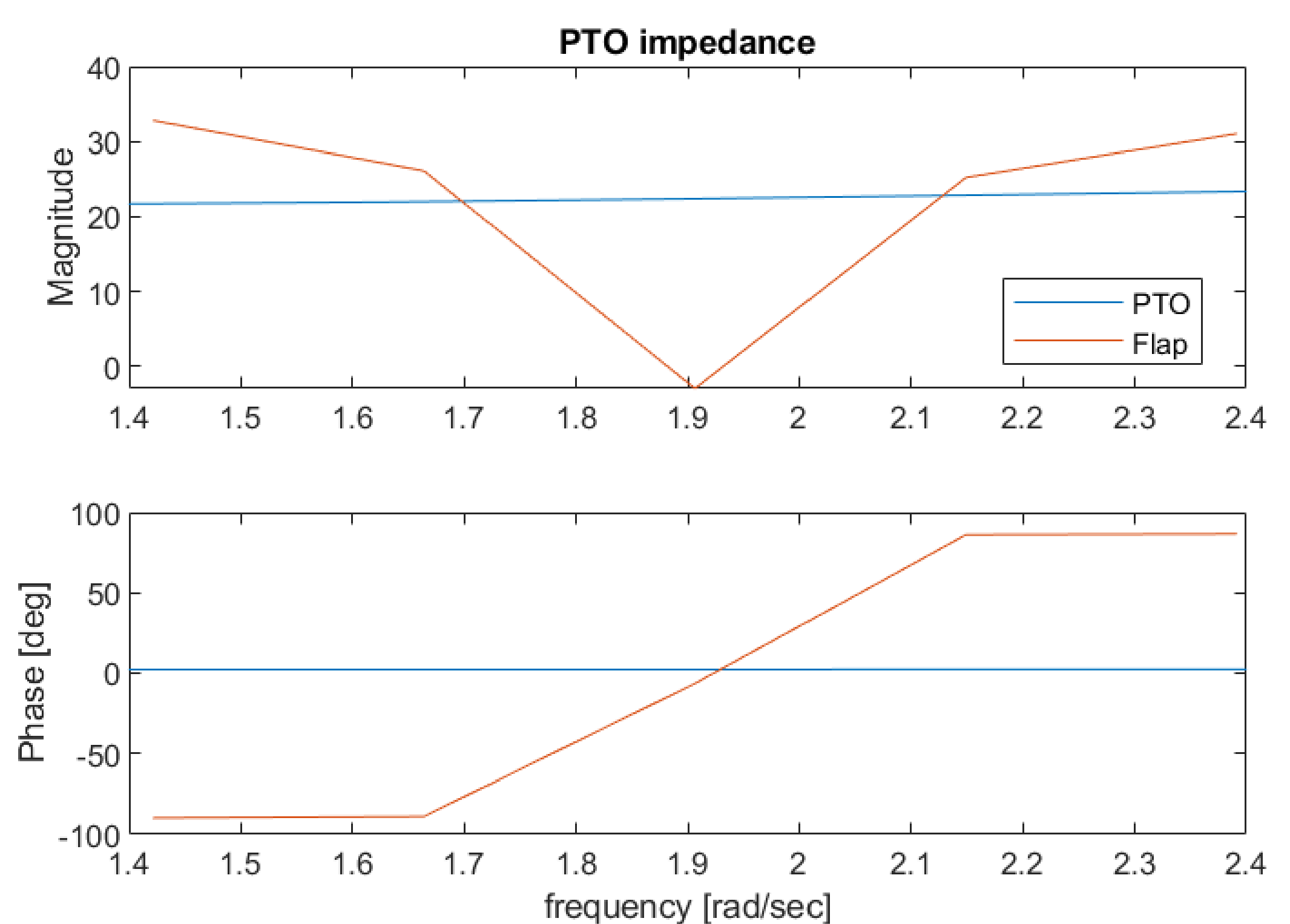


## RESULTS

A best-fit optimization is applied across a model-scale frequency band (2–5s periods), targeting improved performance across a broader range of sea states. The routine successfully minimized impedance mismatch error over the full bandwidth, demonstrating the framework's functionality.

**Flat Impedance Response:** The flap impedance varies strongly with frequency, reaching a minimum near 1.9 rad/s due to hydrodynamic resonance. In contrast, the optimized PTO impedance remains nearly constant across the target frequency band. This is consistent with the initial hydraulic model where no design parameter is yet frequency-dependent. This behavior confirms the current model's anticipated limitation and highlights where future frequency-dependent refinements will have the most impact.

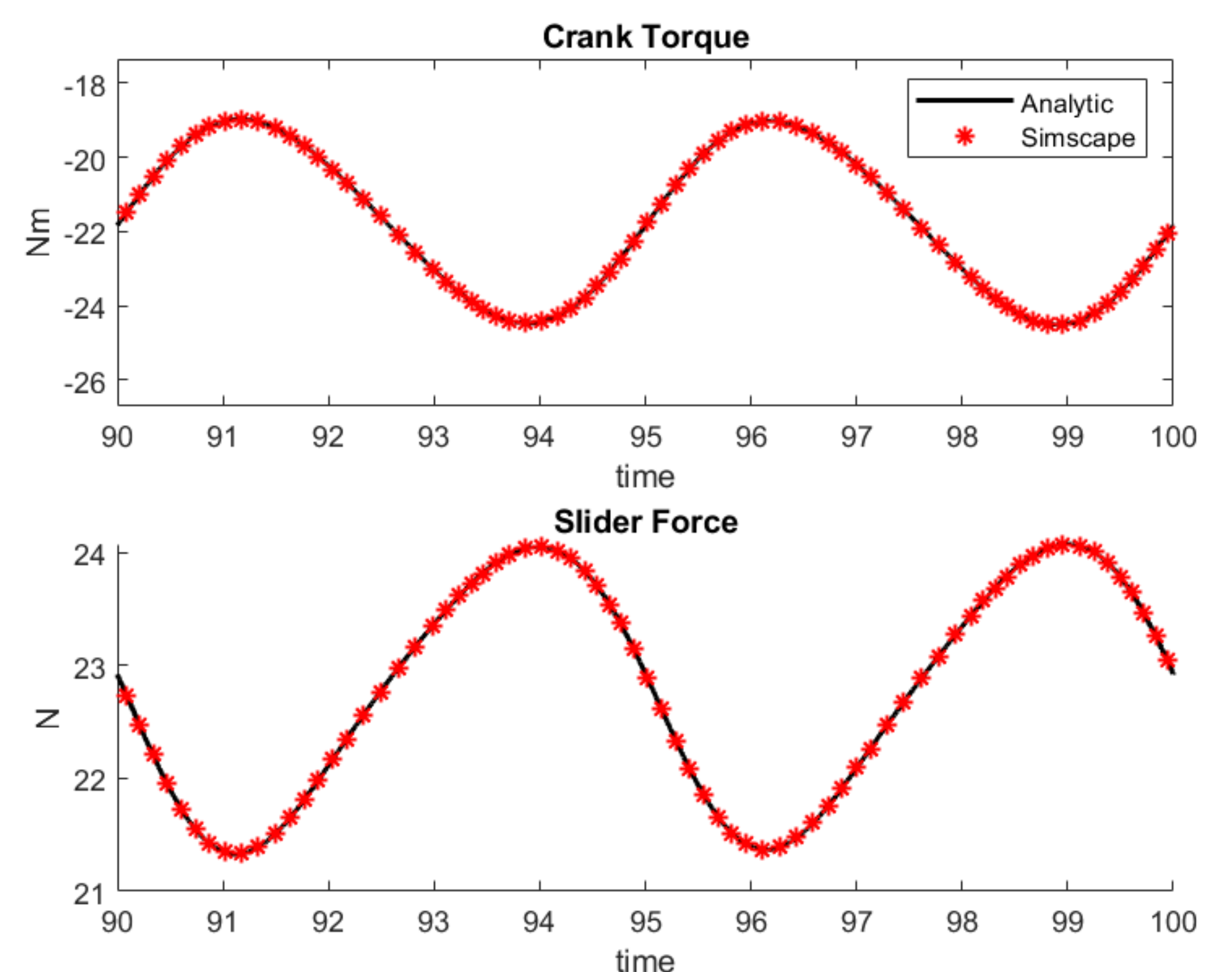
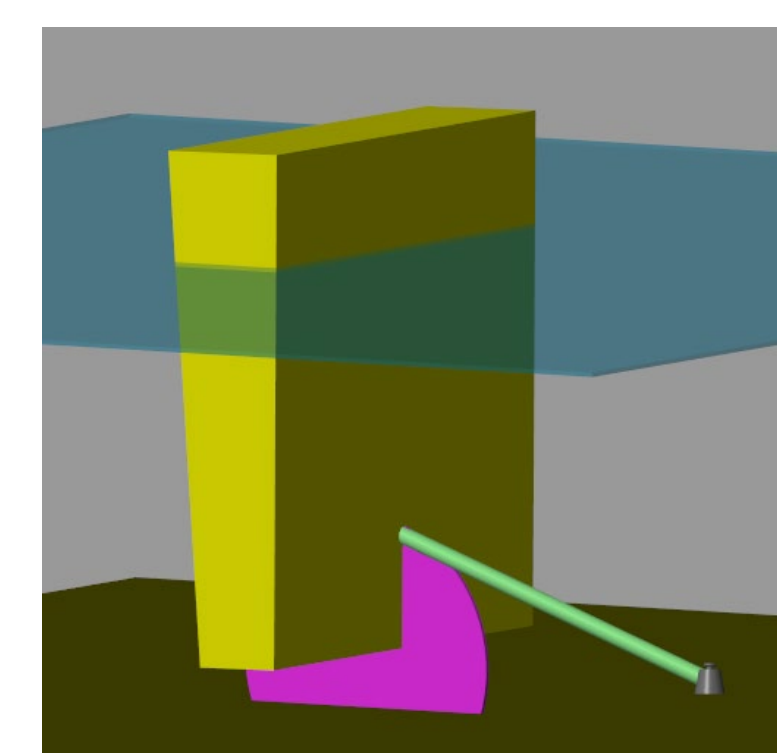
**Geometric Sensitivity:** Variations in crank radius, connecting rod length, piston area, and initial conditions shift the overall level of the impedance curve up or down, rather than changing its shape. Since these components are likely fixed once built (e.g. crank radius not adjustable in operation), they provide baseline control only. This insight points to where alternative designs or parameterizations could improve frequency alignment.



A numerical time-domain model of the PTO was developed using the Matlab Simscape framework. The slider-crank model was developed using Simscape Multibody toolbox and the hydraulic model was developed using Simscape Isothermal Fluids toolbox. The two are coupled by tracking the slider position and internal force constraints.

The time series snippets show strong agreement between the analytic and numeric solutions, for both linear force at the slider and torque at the crankshaft, to verify the numerical implementation.

Coupled with WEC-Sim, the Simscape tool fosters the ability to explore the performance behavior of the high-flow PTO under realistic wave conditions. Coupling is achieved by exchanging torque and angular velocity between the two Multibody models at the axis of rotation.



## Future Work

The hydraulic model used to develop the impedance matching framework considered constant coefficients. Parameters, such as seal friction, do have a dynamic response, thus should exhibit some dependence on frequency. A literature review will be conducted to further improve the model fidelity. The impedance matching should improve as a result, showing less of a flat response.

In addition to frequency dependency, inter-parameter constraints need to be improved to better ground the optimization results in reality. For example, the seal friction coefficient is likely dependent on the piston perimeter, which is related to the area (a free parameter).

Once the PTO design is finalized, future comparisons with the high-head HAWSEC model will reveal the benefits of an impedance matching design approach.



## References

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