On the Development of a Harmonic Balance Model for Wave Energy Converter Arrays

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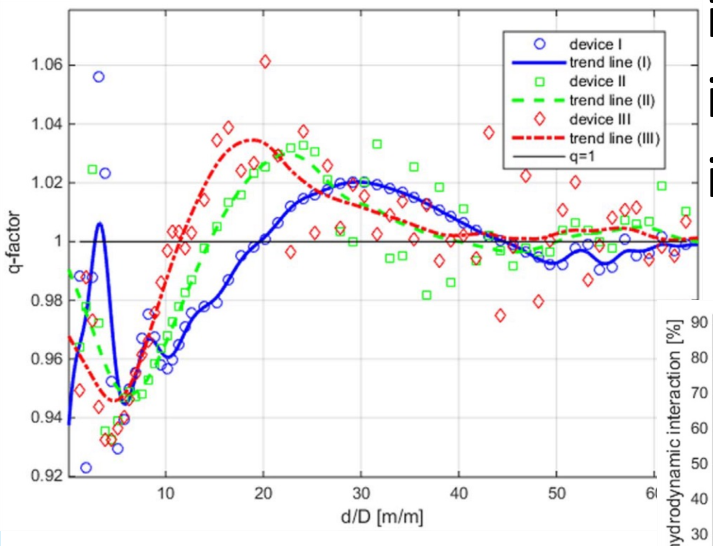
Introduction & motivation

- Low power of isolated Offshore Renewable Energy (ORE) devices
  - FOWTs 10-20 MW approx.
  - WECs < 1 MW
  - TECs ~ 1 MW approx.
- Devices organized in multi-MW arrays
- Array layout conditioned by
  - Mooring & electrical connection configuration
  - Requirements of installation/maintenance/decommissioning operations
  - Aero-hydrodynamic interaction

Introduction & motivation

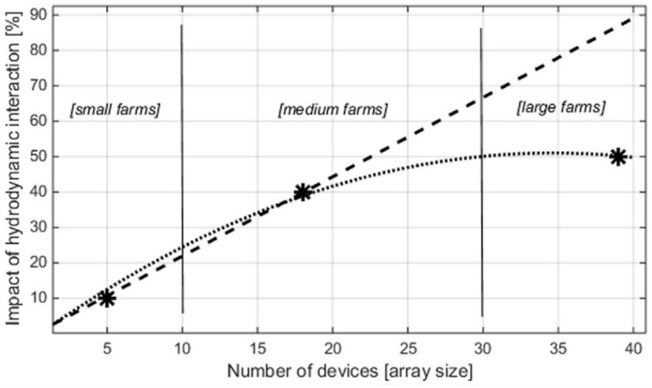
- Hydrodynamic interactions depend on



i.a Device Geometry [1]

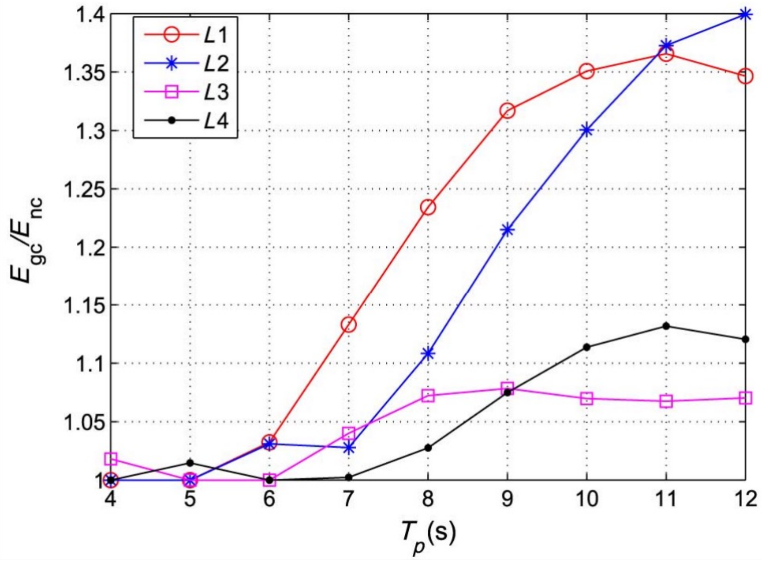
i.b Array size [1]

i.c Inter-device distance [1]



[1] Penalba et al. (2017) A numerical study on the hydrodynamic impact of device slenderness and array size in wave energy farms in realistic wave climates, Ocean Eng., vol. 142, 2017.

ii. Control [2]



[2] Garcia-Rosa et al. (2015). Control-informed optimal array layout for wave farms. IEEE Transactions on Sustainable Energy 6 (2), 575–582.

Introduction & motivation

- WEC array model requirements
  - Capacity to consider hydrodynamic interaction effects
  - Suitable for control implementation
  - Efficient articulation of nonlinear effects
  - Very low computationally cost
- Combination of frequency- and time-domain models

Harmonic Balance (HB)

Methodology – Traditional WEC array model

- Newton's 2nd law for arrays:

$$\mathbf{m}\ddot{z}(t) = \sum_i^N f_i(t), \tag{1}$$

which can be re-written based on Cummins' equation as

Hydrostatic force | PTO force

$$\mathbf{M}\ddot{z}(t) = f_{\text{ex}}(t) - (\mathbf{k}_{\text{h}} + \mathbf{k}_{\text{pto}})z(t) - \mathbf{h}_r \star \dot{z}(t) - \mathbf{h}_{\text{pto}}\dot{z}(t) - \mathbf{h}_{\text{d}}\dot{z}(t)|\dot{z}(t)|, \tag{2}$$

Excitation force | Radiation force | Viscous force

where

- Only one degree of freedom (DoF) is considered: **heave**
- $\mathbf{M} = \mathbf{m} + \boldsymbol{\mu}, \in \mathbb{R}^{n_b \times n_b}$, n_b being the number of devices
- $\star$ denotes convolution
- Parameters in bold font are constant matrix, the rest are time-vectors

Methodology – HB WEC array model

- HB models any systems by means of an **approximate harmonic representation** of system variables
- The system is modelled as a continuous-time state-space as,

$$\dot{x}(t) = f(x(t), f_{\text{ex}}(t)), \qquad (3)$$

where
  - $x(t) = [z(t)^T, \dot{z}(t)^T]^T \in \mathbb{R}^{2n_b \times 1}$
  - f corresponds to the state-transition mapping based on harmonic basis functions (*i.e. sin* & *cos*)

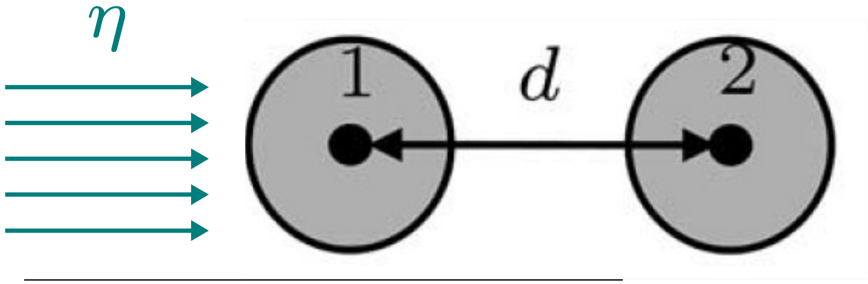
- The steady-state solution can be approximated by a finite-dimensional space

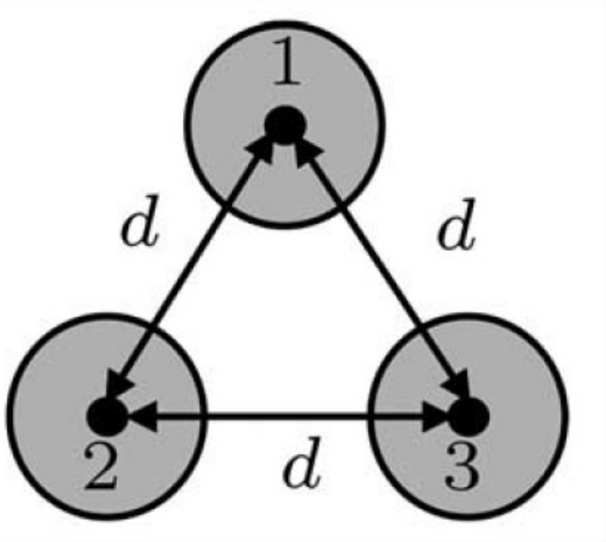
$$\tilde{x}_i(t) = \sum_{p=1}^{N} \alpha_i^p \cos(p\omega t) + \beta_i^p \sin(p\omega t), \qquad (4)$$

computing the auxiliary variables α_i^p and β_i^p via a **Galerkin pseudo-spectral approach**.

Case studies

a. Verification: Generic heaving cylinder
   - $\phi = 10m$
   - $H = 20m$ ($10m$ draft)
   - $m = 7.9\ 10^5 kg$
   - Two array layout configurations:
     i. L1: 2 WECs in line
     ii. L2: 3 WECs in triangle configuration

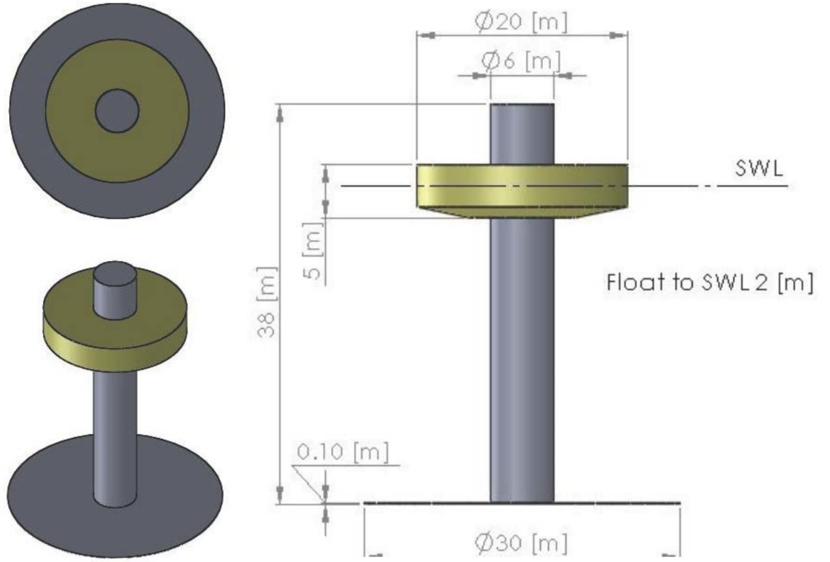


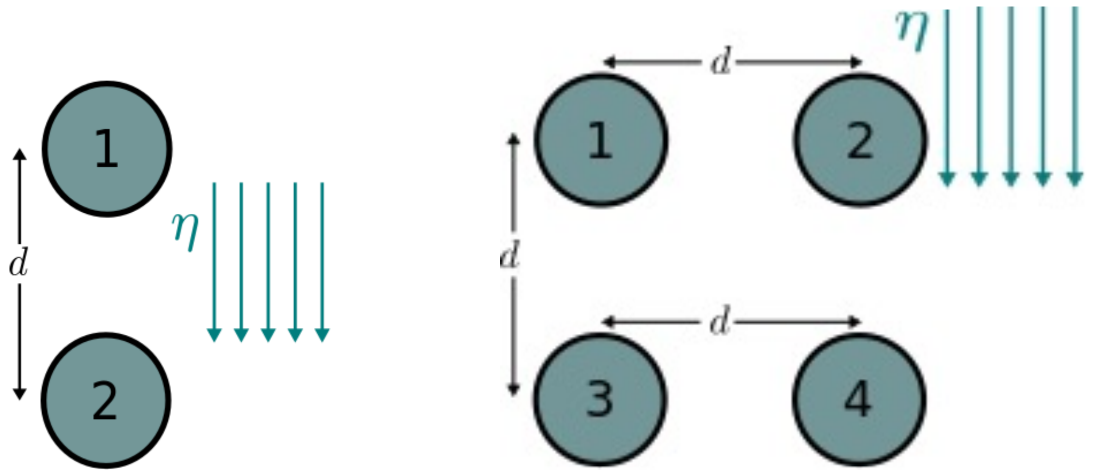


Case studies

b. 2-body RM3 WEC

- $\phi = 20m$ / $6m$ & $30m$
- $H = 5m$ / $38m$ & $0.1m$
- $m = 727.01\ T$ / 878.3 T
- Two array layout configurations:
  i. L4: 2 WECs in line
  ii. L5: 4 devices in two rows





Case studies

- Wave conditions for the different case studies

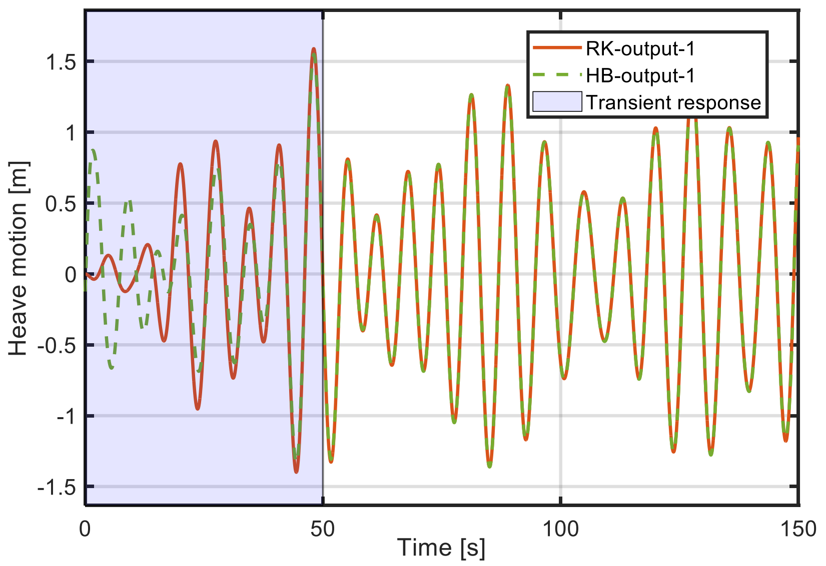
a. Generic heaving cylinder
  - $H_s = 2m$
  - $T_p = 8s$
  - $d \in [2 \;\; 200]\emptyset$
  - Control with optimal PTO coefficients
    - Resistive
    - Reactive

b. 2-body RM3 (PacWave)
  - $H_s = 1.65m$ / $4.33m$ / $1.93m$
  - $T_p =$ $8.81s$ / $13.97s$ / $16.42s$
  - $d_x = 240m$ & $d_y = 276m$
  - Resistive control: non-optimized PTO coefficients

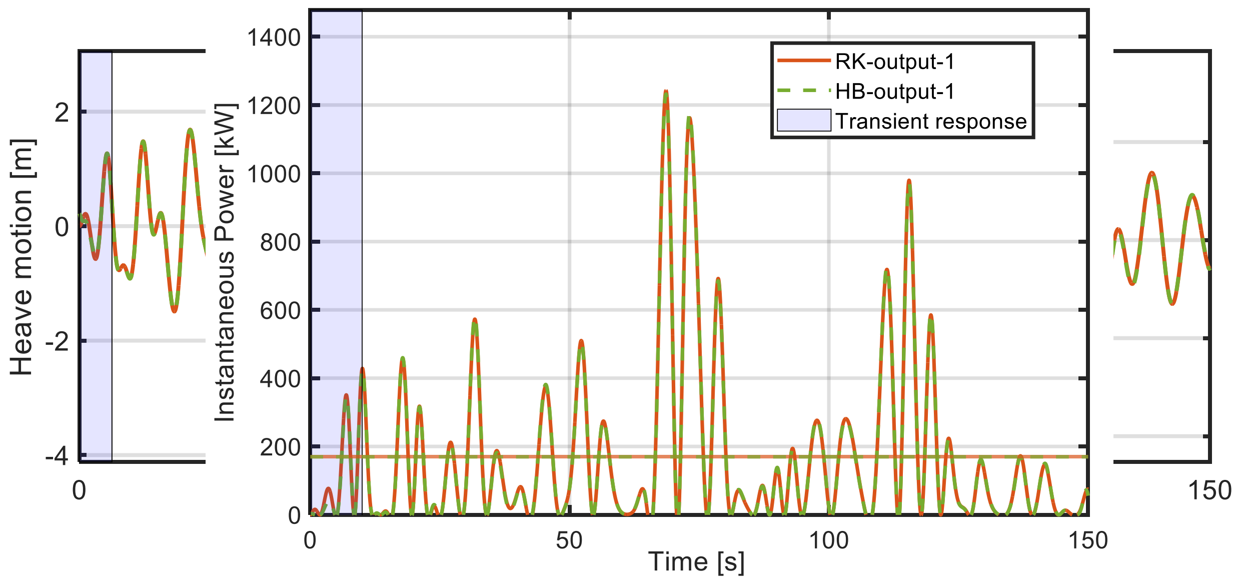
Results – HB model verification

- Single WEC verification: traditional RK vs. HB

i. Generic heaving cylinder



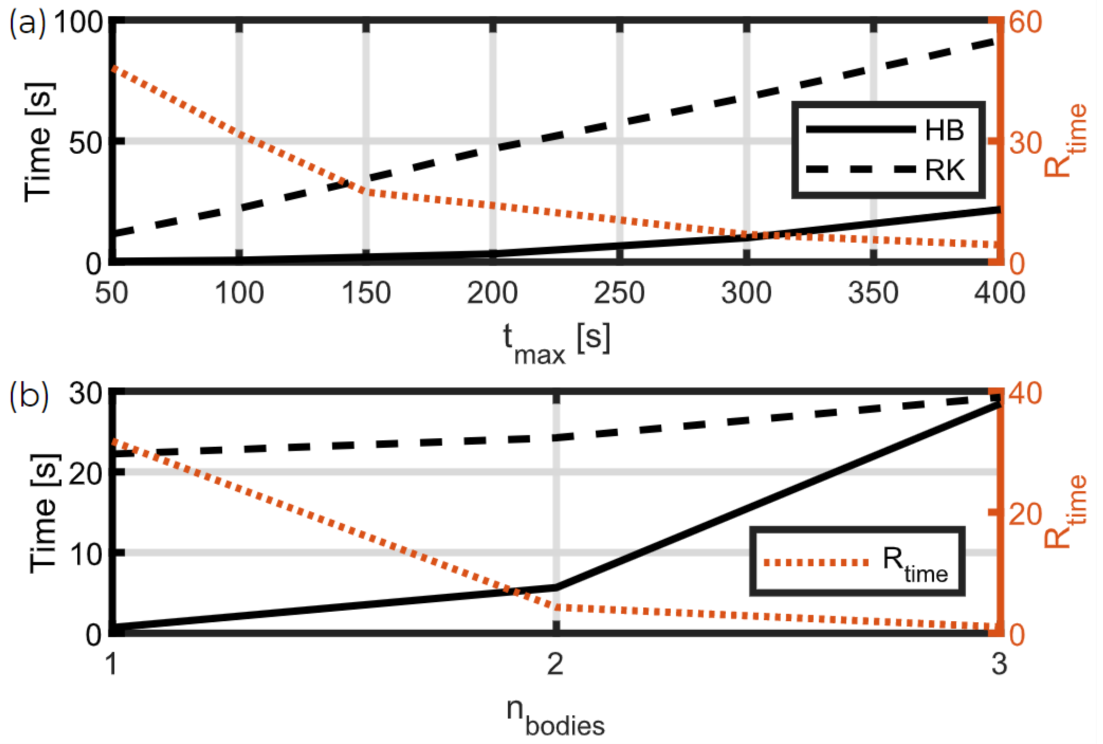
ii. 2-body RM3 WEC



Results – HB model verification

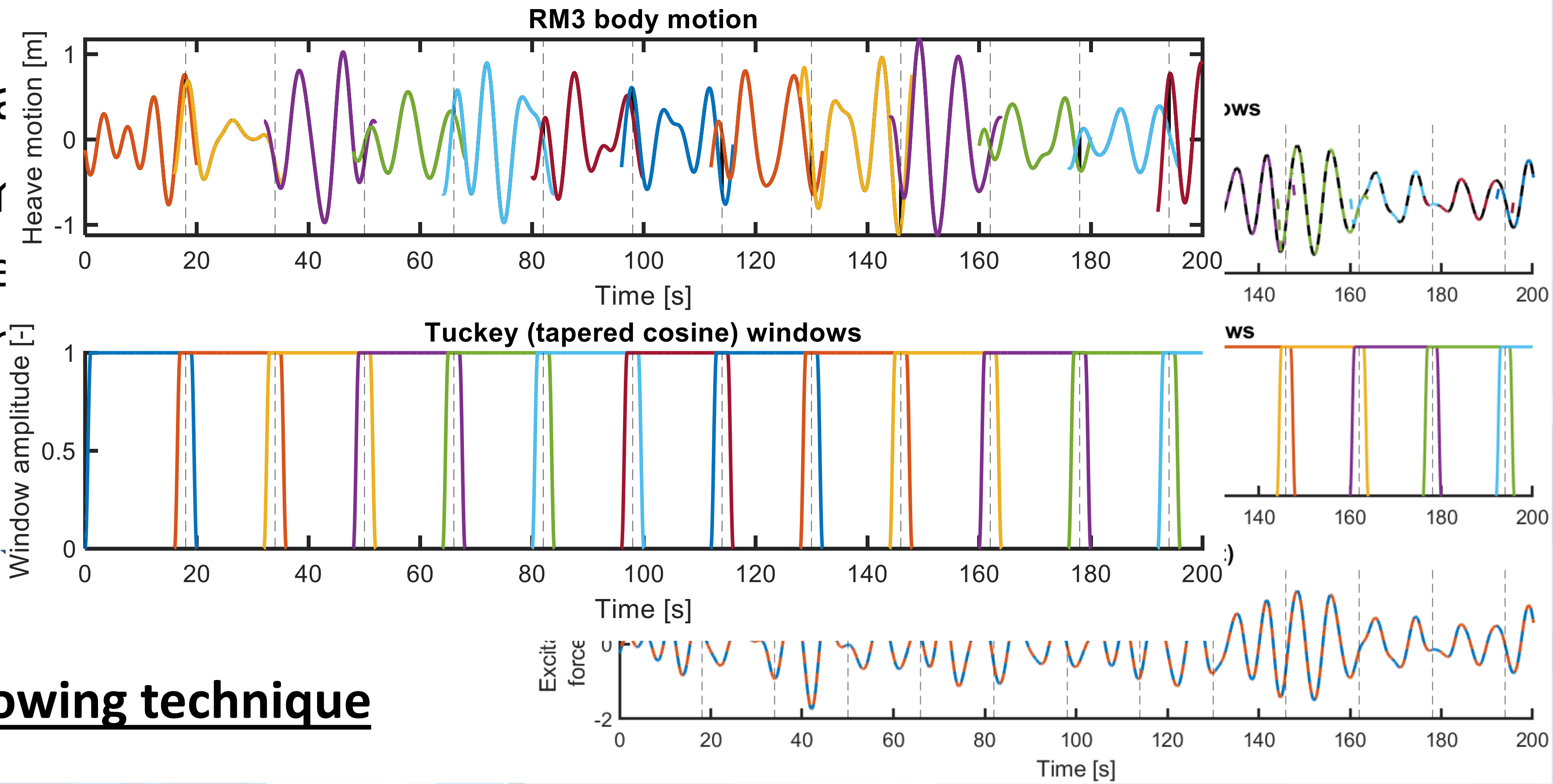
- 2-WEC array verification: traditional RK vs. HB
  - Computational requirements as a function of the
    - duration of the simulation
    - number of devices (100s)

$$\Gamma = \frac{t_{sim_{RK}}}{t_{sim_{HB}}}$$



Results – HB model verification

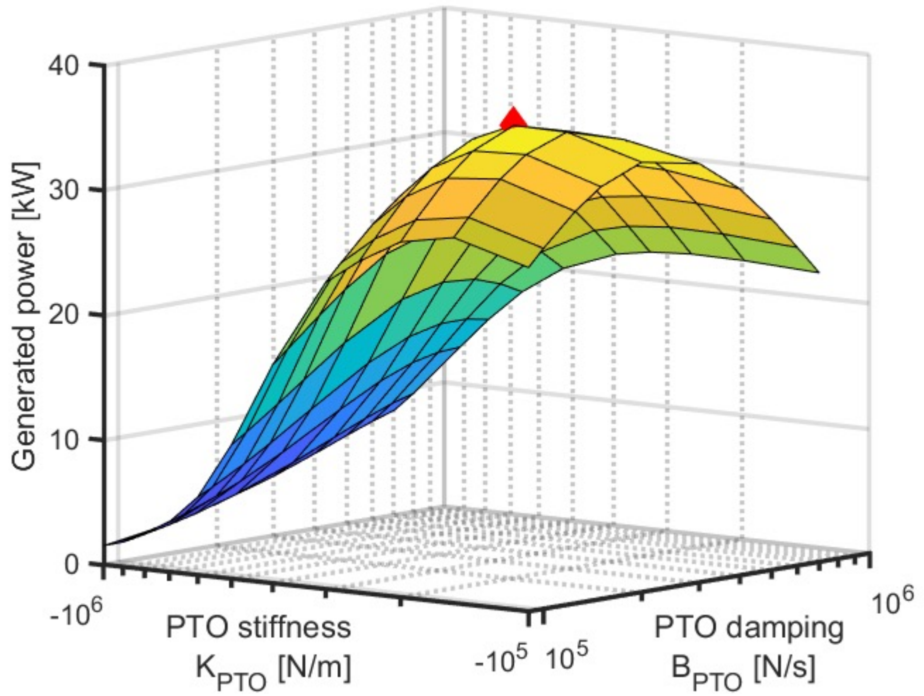
- Larger arra
- Longer sim
  - Non-line
  - Duration

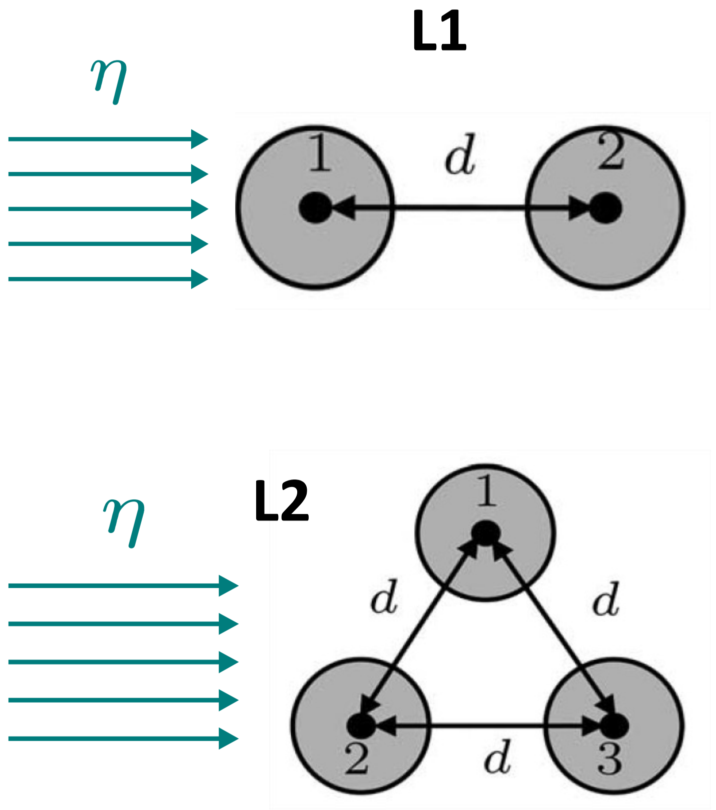


Windowing technique

Results – HB model evaluation

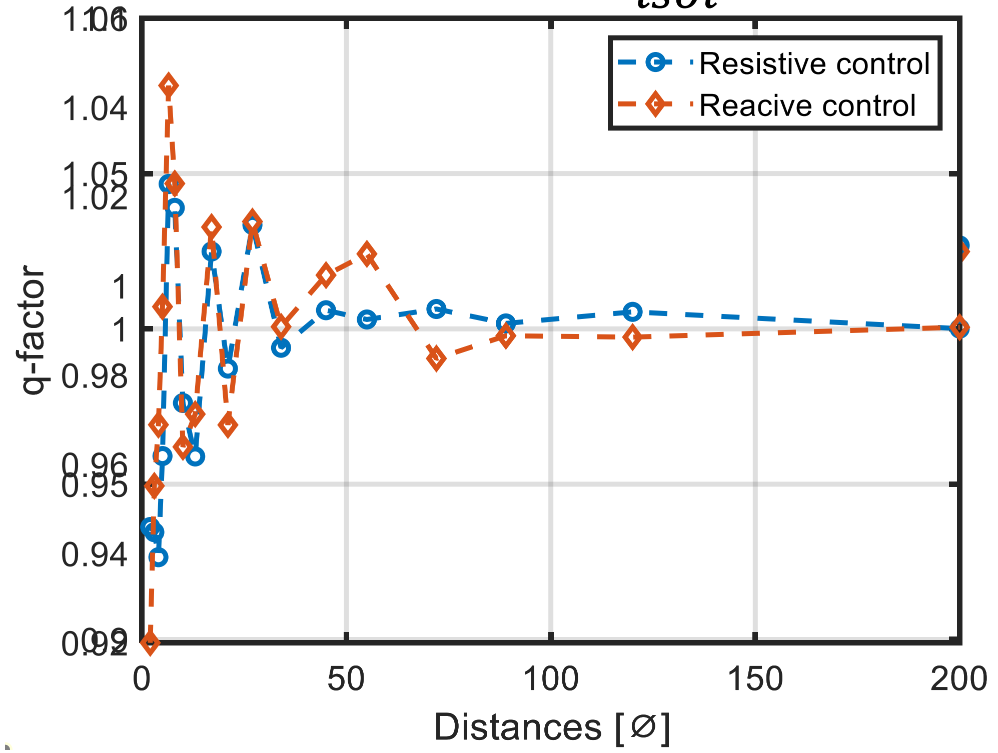
- q-factor: $$q = \frac{P_{array}}{N \cdot P_{isol}}$$

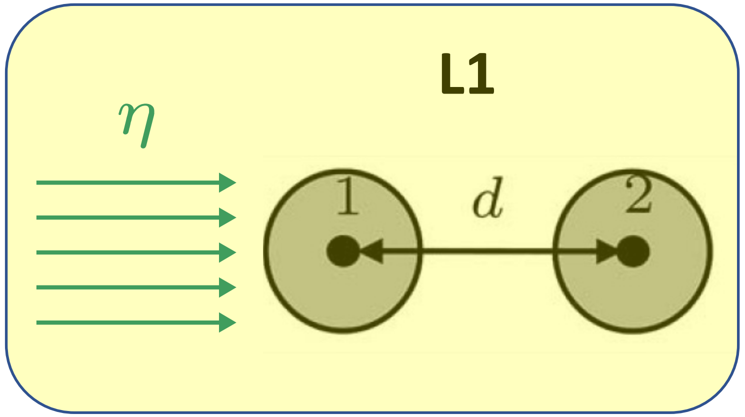


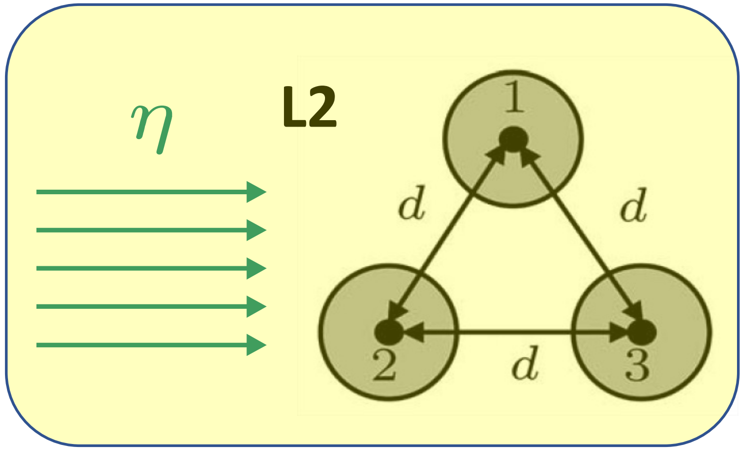


Results – HB model evaluation

- q-factor: $$q = \frac{P_{array}}{N \cdot P_{isol}}$$

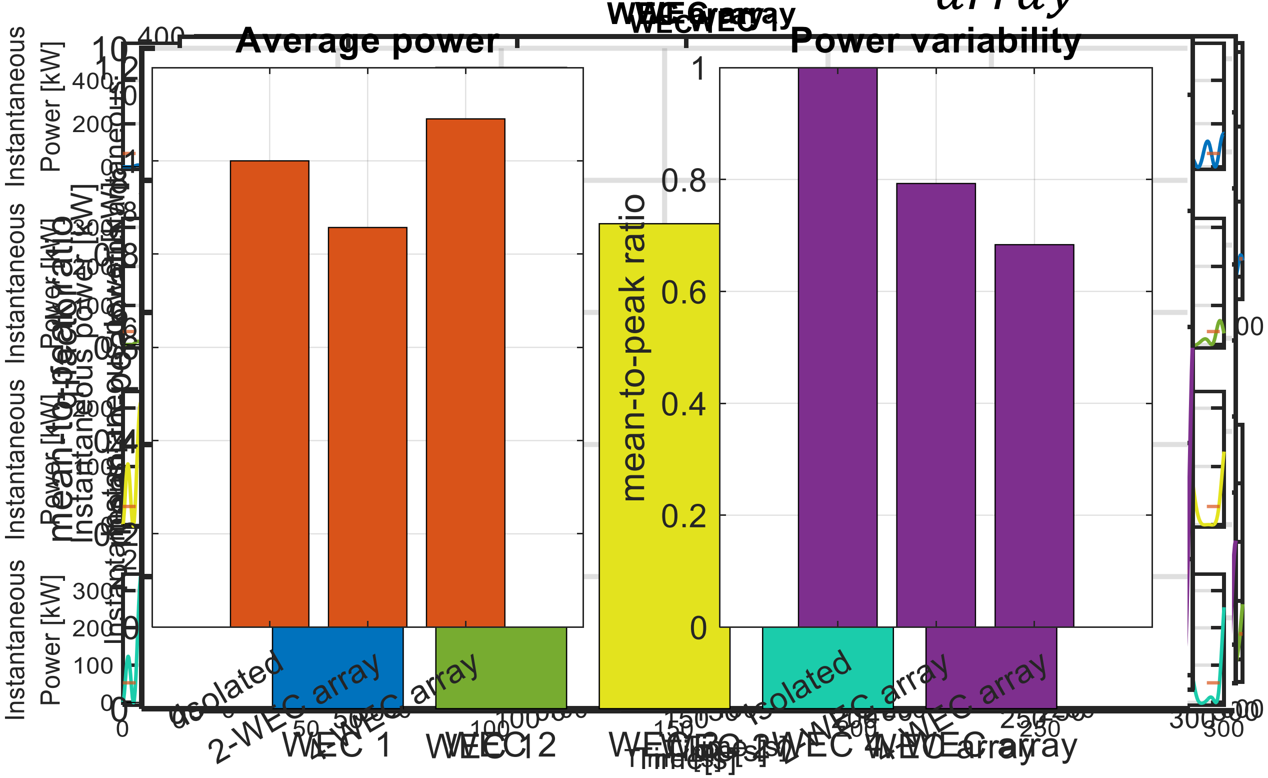


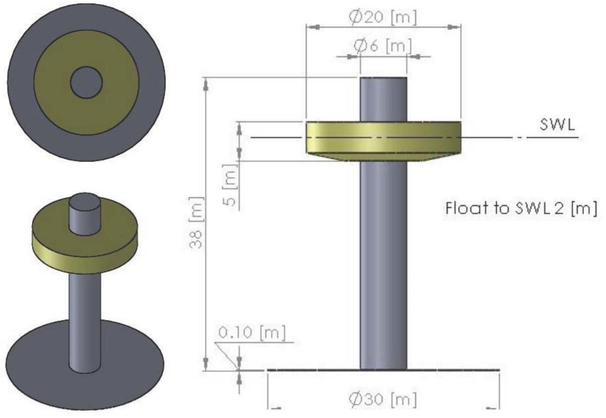


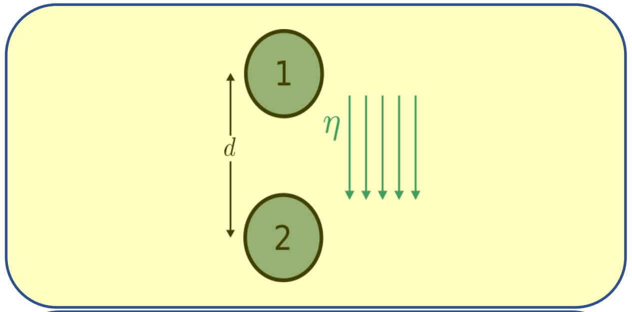


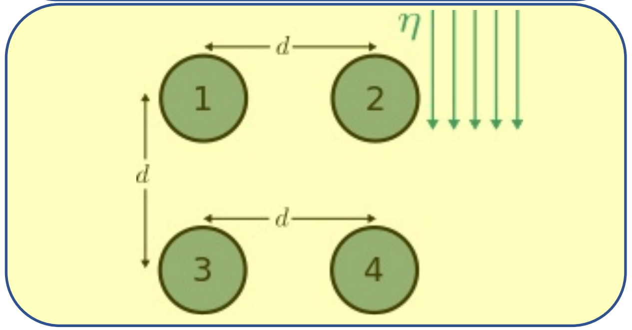
Results – HB model evaluation

- Mean-to-peak: $m2p = \dfrac{max(P_{array}(t))}{\tilde{P}_{array}}$









Conclusions

- A validated HB model for WEC arrays is developed
  - ✓Capacity to articulate nonlinear effects efficiently
  - ✓Controllable mathematical structure
  - ✓Computationally efficient (windowing required!)
  - ✓No transient state / No need for identifying radiation SS

- WEC array layout optimization
  - ✓Improved/conserving final energy generation
  - ✓Reducing energy generation variability

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Thank you for your attention!

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Back up (I) – HB WEC array model

- Traditional WEC array model based on Newton's 2nd law and Cummins' equation

$$\mathbf{M}\ddot{z}(t) = f_{\mathrm{ex}}(t) - \left(\mathbf{k}_{\mathrm{h}} + \mathbf{k}_{\mathrm{pto}}\right)z(t) - \mathbf{h}_r \star \dot{z}(t) - \mathbf{h}_{\mathrm{pto}}\dot{z}(t) - \mathbf{h}_{\mathrm{d}}\dot{z}(t)|\dot{z}(t)|, \qquad \text{(I)}$$

- HB models any systems by means of an **approximate harmonic representation** of system variables as a continuous-time state-space as,

$$\dot{x}(t) = f\left(x(t), f_{\mathrm{ex}}(t)\right), \qquad \text{(II)}$$

where

- $x(t) = [z(t)^T, \dot{z}(t)^T]^T \in \mathbb{R}^{2n_b \times 1}$
- f corresponds to the state-transition mapping based on harmonic basis functions (*i.e. sin* & *cos*)
- $f_{\mathrm{ex}}(t) = \mathrm{F}_{\mathrm{ex}} \cos(\omega t)$

- The steady-state solution approximated by a finite-dimensional space ($H = span(X)$)

$$X = \{\cos(p\omega t), \sin(p\omega t)\}_{p=1}^{N} \qquad \text{(III)}$$

Back up (II) – HB WEC array model

$$X = \{\cos(p\omega t), \sin(p\omega t)\}_{p=1}^{N}. \quad \text{(III)}$$

- This approximation can be given as a linear combination of the elements in X,

$$\tilde{x}_i(t) = \sum_{p=1}^{N} \alpha_i^p \cos(p\omega t) + \beta_i^p \sin(p\omega t), \quad \text{(IV)}$$

with the auxiliary variables,

$$\bar{X}_i = [\alpha_i \beta_i \dots \alpha_i^N \beta_i^N], \text{ and} \quad \text{(V)}$$

$$\Upsilon(t) = [\cos(\omega t)\sin(\omega t) \dots \cos(N\omega t)\sin(N\omega t)]^\top. \quad \text{(VI)}$$

- Thus, the approximation can be written as,

$$\tilde{x}(t) = [\bar{X}_1^\top \dots \bar{X}_n^\top]\Upsilon(t). \quad \text{(VII)}$$

Back up (III) – HB WEC array model

$$\tilde{x}(t) = [\bar{X}_1^\top \dots \bar{X}_n^\top]\Upsilon(t). \quad \text{(VII)}$$

- Similarly, F_{ex} can also be written as a function of $\Upsilon(t)$:

$$\tilde{x}(t) = [F_{\text{ex}}\ 0]\Upsilon(t) = \bar{F}\Upsilon(t), \quad \text{(VIII)}$$

- Now, the residual is given as follows,

$$R(\tilde{x}, f_{ex}) = \dot{x} - f(\tilde{x}, f_{ex}), \quad \text{(IX)}$$

which can also be defined as a function of $\Upsilon(t)$:

$$R(\bar{X}, F, \Upsilon) = \bar{X}\dot{\Upsilon} - f(\bar{X}\Upsilon, \bar{F}\Upsilon). \quad \text{(X)}$$

computing the auxiliary variables α_i^p and β_i^p via a **Galerkin pseudo-spectral approach** that forces the projection of the **residual function to be 0**.