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Invited paper

Control co-design for wave energy systems

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ABSTRACT

Control co-design (CCD) has become a powerful paradigm in the total optimisation of the dynamics of a system against a defined performance goal. Given the impact of control on wave energy systems, including significant effects on motion and energy capture performance, it is imperative that controller and system are optimised together to maximise (economic) system performance. There are a number of design aspects that can usefully be addressed by CCD in wave energy systems, including device geometry, mooring configuration, power take-off (PTO) parameters, and array layout. A number of these design aspects present significant computational challenges and, as a result, wave energy CCD studies typically address a single design aspect at a time. This paper examines the useful roles that CCD can play in wave energy systems, important criteria to consider, and the potential interaction between individual wave energy CCD aspects.

1. Introduction

With increasing concern about the rate of climate change, the need to provide resilient and carbon-neutral energy sources has never been greater. While wind and solar have achieved an encouraging level of penetration, renewable energy grid integration studies show that curtailment levels grow as the penetration of wind and solar energy generation increases (Bird et al., 2016), while times of high demand are not necessarily accompanied by an abundance of wind and solar power availability. However, the significant growth in wind and solar can be attributed not only to their green credentials, but also to their relative cost (per MWh) compared to other renewable, and non-renewable sources. While one solution to demand/supply matching, in addition to demand-side management, is the provision of significant storage capacity on grids, this is expensive (using current technology); a potential alternative lies in the diversification of renewable energy sources.

To that end, wave energy (Guo and Ringwood, 2021b) presents a virtually untapped energy resource, estimated at between 1–20 TW in global instantaneous power (Reguero et al., 2015). It has been shown to complement wind (Fusco et al., 2010) and solar (Pennock et al., 2022) resources, but currently is economically uncompetitive compared with other renewable energy resources (Ringwood, 2022). Note that economic competitiveness for renewable energy systems is normally articulated in terms of €/kWh, i.e. levelised cost of energy (LCoE), though system *efficiency* (e.g. the ratio of delivered energy to the raw energy available) is often employed by technologists as a figure of merit. There are situations where there is direct correspondence between these two metrics, but this is not always the case, and it is important to be mindful of the potential correspondence/disparity in the design, or control co-design, of specific wave energy systems.

While evidence of *concurrent* design of physical systems and control (and other aspects) has been in existence for over a century, beginning with the automatic wind turbine of Brush, and the Wright brothers' design of their Wright Flyer (Garcia-Sanz, 2019), control co-design (CCD) as a moniker was introduced only relatively recently during the 1980s. CCD involves the *simultaneous* design of physical system and controller, both informing the design for the other part towards a common objective function (Allison and Herber, 2014; Allison, 2014). Some further perspectives on CCD are discussed in Section 2.

For wave energy systems, given the strong impact of control on the device performance (in terms of converted energy), and the typically significant effect of control on the device motion (Ringwood et al., 2023b), it is unsurprising that CCD is seen as an important facet of wave energy system design, in the drive to improve the economic competitiveness of wave energy. Since one of the first examples of WEC CCD in 2010 (Gilloteaux and Ringwood, 2010), a wide variety of wave energy converter (WEC) CCD studies have been reported. In particular, it has been shown that not only the controller 'tuning', but also the nature of the control philosophy, can have a significant effect on optimal WEC geometry (Garcia-Rosa and Ringwood, 2015) and optimal array layout (Garcia-Rosa et al., 2015b). As will be seen in Sections 8.1–8.4, CCD has already been applied to many aspects of wave energy technology, including wave energy converter (WEC) geometry (Section 8.1), WEC array layout (Section 8.2), power take-off (PTO) parameters (Section 8.3), and mooring configuration (Section 8.4). This

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paper aims to take a generic view of the particular challenges in wave energy CCD, to collect the various studies addressing wave energy CCD, and to examine the challenges that lie ahead for wave energy CCD. In particular, it is difficult to truly consider control *co-design* for wave energy systems unless most/all aspects of the system design are considered simultaneously, and their inter-dependencies fully considered. Section 2 specifically addresses these considerations.

Other important aspects of wave energy CCD are introduced, including the choice of mathematical models (Section 4) for wave energy CCD (especially given the trade-off between model fidelity and computational efficacy) and the choice of appropriate control methods (Section 5), again considering performance/computational trade-offs. Needless to say, in CCD, as in any other design paradigm, it is vital to choose an appropriate performance function, which is typically a trade-off between the ultimate goal, and one which is computationally tractable (see Section 3).

Finally, while this study on wave energy CCD does not consider the wave resource/climate to be a part of co-design, in that the choice of site is assumed fixed, the chosen site nevertheless is an important part of the design environment, and significantly affects the optimised design. In that sense, the site characteristics are a given, though may evolve with time due, for example, to climate change (Ulazia et al., 2019). However, site selection could also be made part of the WEC CCD problem, and this is made explicit, for example by Azad et al. (2024).

2. Perspectives on CCD

While the 'control co-design' moniker has only been in use for a little over three decades, other labels had previously been used to describe some sense of integrated design, with the terms 'inter-disciplinary', 'multi-disciplinary', and 'concurrent' alternatively used to describe some level of interaction between various system components, including control. Indeed, the 'mechatronics' moniker also involves the infusion of electronics (which typically involves control components) within mechanical systems, and current automobiles owe much of their functionality and performance to the many control loops that work seamlessly in the background.

Ironically, it was possibly not until control science became an established separate discipline that the activities of the application specialist and control engineer became separate. Historically, there are many examples of control integration from the earliest design stages, including the flyball governor, which was an integral (mechanical) part of the drivetrain of early windmills. Equally, some early examples highlighted by Garcia-Sanz (2019) show the fundamental consideration of control at the earliest design stages, including in the unstable aircraft design of the Wright brothers (the pilot was the 'controller'). A more recent analogue is the design of the F-16 fighter aircraft, which is also open-loop unstable (but now impossible to fly by pilot alone, considering the speed of the dynamics), with an electronic controller used for stabilisation, resulting in improved overall performance in agility and economy.

Arguably, the advent of microprocessor technology and digital electronics has also been instrumental in separating the activities of physical system and control design, with two distinct domains appearing, both requiring specialist expertise. Previously, many systems had mechanical (e.g. flyball) or pneumatic (flapper-nozzle) controllers, which were still within the domain of mechanical engineers. Furthermore, since software implementation provided immense flexibility, there was little limit to the complexity of control designs which could be developed, becoming, as a result, quite a specialist activity. It was therefore not surprising that sequential design, as illustrated in Fig. 1, became the norm.

In addition to the iterative CCD design possibility shown in Fig. 1, Allison and Herber (2014) also offer 2 other possible CCD philosophies, as shown in Fig. 2, which involve much closer integration between plant and control design.

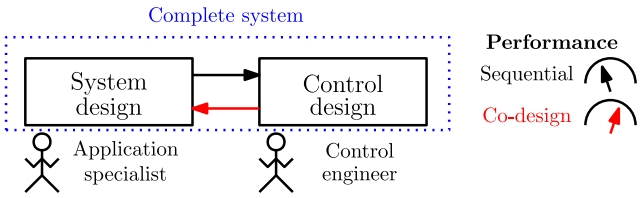

Fig. 1. Sequential system design (black arrow), with separate activities of application specialists and control engineers. The red arrow indicates an iterative co-design approach. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

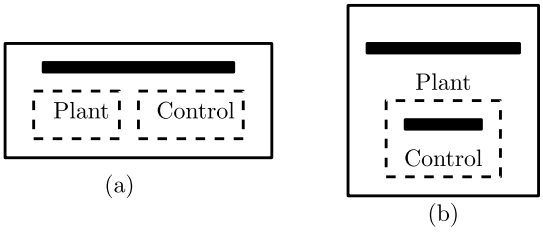

Fig. 2. Additional control co-design philosophies, featuring integrated physical (plant) and control system design: (a) simultaneous design, and (b) nested design.
Source: Adapted from Allison and Herber (2014).

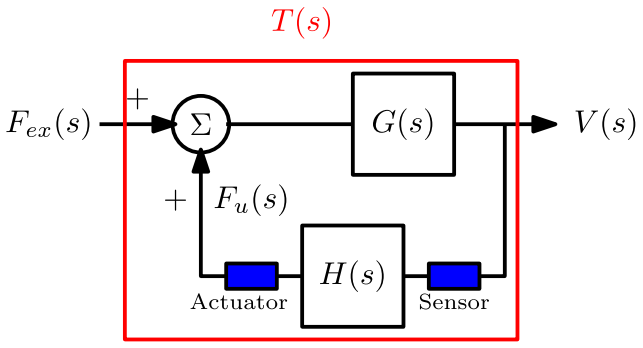

Fig. 3. Closed-loop system, comprising the original open-loop system $G(s)$ and controller $H(s)$.

Given the flexibility offered by computer control systems, and their typically modest cost compared to the capital cost of the physical system, it is tempting to relegate complex design/performance aspects to control design, with the assumption that this will be a lower cost route ("It's only electronics/software"). However, a simple counterexample, quite in the spirit of integrated CCD (specifically simultaneous design), as shown in Fig. 2, illustrates that this may not always be the case. Consider a simple physical mass(M)-spring(K)-damper(B) system, which could represent a simplistic model for a WEC:

$$M\dot{v}(t) + Bv(t) + Kx(t) = f_u(t) + f_{ex}(t) \tag{1}$$

where f_u is the control force and f_{ex} is an external forcing function, e.g. the wave excitation force, with x and v denoting displacement, and velocity, respectively ($v = \dot{x}$). Now parameterise f_u as:

$$f_u(t) = M_c\dot{v}(t) + B_c v(t) + K_c x(t) \tag{2}$$

so that the closed loop system of Fig. 3 results, where:

$$G(s) = \frac{X(s)}{F_e(s) + F_u(s)} = \frac{1}{Ms^2 + Bs + K}, \tag{3}$$

$$H(s) = \frac{F_u(s)}{X(s)} = M_c s^2 + B_c s + K_c, \tag{4}$$

and

$$T(s) = \frac{G(s)}{1 - GH(s)} = \frac{1}{(M - M_c)s^2 + (B - B_c)s + (K - K_c)}. \quad (5)$$

In the spirit of model-reference control, where the closed loop poles of $T(s)$ can be arbitrarily assigned to desired locations, it is clear that this assignment can be alternatively achieved by manipulation of the physical system parameters (M, B, and K) or the controller parameters (M_c, B_c, and K_c).[1] For example, if it is desired that the inertia of the system be increased, this can be achieved by an increase in the physical inertia, represented by M, or the 'virtual' inertia, M_c. At first glace, it seems more cost effective (and always offers more flexibility, for example for different sea states) to manipulate M_c (only software!), but this choice neglects the need for the actuator (see Fig. 3) to generate sufficient force to implement the virtual inertia ($M_c\ddot{x}$). In fact, it may be more cost effective to add physical mass to the system (e.g. concrete) than to require an actuator (e.g. electrical motor) with a greater force capacity. CCD, with a suitable performance objective (which ideally includes some aspect of cost, either as a criterion, or as a constraint) can consider *all* possibilities in an objective way. In relation to this simple example, such considerations are examined, to some extent, in Peña-Sanchez et al. (2022), Ermakov et al. (2024b) within the wave energy application area. However, in general, it can be difficult to implement 'integrated CCD design' (as in Fig. 2) due to the relative opaqueness of WEC models, and the hydrodynamic component in particular.

In essence, control co-design requires two essential ingredients for effective operation:

(a) A common technical language that bridges the application discipline(s) and control science, and
(b) A healthy level of trust between application specialists and control engineers.

For (b) above, the sometimes counter-intuitive suggestions that a control perspective may bring to physical system design (e.g. unstable open-loop design) can be quite challenging. In wave energy system design, given the range of potential disciplines involved (hydrodynamic, mechanical, hydraulic, electrical, electronic) and the number of interacting subsystems (WEC geometry, WEC array layout, PTO, moorings, and control), the challenges in both (a) and (b) above are significantly enhanced.

3. Wave energy system performance objectives

For renewable energy, the raw resource is free, while the cost of conversion to a useful form is not. This has led to the development of LCoE as a potential figure of merit, which articulates (in €/kWh or €/MWh) the cost of provision of useful energy. LCoE, however, does not guarantee that electricity can be provided at any particular point in time, or at the LCoE price. Rather, LCoE is a time averaged measure, typically averaged over the lifetime of a renewable energy device (20–25 years):

$$LCoE = \frac{CapEx + OpEx}{EP} \quad (6)$$

where $CapEx$ and $OpEx$ are capital and operational expenditure, respectively, with EP total energy produced over the lifetime of a device. With the significant difficulty, and uncertainty, in calculating $OpEx$ (e.g. see Teillant et al., 2012), $OpEx$ is often expressed as a percentage of $CapEx$ (see, for example, De Andres et al., 2016), which is more easily articulated, though the suggested fractions vary widely, and are location specific (O'Connor et al., 2013). Some suggested *annual* values are 1%, 3%, 5% and 10% of CapEx (O'Connor et al., 2013). A range of techno-economic metrics are considered for the design of a 20 MW wave farm using CorPower device technology by De Andres et al. (2016), while Castro-Santos et al. (2024) consider the impact of the energy calculation mechanism on economic metrics.

Table 1
Indicative relative sensitivity of LCoE terms to CCD optimisation components (5 = sensitive, 0 = insensitive).

Component	EP	$CapEx$	$OpEx$
WEC geometry	4	4	1
Array layout	2*	4**	1***
PTO parameters	3	3	1
Moorings	2	5	2

* Hydrodynamic interaction not the most important aspect.
** Main issue is cost of moorings and electrical cable sizes.
*** Ability to access WECs for maintenance.

Most wave energy optimisation studies employ a purely technical, rather than economic, metric, but often with a surrogate measure of costs. For example, a typical performance objective may be:

$$Maximise \;\; J(\theta) = \frac{EP(\theta)}{CP(\theta)}, \quad (7)$$

where θ is a parameter vector defining the physical system design (e.g. geometry) and CP is a surrogate 'cost' metric, for example mass, volume, surface area, etc. A range of such metrics are considered in Garcia-Teruel et al. (2021), Guo and Ringwood (2021a), while a range of economic metrics, such as LCoE, net present value (NPV) and discounted cash flow (DCF) are articulated in Teillant et al. (2012). However, some care needs to be taken. Specifically:

- The performance function in (7) is sensitive to location, particularly the range of sea states the wave energy system will be subject to, normally articulated by a scatter plot (Barstow et al., 2008),
- If no cost parameter is included, then the tendency is typically towards a large device, and
- If no constraints on system variables are considered, and linear hydrodynamic models are employed, excessive (unrealistic) device motion may result (see also Section 5).

The above three considerations especially apply to CCD, where the controller is typically enhancing device motion, as well as improving power production performance. In this regard, it may also be important to consider the effect of control action on $OpEx$. In general, it is assumed that $OpEx$ is insensitive to control action and the focus is purely on the sensitivity of EP to control action. However, given the tendency of WEC controllers to enhance device motion (Ringwood et al., 2023b), there is likely to be an upward adjustment in $OpEx$ as a result of control actions.[2] Table 1 attempts to give an idea of the sensitivity of various LCoE terms in relation to the optimisation of various aspects of WEC/array design. It is anticipated that PTO parameter optimisation (red highlighting) may most dramatically affect CCD, since it determines the physical constraints that the controller must work within.

Two final aspects in the choice of an appropriate performance function must be considered. In most wave energy (and many other) CCD applications reported to date, there are significant computational challenges in relation to the nature of the disparate terms in the performance function, which may be difficult to combine. For example, a typical term is captured energy (in kW, to maximise), while other terms related to cost (in €, to minimise) may be included. Combining such terms requires inverting, or negating, one or other so that a common goal (max or min) may be targeted. However, inverting leads to a nonlinear (reciprocal) function, which may be difficult to optimise.

[1] Note that, in the case of wave energy, $H(s)$ is typically chosen to be the complex conjugate of $G(s)$ to maximise energy capture from the waves.

[2] Note that some focus on *health-aware* WEC control is now emerging (Ziaei et al., 2024).

In particular, the inclusion of non-convex terms in the performance function may confound the search for a global optimum, even with considerable computational resources.

One solution to disparate performance function terms, or even when a suitable scaling factor between terms is difficult to find, is to utilise multi-criteria optimisation, resulting in a Pareto front (Giagkiozis and Fleming, 2014), which essentially informs the designer of all possible 'optimal' (non-dominated) answers within the search space. The designer then selects a suitable optimal answer from the set available.

4. Models for WEC control co-design

Ideally, given a performance objective, a specific control design technique, and a particular set of co-design parameters (e.g. hull geometry parameters) it would be possible to simply optimise the objective functional with respect to the design parameters as:

$$\frac{\partial}{\partial p} J(p,c) = 0 \quad , \quad p_{opt} \rightarrow Max\ J \tag{8}$$

where p are the set of (say hull) design parameters and c is the 'set' of controller parameters. There are two issues with (8):

(a) J is not only a function of p, but also a function of c, clearly illustrating the control co-design imperative, and
(b) In general, it is impossible to determine an analytic function J in terms of c.

Focussing on (b) just for the moment ((a) is dealt with in Section 5), the main difficulty here is the WEC hydrodynamic model. While some attempts have been made to determine analytic models for simple geometric WEC shapes (Havelock, 1955; Yeung, 1981; Chau and Yeung, 2012), the relationships between the geometric and associated hydrodynamic parameters rapidly become complex, typically involving series expansions to evaluate pressure integrals. Given the lack of the analytic appearance of the geometric parameters p in J, in (8), it is impossible to calculate analytic partial derivatives. Of course, numerical derivatives can be obtained, but this amounts to an effective iterative procedure typically adopted in control co-design, shown in Fig. 4. Ultimately, therefore, the value of hydrodynamic (or other, e.g. PTO) models based on physics equations, or even having physical interpretations, are of limited value in a co-design context. This opens up WEC control co-design to a variety of modelling approaches, including:

- First principles (physics-based) models,
- Black-box models (including machine learning models) from high-fidelity models, or experimental data, and/or
- Grey-box models,

where the level of physical interpretation decreases in the movement from physics-based (white-box) models to black-box. Grey-box models (various shades of grey in Ljung, 2010) provide some level of physical interpretation. They may be some value in retaining some physical interpretation (e.g. passivity, stability) allowing for broad checks during optimisation iterations. While there are a variety of considerations in choosing a WEC modelling strategy for control co-design, paramount is the need for computational efficacy, given the inevitable iterative nature of the co-design optimisation process. To that end, various forms of *s*urrogate models can be employed as part of a model reduction process. This is dealt with in more detail in Section 6.

Recall that, depending on the scope of the control co-design exercise, more (or less) components of the complete wave-to-wire chain (illustrated in Fig. 5) can be considered. Several comments here are noteworthy:

- For direct-drive PTO systems, the hydraulic section is absent.
- Ideally, even though there may be a co-design focus on particular physical system components (WEC, PTO, etc.), it maybe necessary to model all components, if there are inter-dependencies, in terms of optimality of various parts.

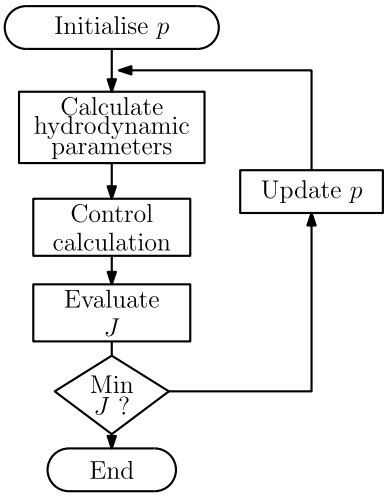

Fig. 4. Trial and error approach to model parameter optimisation.

- Some balance between the fidelity of each model component should be aspired to. However, if one particular component is the focus of a co-design study, there may be value in affording that component a greater level of model fidelity.
- Not completely unrelated to the issue of fidelity balance, but not completed correlated, is the issue of the balance between the speed of dynamics of represented model components. For example, the power converter can be modelled in terms of inter-cycle, or intra-cycle (PWM cycle) behaviour (Vu et al., 2013), but this of little use if the WEC dynamics are included, since the converter time constants are typically of the order of μs/ms, while the dominant (full scale) WEC body time constant is typically in the range 1–10 s. Equally, the dynamics of the electrical side of the generator are generally considerably faster than mechanical/hydraulic system components. Clearly, the slower dynamics dominate the overall system response, so representing dynamics, which an order of magnitude faster than the dominant dynamics, is probably a waste of computational effort. As associated issue is that excessively fast dynamics will require a fast sampling rate (assuming time-domain simulation), with the requirement that the full system must be resolved with the shortest sampling period (with potentially poor numerical conditioning, and excessive computation) or a multi-rate sampling approach adopted (added complexity).

Given their predominance in wave energy control co-design studies, the issues of hydrodynamic modelling and the choice of time- or frequency-domain system representations are given particular treatment in the following subsections.

4.1. Hydrodynamic modelling

Specification of a suitable hydrodynamic model is probably the most challenging component to represent in the power train of Fig. 5, in achieving a suitable compromise in model fidelity and computational simplicity. Fundamentally, the defining equations for hydrodynamic interaction between waves and floating bodies are the Navier–Stokes equations (Fefferman, 2006), in time t and space x:

$$\begin{aligned} &\frac{\partial v_i}{\partial t} + \sum_{j=1}^{3} v_j \frac{\partial v_i}{\partial x_j} = \nu \Delta v_i - \frac{\partial p}{\partial x_i} + f_i(x,t), \\ &\mathrm{div}\{v\} = \sum_{i=1}^{3} \frac{\partial v_i}{\partial x_i} = 0, \end{aligned} \tag{9}$$

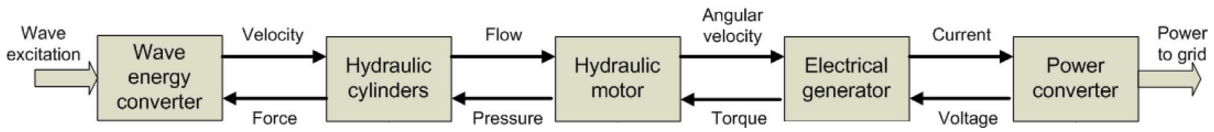

Fig. 5. Full set of potential model components.

assuming an incompressible fluid, where v is velocity, p is pressure, and ν is kinematic viscosity. With no closed-form solution, some form of numerical solution is required, either using a structured mesh, as is typical with computational fluid dynamic (CFD) approaches (Windt et al., 2018) or particle-based methods, such as smoothed particle hydrodynamics (SPH) (Ropero-Giralda et al., 2020). With typical computation times of ~1000 s per second of simulation time, CFD/SPH techniques are largely unsuited to WEC system control co-design, but may have some application in surrogate modelling (see Section 6).

At the other end of the computational scale is the venerated Cummins' equation (Cummins, 1962) which easily the most popular hydrodynamic WEC model, and endorsed by Coe et al. (2021) for co-design purposes, given as:

$$(M + m_\infty)\dot{v} + \int_0^\infty h_r(\tau)v(t-\tau)d\tau + K_r z(t) = f_{ex}(t) + u(t) \tag{10}$$

where v is the WEC velocity, z displacement, and M is the WEC mass. The hydrodynamic parameters m_∞ (infinite-frequency asymptote of added mass), h_r (radiation damping impulse response kernel), K_r (the restoring force constant) and h_{ex} (the excitation force impulse response kernel) are all determined using numerical techniques (as described above) from a variety of numerical solvers, such as WAMIT, Nemoh, or Ansys Aqwa, with:

$$f_{ex}(t) = \int_{-\infty}^{t} h_{ex}(\tau)\eta(t-\tau)d\tau \tag{11}$$

where $\eta(t)$ is the incident free surface elevation variations with time. $u(t)$, in (10), describes the PTO control signal, which may be parameterised, or be left in non-parametric form (see Section 5). (10) and (11) describe the simplest realistic hydrodynamic model for WEC control co-design, though not ideal computationally, due to the presence of convolution terms in both equations. These convolutions terms are usually replaced by a finite-order ordinary differential equation (normally in state-space form) using one of a variety of techniques (Peña-Sanchez et al., 2019). This replacement significantly speeds up computation.

It should be noted that Cummins' equation, in (10), has several restrictive assumptions, not least the assumption of small deviations from equilibrium (facilitating linearisation) and immersion in an inviscid fluid. The issue of small perturbations is rather severe, especially in a control co-design context, since the particular action of energy-maximising control is typically to exaggerate WEC motion (Windt et al., 2021). One possible solution to retain a linear model, which is valid under more substantial motion, is to use a 'representative linear model', but such models need to be determined from higher-fidelity numerical models (Davidson et al., 2015; Farajvand et al., 2023), or tank test data (Bacelli et al., 2017; Giorgi et al., 2019), and are typically amplitude dependent. In terms of the inviscid fluid assumption, a quadratic Morison (Morison et al., 1950) term may be included in Cummins' equation, which could be linearised, if desired, using Lorenz linearisation (Terra et al., 2005).

The principal advantage of utilising linear hydrodynamic models is their potential use in a frequency-domain setting, allowing a 'one-shot' calculation of device performance using a superposition of sinusoids for panchromatic seas. However, between linear boundary-element methods (BEMs) and CFD/SPH, there are a considerable host of other hydrodynamic modelling approaches, including nonlinear BEMs, which offer higher fidelity (Papillon et al., 2020; Visbech et al., 2024), but typically require time-domain solution, using iterative simulation. Some computationally compact nonlinear representations, which retain a frequency-domain flavour, include spectral (Folley and Whittaker, 2010) and harmonic balance (Mérigaud and Ringwood, 2017) models.

One interesting possibility, typically associated with hydrodynamic/aerodynamic models is to use non-dimensional analysis to allow scaling of the device/parameters, having calculated a single set of hydrodynamic parameters. However, this approach is restricted to the case where *all* the device geometric parameters are subject to consistent scaling. An example of such an approach is the study by Gelós et al. (2022).

A final consideration, related to hydrodynamic modelling, is the sensitivity of the controlled WEC system to errors in the hydrodynamic model (usually the principal source of WEC modelling error). It can be noted (Ringwood et al., 2019) that different WEC control structures can be particularly sensitive to various categories of hydrodynamic modelling error (e.g. buoyancy, radiation damping, mass or added mass), which may be an important issue for co-design exercises. However, it is not ultimately important that the correct power production figure is achieved, but rather that the optimum of the design parameter(s) is relatively insensitive to modelling error (more on this in Section 6).

4.2. Other modelling considerations

4.2.1. WEC electrical components

Various mathematical models for WEC electrical subsystems are available, including those in Ozkop and Altas (2017), Penalba and Ringwood (2016), Penalba et al. (2017), Mueller (2002) and the references therein. As discussed earlier in this section, a considerable amount of detail can be entered into on the electrical side, typically down to timescales of μs. Considering that the dominant (slow) dynamics in any WEC wave-to-wire model are due to the mechanics/hydrodynamics, the imperative is to model any inefficiency in the PTO/electrical side, so that a true assessment of power capture, etc may be achieved. In particular, for control co-design studies, it is vital that PTO systems with non-ideal efficiency (i.e. all PTOs!) are correctly represented, since it has been shown that inaccurate representation of PTO efficiency is likely to lead to incorrect controller design, with consequent misrepresentation of power production (Bacelli et al., 2015; Genest et al., 2014; Sergiienko et al., 2022). However, the most appropriate representation of non-ideal PTO characteristics for co-design is not entirely clear, though, a comparison of the models in Lin et al. (2024) may provide some inspiration. Nevertheless, the most appropriate PTO loss model for control co-design remains a relatively open question, though the quadratic loss model in Lin et al. (2024) appears to have a good balance of accuracy and computational efficiency.

4.2.2. Wave climate

While WEC control design may reasonably assess control performance in an indicative number of sea states, co-design of WEC system (physical) parameters, which cannot be adjusted after the design is complete, must be assessed over the full set of possible sea states to ensure optimality in the long term. By way of example, a scatter plot for the Atlantic Marine Energy Test Site (AMETS) is shown in Fig. 6, showing the percentage occurrence of various H_s/T_e combinations over a 12-month period (Cahill and Lewis, 2011). Wave data records over a longer period are preferred for co-design, while some consideration may also be given to the evolution of the wave climate at site(s) of interest (Ulazia et al., 2019).

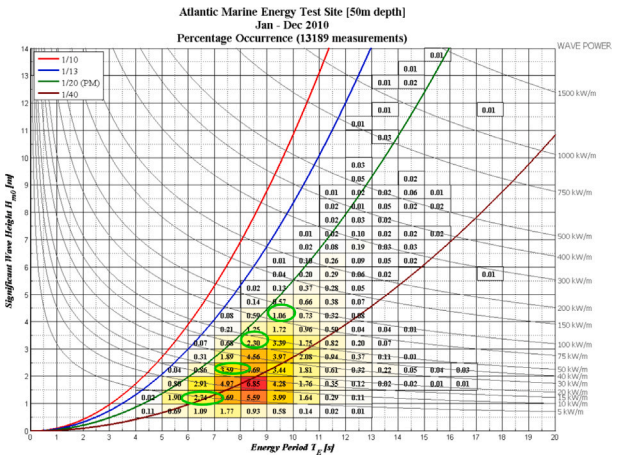

Fig. 6. Scatter plot, showing the percentage occurrence of various H_s/T_e combinations at the AMETS site on the West Coast of Ireland (Cahill and Lewis, 2011).

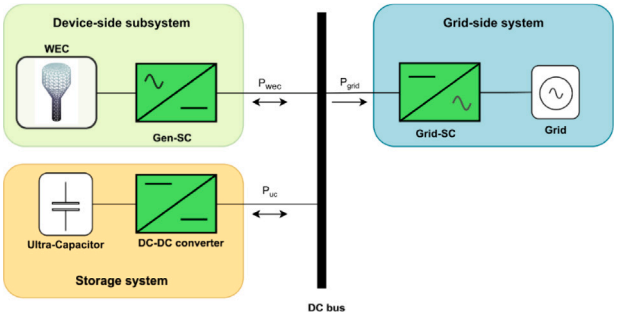

Fig. 7. Back-to-back (B2B) power converter, separating the grid and WEC sides, with optional storage component (Said et al., 2022).

4.2.3. Electrical grid modelling

Depending on the scope of the WEC control co-design exercise, it may be more, or less, important to model the grid side. In general, the WEC control problem can be separated into two domains, one looking towards the WEC side, and the other looking towards the grid, centred at the DC link, as shown in Fig. 7. While this paper is predominantly focussed on the WEC side, control co-design can also be applied to the grid side, for example considering optimal storage sizing for power smoothing of grid-injected energy, usually achieved with battery or other long-term storage devices (not shown in Fig. 7). However, optimal determination of grid-side storage capacity is a complex problem, requiring both short-term and long-term analysis of the wave resource (Rasool et al., 2022).

Regarding the WEC side, typical co-design exercises involve optimal sizing of supercapacitors for supplying reactive power for optimum WEC control. More information on grid connection of WECs, and related aspects, is available in Said and Ringwood (2021).

4.2.4. WEC arrays

WEC arrays present a significant challenge in terms of control co-design, not least in terms of hydrodynamic modelling. Typically, with the added computational burden of multi-body interaction, hydrodynamic modelling is limited to linear BEM models, with finite-order parameterisation of the non-parametric hydrodynamic parameters, e.g. Garcia-Rosa et al. (2015b) while some studies adopt an even simpler surrogate model, e.g. Peña-Sanchez et al. (2024b) (see also Section 6 for more about surrogate models). Some customised array hydrodynamic models, tailored for computational efficacy, are also employed, e.g. Abdulkadir and Abdelkhalik (2023a), where the authors use a semi-analytical model. The study in Abdulkadir and Abdelkhalik (2023a) is also somewhat unique in that an array of *heterogeneous* WECs is considered.

Table 2
Optimal cylindrical WEC parameters for passive control (PC), latching control (LC), declutching control (DC) and (unconstrained) pseudospectral control (PSC). r^*, d^*, T_r are the optimal values for device radius, draught, resonance period, while P_a denotes the average power produced in a Bretschneider sea, with $H_s = 1$ (m) and $T_p = 5$ (s). Note the relative values of T_r^*, compared to T_p, for the various controllers, according with Fig. 8.
Source: Adapted from Garcia-Rosa and Ringwood (2015).

Controller	PC	LC	DC	PSC
r^* (m)	6	2	8	10
d^* (m)	4	2	4	4
T_r^* (s)	5.55	3.68	5.91	6.24
P_a (kW)	9.38	14.67	10.82	23.58

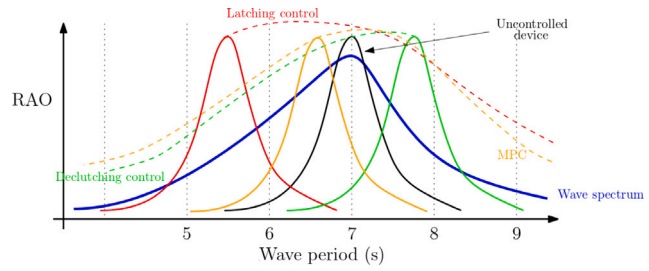

Fig. 8. Overall WEC system response amplitude operators (RAOs) (dotted lines) for different controllers and device RAOs (solid lines) (Ringwood et al., 2023b).

5. WEC control design methods for CCD

5.1. General considerations

Here, it is not intended to cover the full gamut of possible WEC control design methods (adequately covered elsewhere, e.g. Ringwood et al., 2023b), but rather to focus on WEC control methods, and WEC control issues, appropriate to WEC CCD.

Care must be taken in choosing an appropriate control methodology for CCD, considering the strong interaction between the choice of control method and the optimised physical system design. This issue is illustrated in Fig. 8, which shows (conceptually) how the uncontrolled WEC response amplitude operator (RAO) should be placed to take full advantage of various controller characteristics. For example, since latching can essentially only slow the device dynamics down, the (uncontrolled) WEC RAO should be placed relatively high in frequency, the lower frequencies being covered by the latching controller. The converse is true of declutching control, while model-predictive control (MPC), or MPC-like, controllers have the capability to slow down, or speed up (via reactive power injection) the uncontrolled WEC dynamics. This is exemplified in the study by Garcia-Rosa and Ringwood (2015), where radically different optimal device geometries are achieved for different WEC control philosophies. By way of example, Table 2 shows the significant differences between the geometric parameters (radius, draught), resonance frequency, and mean power produced by a cylindrical WEC, for passive control (PC), latching control (LC), declutching control (DC) and a pseudospectral controller (PSC). The important message here is that *the optimality of the overall system (WEC + controller) is highly dependent on the choice of WEC control philosophy*.

5.2. Computational considerations

As in the case of modelling (Section 4), there is an upper limit to the complexity of the controller that can be tolerated, while retaining the ability to perform CCD iterations in reasonable time. Unsurprisingly,

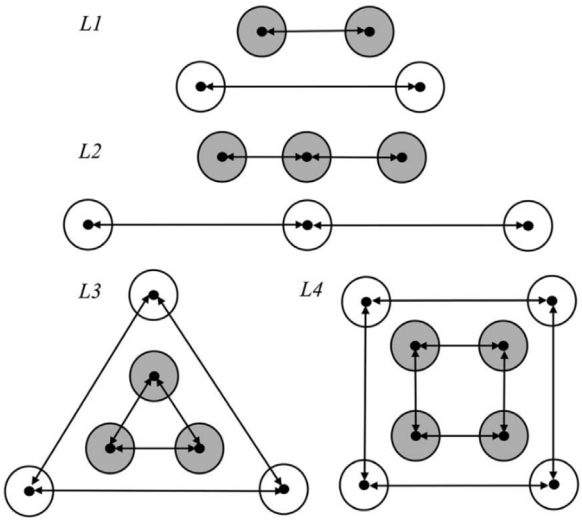

Fig. 9. Different optimal array spacing ($H_s = 1$ m, $T_p = 7$ s), for a passive controller (shaded) and a global pseudospectral controller (unshaded), for four different array configurations of cylindrical WECs (radius $r = 2.5$ m, draught $d = 4$ m), using a single spacing parameter (Garcia-Rosa et al., 2015b).

most CCD studies have tended to focus on simpler WEC control design (García-Violini et al., 2020), typically via a frequency-domain calculation, chosen primarily for speed of calculation. The other alternative is to use a high-performance optimisation-based approach (Faedo et al., 2020b), with a consequent computational overhead. Such an approach includes methods such as MPC (Faedo et al., 2017), pseudospectral control (Bacelli and Ringwood, 2014), and moment-based control (Faedo and Ringwood, 2024). Interestingly, the only purpose-built tool for WEC control co-design, WecOptTool (Coe et al., 2020; Ströfer et al., 2023), uses optimisation-based control, based on a pseudospectral (PS) system representation (Bacelli and Ringwood, 2014). It is noteworthy that the PS control is calculated over the complete evaluation window, rather than in a receding-horizon way, simplifying the computation a little.[3]

Significant care must be exercised if choosing a relatively 'simple' controller for CCD, for computational considerations, with the intention of employing a more complex WEC controller once the optimal WEC design has been arrived at though the CCD exercise. The same is also true when optimising WEC array layout (Garcia-Rosa et al., 2015b), as shown in Fig. 9, with a factor of ~2 change in optimal spacing evident, with a move from passive to pseudospectral control.

The computational efficacy of WEC controllers is brought into particularly sharp focus when addressing CCD of WEC arrays. In particular, for the case of centralised/global WEC array control, the computational load rises geometrically with the array size, proving prohibitive for many CCD applications. Of course, while independent control of array elements results in a considerably lower computational burden (rising linearly with the number of WECs), there may be a significant disparity in control performance (Bacelli et al., 2013), potentially leading to suboptimal CCD designs. One WEC array control technique, specially adapted for CCD, and based on the single WEC 'Simple and Effective' controller (Fusco and Ringwood, 2012), is documented in Ermakov et al. (2024a). This was specially developed for simultaneous CCD of heterogeneous device geometry and array layout, a most demanding wave energy CCD exercise (Ermakov et al., 2025).

[3] However, it should be noted that, in general, time-domain *receding-horizon* MPC algorithms are usually more computationally efficient than their 'global' counterparts.

5.3. Constraint handling

Regardless of the choice of time- or frequency-domain settings for control, it is crucial that physical WEC system constraints are considered by the WEC controller employed, ideally both force and displacement constraints. Otherwise, unrealistic WEC motion, and/or PTO forces will likely occur, but not be visible to the co-designer, since each simulation run will be contained within (possibly nested) optimisation iterations. In particular, if WEC constraint optimisation is the focus (or one of the focii) of the co-design exercise (e.g. in Peña-Sanchez et al., 2022), it is clearly crucial that a constraint-handling controller is employed. While the natural setting for constraint handling is the time-domain setting of MPC (noting that spectral/pseudospectral can also handle constraints), some WEC control approaches, which are ostensibly frequency-domain based, can also handle constraints efficiently, such as Fusco and Ringwood (2012), García-Violini et al. (2023).

5.4. WEC control performance evaluation

With a focus on control co-design, it is important to remember that the objective is not to calculate the iterations of a real-time controller, but rather to evaluate the *control performance*, which provides direction to the system design optimiser. For example, the WEC control algorithm in Fusco and Ringwood (2012) has a simple frequency-domain calculation (which includes constraints), but requires a real-time estimate of the assumed *instantaneous frequency* of the wave excitation force, $f_{ex}(t) = A(t)\sin(\omega(t)t + \phi(t))$ (with $A(t)$, $\omega(t)$, and $\phi(t)$ all required), provided via an extended Kalman filter (EKF) for real-time implementation. In the range of WEC CCD applications considered in this study, the sea state, or selection of sea states (see Section 4.2.2), is assumed known *a priori*, allowing free surface realisations to be calculated, from which f_{ex} may be calculated (for example, via (10)). However, site location could also be included as part of an augmented WEC CCD study, as in Azad et al. (2024). Another WEC controller which may serve CCD is the LiTe-Con controller (García-Violini et al., 2023), which has a simple linear time-invariant (LTI) structure, while handling constraints. The envelope detector, required for real-time operation, may be significantly simplified in the case of CCD, given the advance information available. This is true for all WEC controllers which require excitation force estimation and/or forecasting.

In a similar vein, but developed specially for CCD application, some tools are currently being developed to allow control performance to be efficiently evaluated, but without any aspiration towards real-time capability. These include the algebraic 'half-wave' technique of Lin et al. (2025), which calculates MPC/pseudospectral performance to within 5% accuracy, but with orders of magnitude reduction in computation time. One imagines that there is further scope for the development of such control performance evaluation tools, specifically designed for CCD.

A closely-related development relates to the use of surrogate models for control evaluation, dealt with in more detail in Section 6. Noting that the purpose of a surrogate model is to mimic control action, but with significantly reduced computation, the surrogate model must be trained over a broad range of operational conditions, including active constraints. An example of such an approach is provided by Pasta et al. (2021).

6. Surrogate models in control co-design

Given the potential complexity of the model and controller calculations highlighted in Sections 4 and 5, it may be prudent to consider some complexity reduction in model, controller, or model *and* controller calculations. In this sense, a *surrogate* model, which mimics the essential characteristics of the model, controller, and/or performance function calculation (see Fig. 10) is evolved, usually based on test

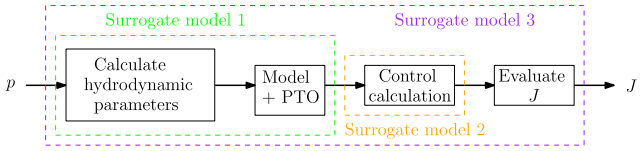

Fig. 10. Scope of possible surrogate models. p is the unknown parameter set, while J is the performance metric.

inputs and corresponding responses recorded from the more complex corresponding component. Such models may be synthesised from (linear and nonlinear) system identification techniques (Ljung, 1999) or machine learning methods (Kudela and Matousek, 2022).

Surrogate modelling has become popular in recent decades as a tool to reduce the computational complexity of simulation models (Alizadeh et al., 2020; Forrester et al., 2008). Importantly, emphasis is placed on a design-of-experiment (DoE), where factorial design is used to help minimise the amount of investigative data from the high-fidelity models needed to train the simpler surrogate models. An interesting example of the use of surrogate WEC modelling in CCD is given in Azad and Herber (2023), where WEC CCD is used to simultaneously optimise device geometry and array layout. In Erdogan et al. (2025), a surrogate model (a 'digital twin') is used in design and control optimisation of marine renewable systems.

The reduction of *dynamical* model complexity falls into the category of *model reduction*, and can involve analytical manipulation of the physical system model (Benner et al., 2015), or composition of a model from data recorded from the system, falling into the category of system identification (Ljung, 1999). Where the model is in nonparametric form (e.g. CFD or BEM hydrodynamic models, or MPC), a system identification approach, either in the time- or frequency-domain must be employed, providing an opportunity to select, in an incremental way, the complexity (e.g. dynamical order, degree of nonlinearity) of the identified model. Within the context of wave energy systems, some efforts have been made to establish somewhat generic complexity reduction approaches, e.g. Penalba and Ringwood (2020), Faedo et al. (2020a). The study by Stavrololou et al. (2025), by way of example, uses physical system data (from a WEC array) to develop models of various levels of fidelity/complexity.

A key aspect in the construction of a surrogate dynamical model is the need to cover the frequency range of interest and, considering hydrodynamic (or other) nonlinearity, to cover the operational amplitude ranges of key variables. Davidson et al. (2016) propose a set of potential test functions which can be injected to cover the amplitude/frequency space, while chirp signals are utilised in a variety of studies (e.g. Farajvand et al., 2023) to obtain a set of *amplitude-specific* representative linear models. Typically, signal injection through the PTO system can offer greater excitation bandwidth, and collect more useful information in a shorter space of time, compared to wave excitation, but the model bandwidth of interest is closely related to the range of frequencies associated with the range of sea states present in the site scatter plot (see Section 4.2.2).

Surrogate models can be constructed for the hydrodynamic (and PTO) subsystem, the controller itself, or a combination of both, as shown in Fig. 10. There are various advantages and disadvantages associated with each of the three possibilities shown:

Physical system model only In this scenario, surrogate models are developed for the hydrodynamic model of the system and/or PTO, recognising the typically time-consuming calculations associated with hydrodynamic parameter calculation, and the fidelity limitations of models based on linear potential theory. Surrogate models can also be created for the PTO, though PTO models (unless complex efficiency characteristics associated with electrical/electronic components are included) are typically fast to execute. One problem associated with surrogate modelling of the hydrodynamic component relates to the format in which the model output is presented. With a view to control co-design, the non-parametric output formats presented by typical BEM platforms is less than ideal, so a further stage of parameterisation may be included, such as in Papillon et al. (2019), Leboucher et al. (2025), where finite-order parametric models, of prescribed order, are fitted to the non-parametric BEM outputs, using one of the methods described in Peña-Sanchez et al. (2019). One particular advantage of the surrogate physical system model only approach is flexibility in the choice of WEC controller (subject to the creation of a suitable interface between system model and controller), and the performance function. A useful study by Zhang et al. (2020) focusses on surrogate modelling of the hydrodynamic interaction between floating bodies in an array.

Controller only Surrogate models for controllers are potentially challenging, since it may be difficult to anticipate the full scope of controller inputs and outputs, in both amplitude and frequency. Pasta et al. (2021) present an example where the actions of a WEC MPC controller are mimicked by an artificial neural network. An advantage of the surrogate controller model only approach is the flexibility in the choice of physical system model and performance function, though there may be a close relationship between the overall CCD objective function and that of the controller. A further difficulty is guaranteeing that the surrogate controller will respect all the physical system constraints, which may need to be adjusted during the CCD process. Equally, the surrogate controller may not be suited to all physical system models. Examples of surrogate receding-horizon controller modelling with traditional regulatory control performance objectives can also be found, e.g. Parisini and Zoppoli (1995).

Complete system model In this option, a surrogate model is created for all components, from the free surface variations all the way to the performance metric. This approach offers the best computational efficacy, but the least flexibility. An example of this approach can be seen in Peña-Sanchez et al. (2024b). This approach shares the same concern regarding physical system constraints as the control-only surrogate approach. One of the main advantages, however, is that the surrogate model has a single output, J, simplifying training and use.

Other surrogate models can, of course, be constructed from other combinations of the components in Fig. 10; only the most obvious possibilities are expanded on here.

In general, the rationale for surrogate modelling, in a control co-design setting, is to shorten the computation time associated with each iteration of the optimisation process. However, this must be considered in tandem with the computational effort in constructing the surrogate model in the first place, as shown in Fig. 11. Clearly, if the physical system model or controller take a considerable time to compute (think CFD/MPC), which the surrogate model is attempting to expedite within a design optimisation loop, it is also likely that such a surrogate model(s) will also take a considerable amount of time to train. However, once the surrogate model is constructed, multiple uses can be made of the model, and a surrogate model may be particularly effective where multiple nested optimisation loops are used, for example WEC geometry, array layout, PTO constraints, mooring configuration, etc. In addition to reducing the computation time for a specific CCD optimisation task, surrogate models also provide the opportunity to explore a larger set of optimisation variables, or to utilise a larger population within stochastic search.

Finally, the impact of surrogate modelling errors on the optimisation process needs to be considered. Fig. 12 demonstrates a rather benign

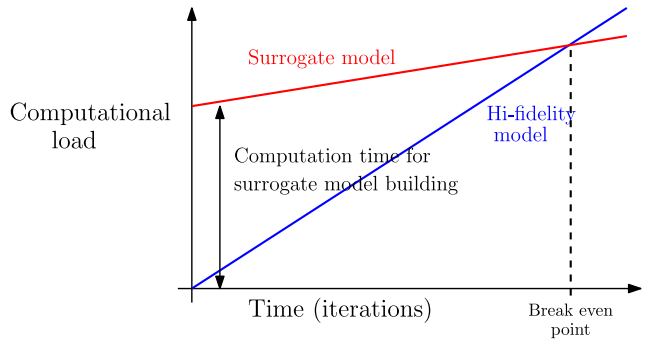

Fig. 11. Computational aspects of surrogate models.

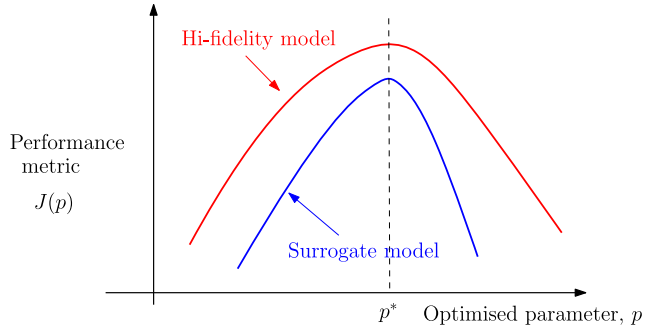

Fig. 12. Perspective on optimisation using surrogate model.

situation where, though the surrogate presents a significant error in the calculation of the performance metric compared to the high-fidelity model/controller, the optima are consistent, leading to an identical result. Unfortunately, it is not possible to assess the consistency of the model optima in advance. Even if the full-fidelity system/controller representation is employed to 'check' if the design is 'good', no information about the relative 'goodness' is easily available. However, the optimum design parameters utilising the surrogate model may be used as a 'hot start' for a small number of iterations of the high-fidelity approach, particularly if the design process is almost complete and deployment is the next step. Some useful information is given by Wilson et al. (2001) in relation to the use of surrogate models in optimisation.

7. Optimisation algorithms

Here, some aspects related to the choice of, and options for, control co-design optimisation are considered. Specifically, this relates to the choice of update algorithm for the 'Update p' block in Fig. 4. It is not intended to be entirely comprehensive, since the topic of optimisation has been well covered elsewhere (Pierre, 2012; Gill et al., 2019; Bertsekas, 2014; Fletcher, 2000). Rather, an attempt is given to explore the nature, and potential benefits, of various optimisation approaches, within a wave-energy control co-design context, and to give a flavour of typical optimisation algorithms currently employed in WEC CCD.

One guiding principle, which this author believes is important, is that the optimisation algorithm chosen should be driven by the specific problem at hand, rather than be prescribed. Some aspects of the WEC CCD that could be considered include:

- Whether the performance function is convex in the parameters to be optimised. This is not always easy to determine, but significant nonlinearity in the system model, or the calculation particular performance functions, may suggest non-convexity.
- Whether the performance function components in (6) can be reasonably combined (with common units), or whether a multi-criterion approach (Giagkiozis and Fleming, 2014) is necessary. In a multi-criteria scenario, separate optima are presented in a Pareto front.
- Whether the evaluation of the performance function is already computationally heavy, which may mitigate against the use of concurrent search algorithms.

Broad classes of optimisation algorithm, which could be employed for WEC CCD, include:

Exhaustive search While exhaustive search (Williams, 2010) is probably the least efficient optimisation technique, it has the significant benefit of allowing the performance surface to be viewed (in up to 3-D, i.e. 2 optimised variables at a time), giving an indication of the convexity, or otherwise, of the problem, and also giving an idea of how sensitive the performance metric is to the parameters to be optimised. Appropriate choice of the search grid is necessary to explore the parameter space of interest, while minimising computational overhead. WEC CCD examples include the studies by Peña-Sanchez et al. (2022), Ströfer et al. (2023), and Rosati and Ringwood (2023).

Gradient-based methods Gradient-based methods (Nesterov, 2013), which may also calculate the Hessian (2nd derivative) are efficient and popular, but can be sensitive to local minima. Some mitigation against local minima, developed for training neural networks, include the use of momentum terms (Qian, 1999) and/or adaptive learning rate (Zeiler, 2012). An example of WEC CCD, comparing line search to a GA, can be found in Liao et al. (2024).

Direct search Direct methods, such as the Nedler–Mead simplex (Olsson and Nelson, 1975), have some immunity to local minima, while being computationally economical. An example of the use of Nedler–Mead optimisation for WEC CCD is Gilloteaux and Ringwood (2010).

Stochastic methods Stochastic search techniques (Hoos and Stützle, 2018) employ some element of uncertainty, or randomness, in the search direction. They can include algorithms that are closer to line searches e.g. stochastic gradient descent (Bottou, 2012), but also algorithms based on *concurrent search* (also termed population-based methods) where multiple solutions are concurrently updated and mixed. Many concurrent search methods are inspired by evolutionary or nature-inspired mechanisms (Holland, 1992), such as genetic algorithms (GAs) (Goldberg, 1989) and particle swarm optimisation (PSO) (Clerc, 2010). Despite their high computational overhead, stochastic/concurrent search is frequently employed in WEC CCD, including by Paduano et al. (2023) and Abdulkadir and Abdelkhalik (2024a) (both GA), with a Bayesian technique employed by Meduri and Kang (2024). A variety of AI-inspired techniques for WEC CCD is reviewed by Peña-Sanchez et al. (2024a).

It is interesting, from the preponderance of WEC CCD studies, that there is a propensity to employ either exhaustive search or a genetic algorithm for WEC CCD optimisation. This can be (at least, partly) explained by the rich amount of information provided by an exhaustive search, while GAs can, in general, deal effectively with non-convex performance surfaces, with a high likelihood of achieving a global optimum.

In some cases, a combination of optimisation tools may be appropriate, with a number of iterations of a concurrent algorithm, or a coarse exhaustive search, used to place the solution in the convex neighbourhood of the global optimum, with the search then refined using

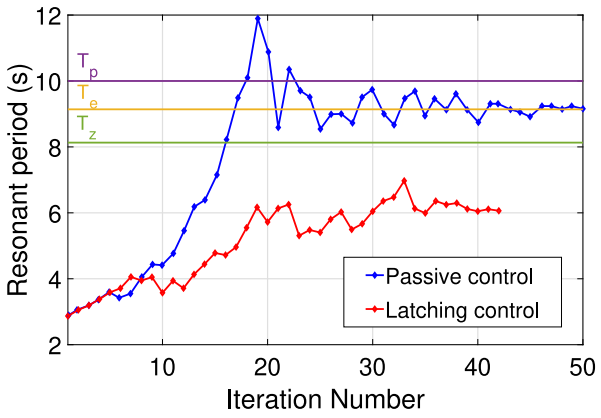

Fig. 13. CCD of a cylindrical WEC, under both passive and latching control. T_e, T_p, and T_z show the values of the wave spectrum energy, peak, and zero-crossing periods, respectively.
Source: Adapted from Gilloteaux and Ringwood (2010).

Table 3
Optimal cylindrical WEC parameters for passive control and latching control. T_r is the device resonant period.

Parameter	T_r	Radius (m)	Draught(m)
Passive control	9.14	12.52	14.08
Latching	6.22	6.38	6.19

a gradient (e.g. line, or simplex) method. Equally, other techniques, including some analytical design intuition, could be used to create a 'hot start' for a numerical optimisation algorithm. However, since some of the results from WEC CCD can be somewhat surprising, care must be taken with the use of intuitive reasoning.

An important consideration is establishment of the search space. Smaller search spaces permit a smaller grid size to be used (in exhaustive search), but may miss the global optimum. If the optimum, following convergence of the optimiser, lies on one of the search space limits, it may be indicative of one of two potential causes:

- The search space is too small, and the global optimum lies outside the original search space limits, and/or
- The performance metric is monotonic, i.e. the 'global' optimum is achieved at an asymptotic value. A simple example of this is where a performance function such as energy/size is employed *with no constraints on control action*. The result is a tendency towards a device of zero size, with infinite oscillation amplitude. Therefore, realistic PTO constraints can play a role in establishing a convex design optimisation problem.

8. A brief literature survey of WEC CCD

Typically, a research paper has the literature survey at the start. Since this is more of a guidebook, the process has been somewhat inverted, so that the available literature can be considered in the light of the forgoing discussion. The following subsections illustrate some of the published literature for wave energy control co-design in the areas of WEC geometry optimisation, array layout optimisation, PTO optimisation, and mooring system optimisation. In some cases, multiple aspects are considered within a single study; these cases are cross-referenced, as appropriate. While some attempt is made to make this a reasonably complete review, the author apologises in advance for any omissions, and would be delighted to be made aware of other studies from the offended parties!

8.1. WEC geometry co-design

Initial work exploring the interaction between WEC control design and geometry optimisation began with the 2010 study by Gilloteaux and Ringwood (2010) which showed a significant disparity between the optimal WEC geometries achieved with latching control and passive control. For illustration, Fig. 13 shows the convergence of the optimal resonant period of a cylindrical WEC, using Nedler–Mead simplex optimisation, while Table 3 shows the converged geometry. Note the significant disparity between the optimal T_r/radius/draught for the passively- and latching-controlled devices. Since then, a wide variety of WEC geometry optimisation studies have been undertaken, though many WEC geometry optimisation studies have not included CCD in the methodology. Broad reviews of WEC geometry optimisation include the reviews by Guo and Ringwood (2021a) and Garcia-Teruel and Forehand (2021), with the comprehensive reference lists including a significant number of CCD studies. In addition, Garcia-Teruel et al. (2021) provide much useful information on the choice of different objective functions and optimisation algorithms (e.g. GA, PSO), but do not adapt the control, though this is indicated as an area for future research. Specific CCD WEC geometry studies include a number of studies using the WecOptTool developed by Sandia Labs., including a Wavebot conceptual geometric study, with linear damping, complex conjugate, and pseudospectral control presented in Coe et al. (2020).

An early WEC geometry CCD study (Garcia-Rosa and Ringwood, 2015) also shows the radically different cylindrical WEC designs resulting from the co-optimisation of various control strategies, including no control, latching control, declutching control, and pseudospectral control, with results summarised in Table 2. Another point-absorber geometric optimisation study, using pseudospectral control and genetic optimisation, was undertaken in Abdelkhalik et al. (2017), while a genetic algorithm was also used to optimise cylindrical and spherical point absorbers, using nonlinear polynomial damping control (Abdulkadir and Abdelkhalik, 2024a,b). In Grasberger et al. (2024), an oscillating wave surge converter (OSWEC) was optimised using a parameter (width and height) sweep, using a variety of performance metrics, including a comparison between electrical and mechanical power maximisation, with co-design of a PI controller. In Sergiienko et al. (2020), a CETO WEC system is comprehensively optimised, where a 3 DoF spectral hydrodynamic model (incorporating linearised viscous drag) is used. The design parameters for optimisation include the buoy radius, buoy height, tether inclination angles, and control variables (damping and stiffness). A unique study, which optimises ballast, in addition to geometry in pseudo-spectral WEC CCD, for a hinge-barge device, is described in Wang and Ringwood (2021).

A number of studies have examined the simultaneous optimisation of array layout and geometry optimisation, including Azad et al. (2024), Azad and Herber (2023), Azad et al. (2025), which employs varying levels of design freedom to optimise array layout and geometry, using evolutionary optimisation, with surrogate modelling, while optimising an array-level PI control strategy. Optimisation of heterogeneous arrays of WEC arrays (individual geometry and array layout) is considered in studies by Abdulkadir and Abdelkhalik (2023a,b) and Ermakov et al. (2025).

Finally, the effect of control constraints on optimal WEC geometry, with concurrent control design, are examined in Garcia-Rosa et al. (2015a).

8.2. WEC array layout co-design

Perhaps one of the starkest reminders of the importance of wave energy CCD are radically different array layouts that result from different array level control philosophies, as illustrated in Garcia-Rosa et al. (2015b), and summarised in Fig. 9. Karani et al. (2024) optimise the layout of an array of WEPTOS devices, with simultaneous optimisation of a linear (independent) damping parameter for each device,

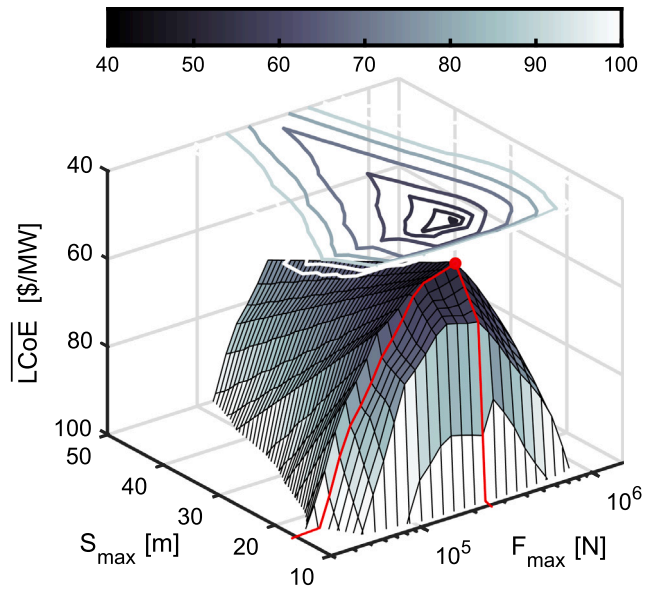

Fig. 14. Approximate LCoE ($\overline{\text{LCoE}}$) for different stroke (S_{max}) and force (F_{max}) limits (Peña-Sanchez et al., 2022).

while Mercadé Ruiz et al. (2017) parameterise a WEC array in terms of inter-row and inter-column spacing, along with orientation of the array and relative row/column angle. However, an independent fixed spring/damper controller is assigned to each WEC, limiting the scope of the control CCD influence. Optimisation is provided by a comparison of GA and glowworm swarm methods. As mentioned in Section 8.1, a number of studies optimise array layout, in addition to WEC geometry, including Azad et al. (2024), Azad and Herber (2023), Abdulkadir and Abdelkhalik (2023a,b), Ermakov et al. (2025).

Two studies perform array optimisation, but it is not entirely clear if co-optimisation of control takes place. In the study from Meduri and Kang (2024), a dual Bayesian optimisation strategy is used to optimise array layout (in Step 1) and the independent controller PI parameters (in Step 2), but it appears that each step is performed sequentially. Neshat et al. (2020) optimise the positions of a number ($\leq$ 16) of submerged WECs. Again, independent spring/damper controllers are employed, but it appears that the controller parameters are outside the optimisation loop. A combination of stochastic optimisation and local search was used, along with surrogate modelling.

8.3. WEC PTO co-design

PTO CCD has received less attention than the more popular focus on geometry and array layout, though there is not total clarity on what comprises the PTO since, in some cases, the controller 'parameters' are considered as PTO parameters. Here, the distinction is made between control aspects and physical PTO parameters, such as torque (or force), velocity, and displacement constraints, and any additional physical inertia, such as internal mass (or flywheel), or additional physical springs.

Typical examples, where stroke and torque/force constraints are co-optimised include the studies by Peña-Sanchez et al. (2022) and Ströfer et al. (2023). By way of example, Fig. 14 shows (on an inverted scale), the convex performance surface of an approximate LCoE metric (termed $\overline{LCoE}$) for various PTO force and displacement constraints. In Devin et al. (2024), five aspects of PTO (flywheel inertia, magnetic spring stiffness, spring gear ratio, motor gear ratio, and coulomb friction) are co-optimised while, for the M4 device, the mechanical gearbox ratio and the generator rated torque are optimised in Liao et al. (2024). WecOptTool is also used ina study by Gaebele et al. (2025) using Technology Performance Level (TPL) as a metric to improve power-to-cost ratio. A parameter sweep is carried out over gear ratios and spring stiffness for various generators, in irregular wave conditions.

The role of an accurate loss model in WEC CCD is also highlighted in Lin et al. (2024). Specifically, the effect of increasing losses with increasing displacement/velocity/force needs to critically inform the economic specification of PTO constraints. In addition, the study in Huang et al. (2024) also considers variable DC bus voltage as a potential co-design parameter.

8.4. WEC moorings co-design

Of all the WEC CCD possibilities, moorings CCD has attracted the least interest, no doubt reflecting the paucity of studies which consider moorings to be an active aspect of the WEC control design problem, in general. Nevertheless, there are a number of WEC mooring CCD studies of note, including Papini et al. (2023), which analyses the effect of including a mooring model in the WEC control formulation, and Paduano et al. (2023), which optimises mooring layout for a 3-point tethered WEC.

Other moorings-related WEC CCD studies include Peña-Sanchez et al. (2024b), where both array layout *and* mooring configurations are optimised, while Gaebele et al. (2023) consider mooring line pretension as a co-design variable.

9. Conclusions

Given the very strong interaction between optimal physical WEC system design and control philosophy/tuning, it is not surprising that control co-design is starting to feature strongly in the wave energy literature. This is particularly so for resonant devices, where controllers are highly tuned to the device dynamics, creating a sharply focussed bond that is generally in contrast to physical systems controlled by typical setpoint-following feedback controllers. One aspect of WEC CCD, which is significant, is the radically different WEC design parameters resulting from co-design with different WEC controllers. This is evident from a range of studies, and clearly illustrated in Figs. 8 and 13. Thus, it is not sufficient to perform co-design with an arbitrary controller (many WEC CCD studies rely on the simplest PI controller structure); rather, an optimal controller should be chosen and, given a requirement to optimise PTO parameters (including physical constraints), a constraint-handling WEC controller should, ideally, be chosen. While this brings the challenge of increased computation within the CCD optimisation loop, methods that evaluate control *performance*, without actually implementing an optimising controller, may be a useful way forward, as exemplified in Ermakov et al. (2024a), Lin et al. (2025). In terms of optimal WEC control, some clue as to the best performing WEC controllers may be had from the recent Wave Energy Converter Control Competition (WECCCOMP) (Ringwood et al., 2023a), where competitors uniformly chose an MPC, or MPC-like, controller.

One aspect, not covered in this study, is the issue of uncertainty in the physical system model and its components (hydrodynamics, actuators, sensors). While uncertainty related to the calculation of $OpEx$ was highlighted in Section 3, uncertainty in the physical system model has been shown to be particularly problematic for control calculations with an energy-maximising control objective (Ringwood et al., 2019). Uncertainty is not completely unrelated to the issue of physical model fidelity, and may be addressed through the use of robust control techniques (Faedo et al., 2019; Garcia-Violini and Ringwood, 2021; Faedo et al., 2022). However, robust control (optimally) trades off robustness for some nominal performance, so involves some level of compromise.

A wide variety of WEC design aspects (geometry, array layout, PTO, moorings) have been addressed in various WEC CCD studies, with a number addressing multiple aspects simultaneously, e.g. moorings and array layout, or geometry (homogeneous and heterogeneous)

and array layout. However, what is not entirely clear, at this point, is the degree to which these different WEC design aspects interact. One reason for this uncertainty is the current limit on computational capacity to address two, or more, design aspects simultaneously, as well as concurrently co-optimising the control. This is an area for future study and may be facilitated by improvements in computational techniques, as well as computational performance. Ultimately, a suitable compromise needs to be arrived at between computational simplicity and model/controller fidelity, along with efficient optimisation algorithms which can deal with the (potentially non-convex) nature of the range/combination of WEC CCD optimisation problems. While surrogate models can play a useful role in this respect, dedicated algorithms may need to be developed, also an area for further research.

Equally, the search for suitable performance functions, which closely approximate LCoE, is still incomplete. Garcia-Teruel et al. (2021) has much useful information on the choice of different objective functions and optimisation algorithms (GA, PSO), but does not adapt the control and employs a constrained optimal control (constraints are somewhat crudely applied after the optimal control is calculated), rather than optimal constrained control (the problem is optimally solved as a constrained problem), leading to an uncertain design optimum.

As a final counterpoint to WEC CCD, Meduri and Kang (2025) adopt a *sequential* design approach, with LCoE minimisation as the final performance goal, though some iterations are performed on the controller spring and damping terms.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

Abdelkhalik, O., Coe, R.G., Bacelli, G., Wilson, D.G., 2017. WEC geometry optimization with advanced control. In: Intl. Conf. on Offshore Mechanics and Arctic Eng., Trondheim. vol. 57786, American Society of Mechanical Engineers, V010T09A031.

Abdulkadir, H., Abdelkhalik, O., 2023a. Heterogeneous WEC array optimization using the hidden genes genetic algorithm. In: Proc. European Wave and Tidal Energy Conf. (EWTEC), Bilbao. pp. 286.1–286.8.

Abdulkadir, H., Abdelkhalik, O., 2023b. Optimization of heterogeneous arrays of wave energy converters. Ocean Eng. 272, 113818.

Abdulkadir, H., Abdelkhalik, O., 2024a. Control co-design of linear and nonlinear wave energy converters. In: 2024 4th International Conference on Smart Grid and Renewable Energy. SGRE, IEEE, pp. 1–7.

Abdulkadir, H., Abdelkhalik, O., 2024b. Control co-design optimization of nonlinear wave energy converters. Ocean Eng. 304, 117827.

Alizadeh, R., Allen, J.K., Mistree, F., 2020. Managing computational complexity using surrogate models: A critical review. Res. Eng. Des. 31 (3), 275–298.

Allison, J.T., 2014. Engineering system co-design with limited plant redesign. Eng. Optim. 46 (2), 200–217.

Allison, J.T., Herber, D.R., 2014. Special section on multidisciplinary design optimization: multidisciplinary design optimization of dynamic engineering systems. AIAA J. 52 (4), 691–710.

Azad, S., Herber, D.R., 2023. Concurrent probabilistic control co-design and layout optimization of wave energy converter farms using surrogate modeling. In: International Design Engineering Technical and Computers and Information in Engineering Conference. vol. 87318, American Society of Mechanical Engineers, V03BT03A035.

Azad, S., Herber, D.R., Khanal, S., Jia, G., 2024. Site-dependent solutions of wave energy converter farms with surrogate models, control co-design, and layout optimization. In: Proc. American Control Conf., Toronto. pp. 2572–2579.

Azad, S., Herber, D.R., Khanal, S., Jia, G., 2025. Concurrent geometry, control, and layout optimization of wave energy converter farms in probabilistic irregular waves using surrogate modeling. Ocean Eng. 320, 120183.

Bacelli, G., Balitsky, P., Ringwood, J.V., 2013. Coordinated control of arrays of wave energy devices—Benefits over independent control. IEEE Trans. Sustain. Energy 4 (4), 1091–1099.

Bacelli, G., Coe, R.G., Patterson, D., Wilson, D., 2017. System identification of a heaving point absorber: Design of experiment and device modeling. Energies 10 (4), 472.

Bacelli, G., Genest, R., Ringwood, J.V., 2015. Nonlinear control of flap-type wave energy converter with a non-ideal power take-off system. Annu. Rev. Control. 40, 116–126.

Bacelli, G., Ringwood, J.V., 2014. Numerical optimal control of wave energy converters. IEEE Trans. Sustain. Energy 6 (2), 294–302.

Barstow, S., Mørk, G., Mollison, D., Cruz, J., 2008. The wave energy resource. In: Ocean Wave Energy: Current Status and Future Prespectives. pp. 93–132.

Benner, P., Gugercin, S., Willcox, K., 2015. A survey of projection-based model reduction methods for parametric dynamical systems. SIAM Rev. 57 (4), 483–531.

Bertsekas, D.P., 2014. Constrained Optimization and Lagrange Multiplier Methods. Academic Press.

Bird, L., Lew, D., Milligan, M., Carlini, E.M., Estanqueiro, A., Flynn, D., Gomez-Lazaro, E., Holttinen, H., Menemenlis, N., Orths, A., et al., 2016. Wind and solar energy curtailment: A review of international experience. Renew. Sustain. Energy Rev. 65, 577–586.

Bottou, L., 2012. Stochastic gradient descent tricks. In: Neural Networks: Tricks of the Trade: Second Edition. Springer, pp. 421–436.

Cahill, B., Lewis, T., 2011. Wave energy resource characterization and the evaluation of potential wave farm sites. In: Proc. MTS/IEEE OCEANS, Kona. pp. 1–10.

Castro-Santos, L., Cordal-Iglesias, D., Filgueira-Vizoso, A., 2024. Floating wave energy farms: How energy calculations shape economic feasibility? Heliyon 10 (22), e39672.

Chau, F.P., Yeung, R.W., 2012. Inertia, damping, and wave excitation of heaving coaxial cylinders. In: ASMA Intl. Conf. on Offshore Mechanics and Arctic Eng. (OMAE), Rio de Janeiro. vol. 44915, pp. 803–813.

Clerc, M., 2010. Particle Swarm Optimization. John Wiley & Sons.

Coe, R.G., Bacelli, G., Forbush, D., 2021. A practical approach to wave energy modeling and control. Renew. Sustain. Energy Rev. 142, 110791.

Coe, R.G., Bacelli, G., Olson, S., Neary, V.S., Topper, M.B., 2020. Initial conceptual demonstration of control co-design for WEC optimization. J. Ocean. Eng. Mar. Energy 6 (4), 441–449.

Cummins, W., 1962. The impulse response function and ship motions. Schiffstechnik 9, 101—-109.

Davidson, J., Giorgi, S., Ringwood, J.V., 2015. Linear parametric hydrodynamic models for ocean wave energy converters identified from numerical wave tank experiments. Ocean Eng. 103, 31–39.

Davidson, J., Giorgi, S., Ringwood, J.V., 2016. Identification of wave energy device models from numerical wave tank data — Part 1: Numerical wave tank identification tests. IEEE Trans. Sustain. Energy 7 (3), 1012–1019.

De Andres, A., Maillet, J., Hals Todalshaug, J., Möller, P., Bould, D., Jeffrey, H., 2016. Techno-economic related metrics for a wave energy converters feasibility assessment. Sustainability 8 (11), 1109.

Devin, M.C., Gaebele, D.T., Ströfer, C.A.M., Grasberger, J.T., Lee, J., Coe, R.G., Bacelli, G., 2024. High-dimensional control co-design of a wave energy converter with a novel pitch resonator power takeoff system. Ocean Eng. 312, 119124.

Erdogan, N., Bhinder, M., Murphy, J., Cali, U., Aghaei, M., 2025. Digital twin in design and control optimization of marine renewable energy and offshore wind energy systems. In: Digital Twin Technology for the Energy Sector. Elsevier, pp. 143–160.

Ermakov, A., Ali, Z.A., Mahmoodi, K., Mason, O., Ringwood, J.V., 2024a. A frequency domain-based control methodology for performance assessment and optimisation of heterogeneous arrays of wave energy converters. In: 2024 IEEE Conf. on Control Technology and Applications (CCTA), Newcastle, UK. pp. 584–589.

Ermakov, A., Ali, Z., Mahmoodi, K., Mason, O., Ringwood, J., 2025. Optimization of heterogeneous wave energy converter arrays: A control co-design strategy for optimal geometry and layout. Renew. Energy In press.

Ermakov, A.M., Rose-Butcher, J.L., Stepanyants, Y.A., Ringwood, J.V., 2024b. Exploiting Fano resonance in wave energy systems. IFAC- Pap. 58 (20), 380–385.

Faedo, N., Dores Piuma, F.J., Giorgi, G., Ringwood, J.V., 2020a. Nonlinear model reduction for wave energy systems: A moment-matching-based approach. Nonlinear Dynam. 102 (3), 1215–1237.

Faedo, N., García-Violini, D., Peña-Sanchez, Y., Ringwood, J.V., 2020b. Optimisation-vs. non-optimisation-based energy-maximising control for wave energy converters: A case study. In: 2020 European Control Conf. (ECC), St. Petersburg. IEEE, pp. 843–848.

Faedo, N., García-Violini, D., Scarciotti, G., Astolfi, A., Ringwood, J.V., 2019. Robust moment-based energy-maximising optimal control of wave energy converters. In: 2019 IEEE 58th Conf. on Decision and Control (CDC), Nice. IEEE, pp. 4286–4291.

Faedo, N., Mattiazzo, G., Ringwood, J.V., 2022. Robust energy-maximising control of wave energy systems under input uncertainty. In: Proc. 2022 European Control Conference (ECC), London. IEEE, pp. 614–619.

Faedo, N., Olaya, S., Ringwood, J.V., 2017. Optimal control, MPC and MPC-like algorithms for wave energy systems: An overview. IFAC J. Syst. Control. 1, 37–56.

Faedo, N., Ringwood, J.V., 2024. A control framework for ocean wave energy conversion systems: The potential of moments. Annu. Rev. Control. Robot. Auton. Syst. 7.

Farajvand, M., García-Violini, D., Ringwood, J.V., 2023. Representative linearised models for a wave energy converter using various levels of force excitation. Ocean Eng. 270, 113635.
Fefferman, C.L., 2006. Existence and smoothness of the Navier-Stokes equation. Millenn. Prize. Probl. - Clay Math. Inst. 57, 67.
Fletcher, R., 2000. Practical Methods of Optimization. John Wiley & Sons.
Folley, M., Whittaker, T., 2010. Spectral modelling of wave energy converters. Coast. Eng. 57 (10), 892–897.
Forrester, A., Sobester, A., Keane, A., 2008. Engineering Design Via Surrogate Modelling: a Practical Guide. John Wiley & Sons.
Fusco, F., Nolan, G., Ringwood, J.V., 2010. Variability reduction through optimal combination of wind/wave resources–An Irish case study. Energy 35 (1), 314–325.
Fusco, F., Ringwood, J.V., 2012. A simple and effective real-time controller for wave energy converters. IEEE Trans. Sustain. Energy 4 (1), 21–30.
Gaebele, D.T., Anderson, M.L., Roach, A.L., Forbush, D.D., Roberts, J.D., Weber, J., 2025. From TPL assessment to design optimization: Wave energy converter control co-design applied to the RM3. Renew. Energy 122338.
Gaebele, D.T., Michelén Ströfer, C.A., Devin, M.C., Grasberger, J.T., Coe, R.G., Bacelli, G., 2023. Incorporating empirical nonlinear efficiency into control co-optimization of a real world heaving point absorber using WecOptTool. In: Intl. Conf. on Offshore Mechanics and Arctic Eng. (OMAE), Melbourne. vol. 86908, ASME, V008T09A067.
Garcia-Rosa, P.B., Bacelli, G., Ringwood, J.V., 2015a. Control-informed geometric optimization of wave energy converters: The impact of device motion and force constraints. Energies 8 (12), 13672–13687.
Garcia-Rosa, P.B., Bacelli, G., Ringwood, J.V., 2015b. Control-informed optimal array layout for wave farms. IEEE Trans. Sustain. Energy 6 (2), 575–582.
Garcia-Rosa, P.B., Ringwood, J.V., 2015. On the sensitivity of optimal wave energy device geometry to the energy maximizing control system. IEEE Trans. Sustain. Energy 7 (1), 419–426.
Garcia-Sanz, M., 2019. Control co-design: An engineering game changer. Adv. Control. Applications: Eng. Ind. Syst. 1 (1), e18.
Garcia-Teruel, A., DuPont, B., Forehand, D.I., 2021. Hull geometry optimisation of wave energy converters: On the choice of the objective functions and the optimisation formulation. Appl. Energy 298, 117153.
Garcia-Teruel, A., Forehand, D., 2021. A review of geometry optimisation of wave energy converters. Renew. Sustain. Energy Rev. 139, 110593.
García-Violini, D., Faedo, N., Jaramillo-Lopez, F., Ringwood, J.V., 2020. Simple controllers for wave energy devices compared. J. Mar. Sci. Eng. 8 (10), 793.
García-Violini, D., Peña-Sanchez, Y., Faedo, N., Ferri, F., Ringwood, J.V., 2023. A broadband time-varying energy maximising control for wave energy systems (LiTe-Con+): Framework and experimental assessment. IEEE Trans. Sustain. Energy 14 (3), 1516–1525.
Garcia-Violini, D., Ringwood, J.V., 2021. Energy maximising robust control for spectral and pseudospectral methods with application to wave energy systems. Internat. J. Control 94 (4), 1102–1113.
Gelós, E.M., Judewicz, M.G., Sosa, A.D., Montanelli, L.N., Carrica, D.O., 2022. Dimensioning method for oscillating arm-float wave energy converters. In: IEEE Biennial Congress of Argentina (ARGENCON), San Juan. pp. 1–8.
Genest, R., Bonnefoy, F., Clément, A.H., Babarit, A., 2014. Effect of non-ideal power take-off on the energy absorption of a reactively controlled one degree of freedom wave energy converter. Appl. Ocean Res. 48, 236–243.
Giagkiozis, I., Fleming, P.J., 2014. Pareto front estimation for decision making. Evol. Comput. 22 (4), 651–678.
Gill, P.E., Murray, W., Wright, M.H., 2019. Practical Optimization. SIAM.
Gilloteaux, J.-C., Ringwood, J., 2010. Control-informed geometric optimisation of wave energy converters. IFAC Proc. Vol. 43 (20), 366–371.
Giorgi, S., Davidson, J., Jakobsen, M., Kramer, M., Ringwood, J.V., 2019. Identification of dynamic models for a wave energy converter from experimental data. Ocean Eng. 183, 426–436.
Goldberg, D.E., 1989. Genetic Algorithms in Search, Optimization, and Machine Learning. Addison Wesley.
Grasberger, J., Yang, L., Bacelli, G., Zuo, L., 2024. Control co-design and optimization of oscillating-surge wave energy converter. Renew. Energy 225, 120234.
Guo, B., Ringwood, J.V., 2021a. Geometric optimisation of wave energy conversion devices: A survey. Appl. Energy 297, 117100.
Guo, B., Ringwood, J.V., 2021b. A review of wave energy technology from a research and commercial perspective. IET Renew. Power Gener. 15 (14), 3065–3090.
Havelock, T.H., 1955. Waves due to a floating sphere making periodic heaving oscillations. Proc. R. Soc. Lond. Ser. A. Math. Phys. Sci. 231 (1184), 1–7.
Holland, J.H., 1992. Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence. MIT Press.
Hoos, H.H., Stvtzle, T., 2018. Stochastic local search. In: Handbook of Approximation Algorithms and Metaheuristics. Chapman and Hall/CRC, pp. 297–307.
Huang, X., Lin, Z., Chen, K., Zhou, J., Xiao, X., 2024. Improving wave-to-wire efficiency of direct-drive wave energy converters with loss-aware model predictive control and variable DC voltage control. IEEE Trans. Ind. Electron. , Early access.
Karani, M.M.H., Nikoo, M.R., Pirooz, H.D., Shadmani, A., Al-Saadi, S., Gandomi, A.H., 2024. Multi-objective evolutionary framework for layout and operational optimization of a multi-body wave energy converter. Energy 134045.
Kudela, J., Matousek, R., 2022. Recent advances and applications of surrogate models for finite element method computations: a review. Soft Comput. 26 (24), 13709–13733.
Leboucher, S., Fitzgerald, C.J., Ringwood, J.V., 2025. Surrogate modelling for wave energy hydrodynamics and optimisation. Appl. Ocean Res. In press.
Liao, Z., Li, G., Apsley, J., Iacchetti, M., Stansby, P.K., 2024. Power take-off co-design and LCoE reduction of a hinged-type wave energy converter using a genetic algorithm. Available At SSRN 4933133.
Lin, Z., Huang, X., Xiao, X., Ringwood, J.V., 2025. Fast optimal control performance evaluation for wave energy control co-design. Renew. Energy 239, 121974.
Lin, Z., Zhou, J., Huang, X., Chen, K., Xiao, X., Ringwood, J.V., 2024. On loss-aware optimal control of wave energy converters with electrical power take-offs. IEEE Trans. Sustain. Energy , Early access.
Ljung, L., 1999. System Identification: Theory for the User, 2nd ed. Prentice-Hall.
Ljung, L., 2010. Perspectives on system identification. Annu. Rev. Control. 34 (1), 1–12.
Meduri, A., Kang, H., 2024. Array optimization for a wave energy converter with adaptive resonance using dual Bayesian optimization. J. Mar. Sci. Eng. 12, 2143.
Meduri, A., Kang, H., 2025. Sequential design optimization with Bayesian approach for cost-competitive levelized cost of energy of a wave energy converter with adaptive resonance. Appl. Energy 382, 125166.
Mercadé Ruiz, P., Nava, V., Topper, M.B., Ruiz Minguela, P., Ferri, F., Kofoed, J.P., 2017. Layout optimisation of wave energy converter arrays. Energies 10 (9), 1262.
Mérigaud, A., Ringwood, J.V., 2017. A nonlinear frequency-domain approach for numerical simulation of wave energy converters. IEEE Trans. Sustain. Energy 9 (1), 86–94.
Morison, J.R., Johnson, J.W., Schaaf, S.A., 1950. The force exerted by surface waves on piles. J. Pet. Technol. 2 (05), 149–154.
Mueller, M., 2002. Electrical generators for direct drive wave energy converters. IEE Proc., Gener. Transm. Distrib. 149 (4), 446–456.
Neshat, M., Alexander, B., Sergiienko, N.Y., Wagner, M., 2020. New insights into position optimisation of wave energy converters using hybrid local search. Swarm Evol. Comput. 59, 100744.
Nesterov, Y., 2013. Gradient methods for minimizing composite functions. Math. Program. 140 (1), 125–161.
O'Connor, M., Lewis, T., Dalton, G., 2013. Operational expenditure costs for wave energy projects and impacts on financial returns. Renew. Energy 50, 1119–1131.
Olsson, D.M., Nelson, L.S., 1975. The Nelder-Mead simplex procedure for function minimization. Technometrics 17 (1), 45–51.
Ozkop, E., Altas, I.H., 2017. Control, power and electrical components in wave energy conversion systems: A review of the technologies. Renew. Sustain. Energy Rev. 67, 106–115.
Paduano, B., Pasta, E., Carapellese, F., Papini, G., Baltazar, J., Faedo, N., Mattiazzo, G., 2023. Control co-design mooring optimisation for wave energy systems: a three-tethered point absorber case. IFAC- Pap. 56 (2), 11717–11722.
Papillon, L., Costello, R., Ringwood, J.V., 2020. Boundary element and integral methods in potential flow theory: A review with a focus on wave energy applications. J. Ocean. Eng. Mar. Energy 6 (3), 303–337.
Papillon, L., Wang, L., Tom, N., Weber, J., Ringwood, J., 2019. Parametric modelling of a reconfigurable wave energy device. Ocean Eng. 186, 106105.
Papini, G., Paduano, B., Pasta, E., Carapellese, F., Mattiazzo, G., Faedo, N., 2023. On the influence of mooring systems in optimal predictive control for wave energy converters. Renew. Energy 218, 119242.
Parisini, T., Zoppoli, R., 1995. A receding-horizon regulator for nonlinear systems and a neural approximation. Automatica 31 (10), 1443–1451.
Pasta, E., Carapellese, F., Mattiazzo, G., 2021. Deep neural network trained to mimic nonlinear economic model predictive control: An application to a pendulum wave energy converter. In: 2021 IEEE Conference on Control Technology and Applications (CCTA), San Diego. pp. 295–300.
Peña-Sanchez, Y., Centeno-Telleria, M., Zarketa-Astigarraga, A., Penalba, M., García-Violini, D., Faedo, N., Ringwood, J., 2024a. Efficient wave energy converter optimisation via control co-design: A comparison of AI-based algorithms. In: Innovations in Renewable Energies Offshore, Lisbon. CRC Press, pp. 535–542.
Peña-Sanchez, Y., Faedo, N., Ringwood, J.V., 2019. A critical comparison between parametric approximation methods for radiation forces in wave energy systems. In: Proc. ISOPE International Ocean and Polar Engineering Conference, Hawaii. pp. 174–181.
Peña-Sanchez, Y., García-Violini, D., Penalba, M., Zarketa-Astigarraga, A., Ferri, F., Nava, V., Ringwood, J.V., 2024b. Control co-design for wave energy farms: Optimisation of array layout and mooring configuration in a realistic wave climate. Renew. Energy 227, 120506.
Peña-Sanchez, Y., García-Violini, D., Ringwood, J.V., 2022. Control co-design of power take-off parameters for wave energy systems. IFAC- Pap. 55 (27), 311–316.
Penalba, M., Cortajarena, J.-A., Ringwood, J.V., 2017. Validating a wave-to-wire model for a wave energy converter—Part II: The electrical system. Energies 10 (7), 1002.
Penalba, M., Ringwood, J.V., 2016. A review of wave-to-wire models for wave energy converters. Energies 9 (7), 506.
Penalba, M., Ringwood, J.V., 2020. Systematic complexity reduction of wave-to-wire models for wave energy system design. Ocean Eng. 217, 107651.
Pennock, S., Coles, D., Angeloudis, A., Bhattacharya, S., Jeffrey, H., 2022. Temporal complementarity of marine renewables with wind and solar generation: implications for GB system benefits. Appl. Energy 319, 119276.

Pierre, D.A., 2012. Optimization Theory with Applications. Courier Corporation.
Qian, N., 1999. On the momentum term in gradient descent learning algorithms. Neural Netw. 12 (1), 145–151.
Rasool, S., Muttaqi, K.M., Sutanto, D., 2022. A multi-filter based dynamic power sharing control for a hybrid energy storage system integrated to a wave energy converter for output power smoothing. IEEE Trans. Sustain. Energy 13 (3), 1693–1706.
Reguero, B., Losada, I., Méndez, F., 2015. A global wave power resource and its seasonal, interannual, and long-term variability. Appl. Energy 148, 366–380.
Ringwood, J.V., 2022. Marine renewable energy devices and their control: An overview. IFAC- Pap. 55 (31), 136–141.
Ringwood, J.V., Mérigaud, A., Faedo, N., Fusco, F., 2019. An analytical and numerical sensitivity and robustness analysis of wave energy control systems. IEEE Trans. Control Syst. Technol. 28 (4), 1337–1348.
Ringwood, J.V., Tom, N., Ferri, F., Yu, Y.-H., Coe, R.G., Ruehl, K., Bacelli, G., Shi, S., Patton, R.J., Tona, P., et al., 2023a. The wave energy converter control competition (WECCCOMP): Wave energy control algorithms compared in both simulation and tank testing. Appl. Ocean Res. 138, 103653.
Ringwood, J.V., Zhan, S., Faedo, N., 2023b. Empowering wave energy with control technology: Possibilities and pitfalls. Annu. Rev. Control. 55, 18–44.
Ropero-Giralda, P., Crespo, A.J., Tagliafierro, B., Altomare, C., Domínguez, J.M., Gómez-Gesteira, M., Viccione, G., 2020. Efficiency and survivability analysis of a point-absorber wave energy converter using DualSPHysics. Renew. Energy 162, 1763–1776.
Rosati, M., Ringwood, J.V., 2023. Control co-design of power take-off and bypass valve for OWC-based wave energy conversion systems. Renew. Energy 219, 119523.
Said, H.A., García-Violini, D., Ringwood, J.V., 2022. Wave-to-grid (W2G) control of a wave energy converter. Energy Convers. Management: X 14, 100190.
Said, H.A., Ringwood, J.V., 2021. Grid integration aspects of wave energy—Overview and perspectives. IET Renew. Power Gener. 15 (14), 3045–3064.
Sergiienko, N.Y., Bacelli, G., Coe, R.G., Cazzolato, B.S., 2022. A comparison of efficiency-aware model-predictive control approaches for wave energy devices. J. Ocean. Eng. Mar. Energy 8, 17–29.
Sergiienko, N.Y., Neshat, M., Da Silva, L.S., Alexander, B., Wagner, M., 2020. Design optimisation of a multi-mode wave energy converter. In: Intl. Conf. on Offshore Mechanics and Arctic Eng. (OMAE), Fort Lauderdale. vol. 84416, ASME, V009T09A039.
Stavrololou, C., Katsidoniotaki, Faedo, N., Goteman, M., 2025. Multi-fidelity surrogate modeling of nonlinear dynamic responses in wave energy farms. Appl. Energy 380, 125011.
Ströfer, C.A.M., Gaebele, D.T., Coe, R.G., Bacelli, G., 2023. Control co-design of power take-off systems for wave energy converters using WecOptTool. IEEE Trans. Sustain. Energy 14, 2157–2167.
Teillant, B., Costello, R., Weber, J., Ringwood, J., 2012. Productivity and economic assessment of wave energy projects through operational simulations. Renew. Energy 48, 220–230.
Terra, G.M., van de Berg, W.J., Maas, L.R., 2005. Experimental verification of Lorentz' linearization procedure for quadratic friction. Fluid Dyn. Res. 36 (3), 175–188.
Ulazia, A., Penalba, M., Ibarra-Berastegui, G., Ringwood, J., Sáenz, J., 2019. Reduction of the capture width of wave energy converters due to long-term seasonal wave energy trends. Renew. Sustain. Energy Rev. 113, 109267.
Visbech, J., Engsig-Karup, A.P., Bingham, H.B., Amini-Afshar, M., Ricchiuto, M., 2024. A high-order shifted boundary method for water waves and floating bodies. In: 39th International Workshop on Water Waves and Floating Bodies, St. Andrews.
Vu, T.T., O'Driscoll, S., Ringwood, J.V., 2013. Nonlinear dynamic transformer time-domain identification for power converter applications. IEEE Trans. Power Electron. 29 (1), 318–327.
Wang, L., Ringwood, J.V., 2021. Control-informed ballast and geometric optimisation of a three-body hinge-barge wave energy converter using two-layer optimisation. Renew. Energy 171, 1159–1170.
Williams, R., 2010. Improving exhaustive search implies superpolynomial lower bounds. In: Proc. of the 42nd ACM Symposium on Theory of Computing. pp. 231–240.
Wilson, B., Cappelleri, D., Simpson, T.W., Frecker, M., 2001. Efficient Pareto frontier exploration using surrogate approximations. Optim. Eng. 2, 31–50.
Windt, C., Davidson, J., Ringwood, J.V., 2018. High-fidelity numerical modelling of ocean wave energy systems: A review of computational fluid dynamics-based numerical wave tanks. Renew. Sustain. Energy Rev. 93, 610–630.
Windt, C., Faedo, N., Penalba, M., Dias, F., Ringwood, J.V., 2021. Reactive control of wave energy devices–the modelling paradox. Appl. Ocean Res. 109, 102574.
Yeung, R.W., 1981. Added mass and damping of a vertical cylinder in finite-depth waters. Appl. Ocean Res. 3 (3), 119–133.
Zeiler, M.D., 2012. ADADELTA: an adaptive learning rate method. arXiv preprint arXiv:1212.5701.
Zhang, J., Taflanidis, A.A., Scruggs, J.T., 2020. Surrogate modeling of hydrodynamic forces between multiple floating bodies through a hierarchical interaction decomposition. J. Comput. Phys. 408, 109298.
Ziaei, A., Said, H., Ringwood, J., 2024. Health-sensitive control of wave energy converters: A primer. Ocean Eng. 311, 118893.