



A Robust Optimal Control for Docking and Charging Unmanned Underwater Vehicles Powered by Wave Energy

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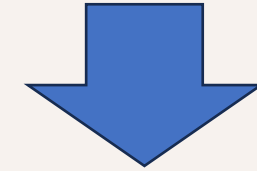
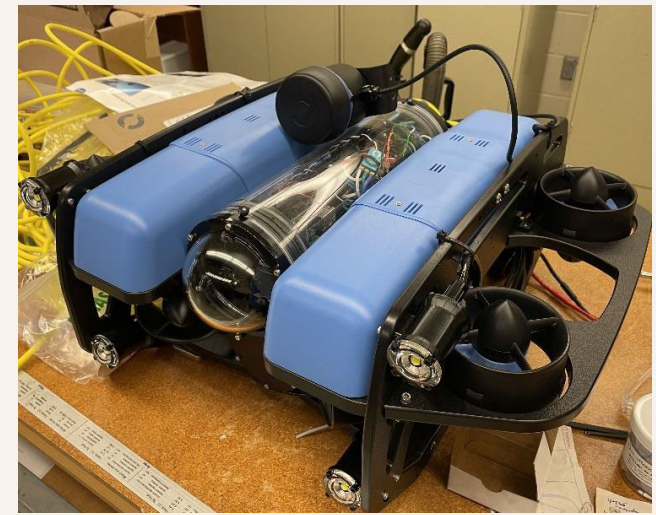
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Introduction

- Unmanned Underwater Vehicles (UUV)
 - Increasingly used to **conduct dangerous and remote missions** such as ocean mapping, offshore structure maintenance, detecting and clearing mines, maritime security, recovery, and so forth.
 - Encounter significant **operational challenges**, including the need for manual retrieval, recharging, and redeployment.

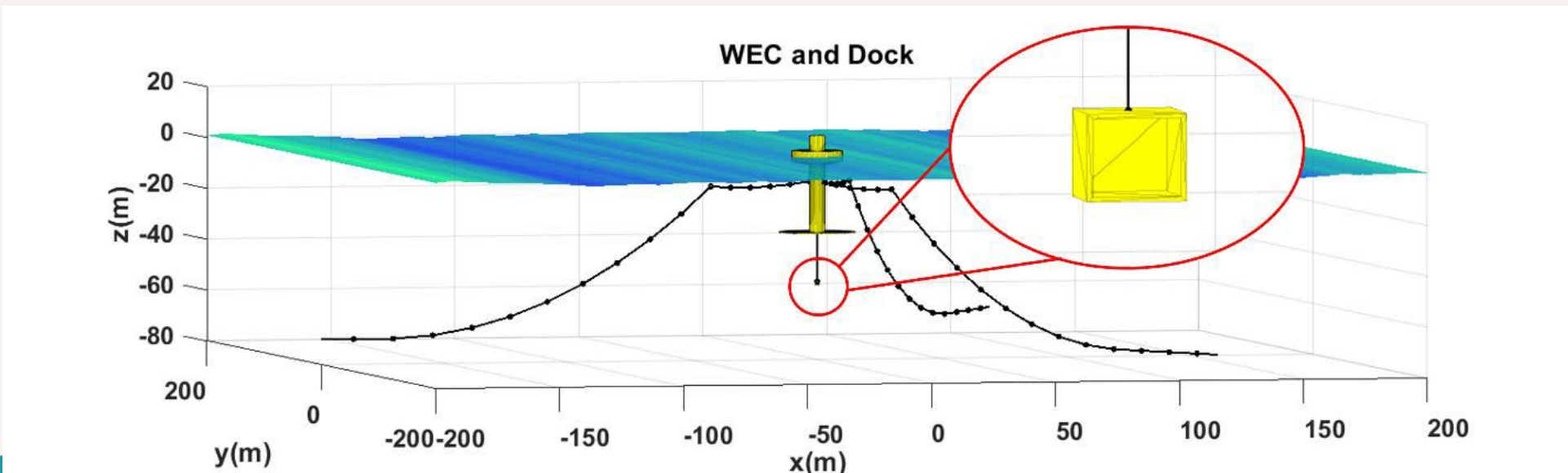


Mooring line vision-based fault diagnosis (one of the focus applications)



Introduction

- Wave Energy Converters can be used to charge the UUVs
 - Wave Energy is abundant, consistent, and offers high-power density.
 - WEC can be coupled with a docking/charging station beneath to continuously supply wave power to UUVs.
 - This can enable on-demand, at-sea, and autonomous recharging and surface communication for UUV.



Introduction

- **Motivation**

- Autonomous control of UUV coupled with WEC system is insufficiently studied.
- Lack of an accurate and efficient integrated simulation framework to describe WEC-UUV behavior.
- Lack of an effective and robust control system that can guide the UUV to dock with the docking station with minimal fuel consumption and time, subject to uncertainties and disturbances.

- **Research Focus**

- Develop a detailed simulation framework that integrates the WEC, docking station, and UUV, which can simulate the UUV docking performance efficiently and accurately.
- Develop a robust optimal control to optimize the UUV docking performance subject to dynamic ocean environments and uncertainties.



Methodology: WEC modeling

- A generic point absorber WEC, Reference Model 3 (RM3), is used to represent the WEC.

Cummins Equation (Newtons 2nd Law)

$$(\mathbf{M}_r + \mathbf{M}_\infty)\ddot{\vec{x}} = \vec{F}_e + \vec{F}_{PTO} + \vec{F}_r + \vec{F}_s + \vec{F}_m + \vec{F}_v + \vec{F}_c$$

- $\vec{x} = [x, y, z, \phi, \theta, \psi]$ is the state vector which represents the 6 DoF displacement in the body-fixed frame.
- The mooring force is simulated by using MoorDyn.

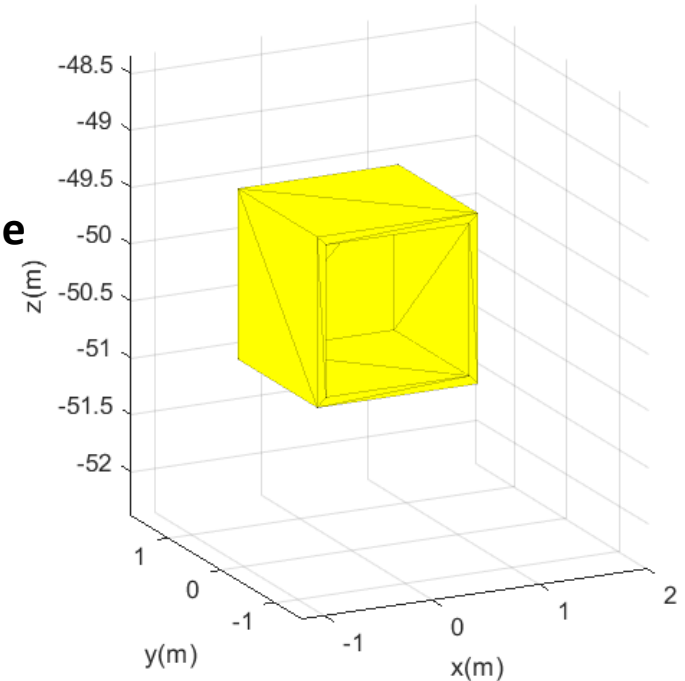
- Docking station is modelled as a **drag body** in WecSim
- The hydrostatic force, cable reaction forces, and quadratic drag force are considered.
- The hydrodynamic forces are neglected given that they are deeply submerged in the water

Docking Station

Mass (kg)	1064.621
Volume (m^3)	0.394
Inertia [I_x, I_y, I_z] ($kg\ m^2$)	[733.352, 733.352, 457.979]
Quadratic Drag Area (m^2)	[2.25, 2.25, 2.25, 0, 0, 0]
Quadratic Drag Coefficient (Cd)	[1.2, 1.2, 1.2, 0, 0, 0]
Center of Gravity (m) in global frame	[0, 0, -50]
Center of Buoyancy (m) in body fixed frame	[0, 0, 0]

Force between the WEC and the cocking station

Wave Elevation and Geometry Visualization



Suspending cable

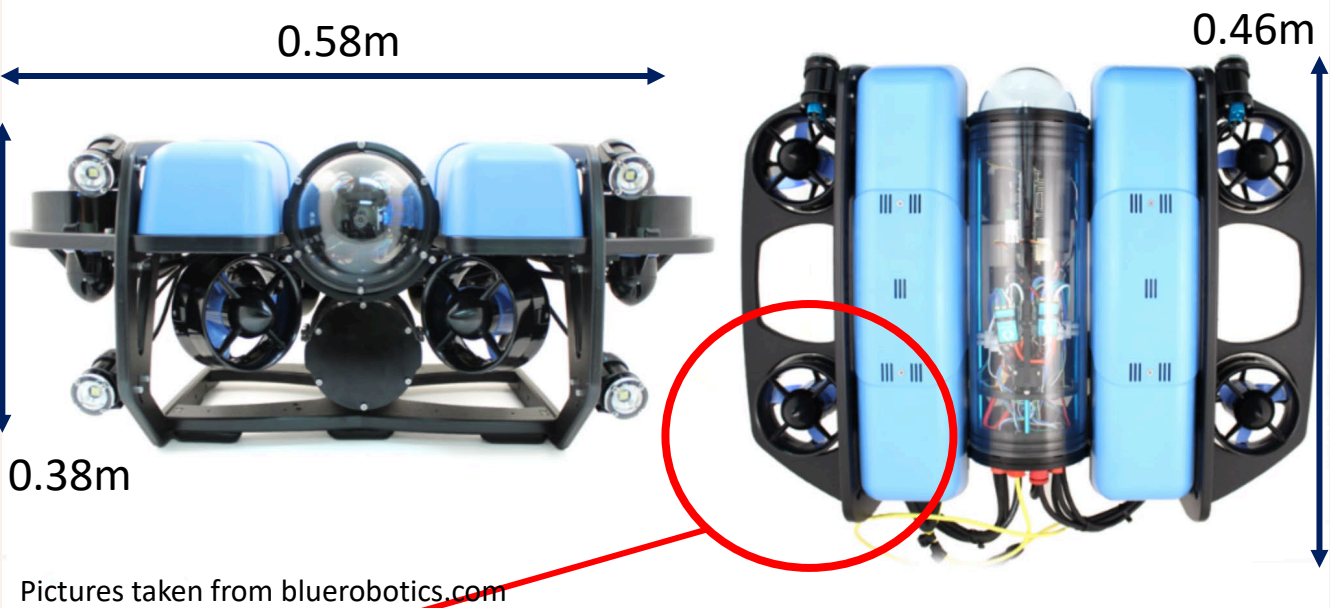
Stiffness	1000000
Damping	100
Quadratic Drag Area (m^2)	[10, 10, 10, 0, 0, 0]
Quadratic Drag Coefficient (Cd)	[1.4, 1.4, 1.4, 0, 0, 0]
Cable top coordinates in global frame (m)	[0, 0, -30]
Cable bottom coordinates in global frame (m)	[0, 0, -49.25]

Methodology: UUV modeling

Nonlinear Dynamics of UUV

$$\dot{\vec{\eta}} = J(\vec{\eta})\vec{v}$$
$$\mathbf{M}\dot{\vec{v}} + \mathbf{C}(\vec{v}) + \mathbf{D}(\vec{v})\vec{v} + \mathbf{g}(\vec{\eta}) = \vec{\tau}$$

- $\vec{\eta} = [x, y, z, \phi, \theta, \psi]$ represents position of the UUV in the global frame
- $\vec{v} = [u, v, w, p, q, r]$ denotes the velocity of the vehicle expressed in the body-fixed frame.
- The model parameters are calibrated with experiments in [1].



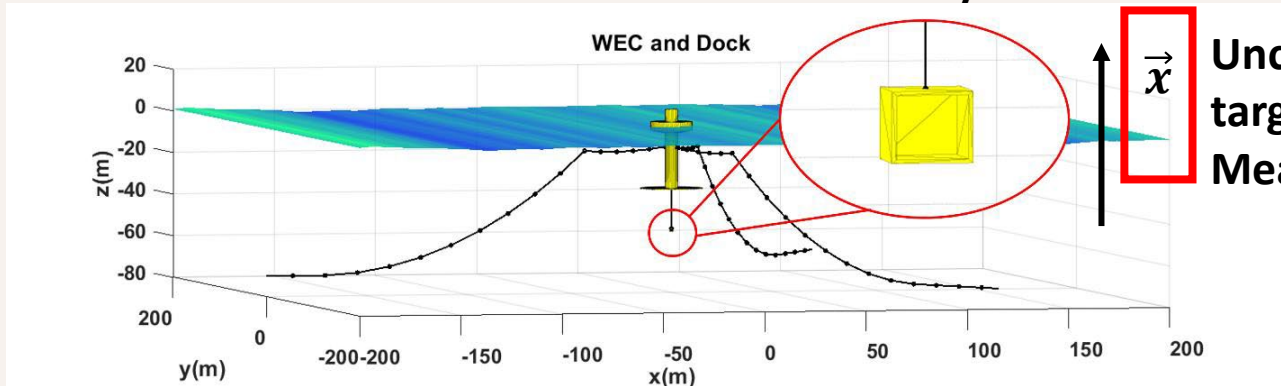
Direction	Max Thrust
Surge	85 N
Sway	85 N
Heave	120 N
Roll	26 Nm
Pitch	14 Nm
Yaw	22 Nm

Mass (kg)	13.5
Moment of Inertia (kg m^2)	(0.26, 0.23, 0.37)
Damping Coefficients (Nsm^(-1))	(137.7, 0, 33, 0, 0.8, 0)
Quadratic Damping Coefficients (Ns^2 m^(-2))	(141, 217, 190, 1.19, 0.47, 1.5)
Added Mass (kg)	(6.36, 7.12, 18,68)
Added Mass Moment (kg m^2)	(0.189, 0.135, 0.222)

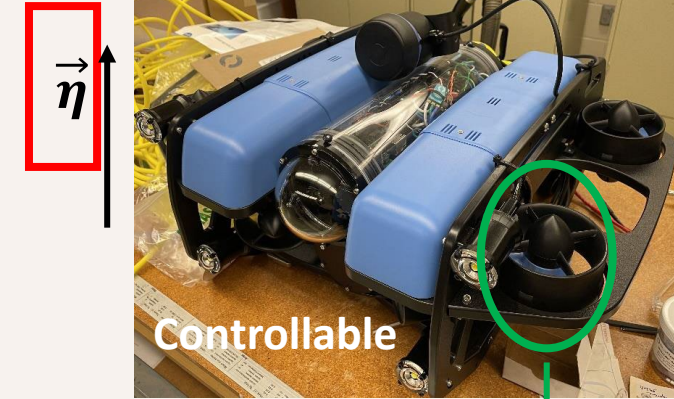


[1] von Benzon, Malte, et al. "An open-source benchmark simulator: Control of a bluerov2 underwater robot." *Journal of Marine Science and Engineering* 10.12 (2022): 1898

Methodology: Docking control



Position/Orientation of the UUV



- The control problem can be formulated as:

$$\text{Min: } \vec{e} = \vec{\eta} - \vec{x}$$

Subject to:

$$\dot{\vec{x}}_1 = \mathbf{J}(\vec{x}_1) \vec{x}_2$$

$$\dot{\vec{x}}_2 = -\mathbf{M}^{-1}(\mathbf{C}(\vec{x}_2) + \mathbf{D}(\vec{x}_2) \vec{x}_2 + \mathbf{g}(\vec{x}_1)) + \Delta f(\vec{x}_1, \vec{x}_2) + \mathbf{M}^{-1} \left(\vec{\tau} + \delta(t, \vec{x}_1, \vec{x}_2, \vec{\tau}) \right)$$

Model Uncertainties

Actuation disturbances

Control that needs to be designed

- It is difficult to design a linear control subject to this complex dynamics, which also cannot regulate the uncertainties and disturbances. An **input-output linearization technique** is therefore applied.



Methodology: Input-Output Linearization

Nonlinear state transformation

- Define **state**:

$$\vec{e}_1 = \vec{\eta} - \vec{x}$$

$$\vec{e}_2 = \dot{\vec{\eta}} - \dot{\vec{x}}$$

- According to the UUV dynamics before, we have:

$$\dot{\vec{e}}_1 = \vec{e}_2$$

$$\dot{\vec{e}}_2 = \mathbf{S}_1 \mathbf{J}(\vec{x}_1) \vec{x}_2 + \mathbf{S}_2 \left(-\mathbf{M}^{-1} \left(\mathbf{C}(\vec{x}_2) + \mathbf{D}(\vec{x}_2) \vec{x}_2 + \mathbf{g}(\vec{x}_1) \right) + \Delta f(\vec{x}_1, \vec{x}_2) + \mathbf{M}^{-1} \left(\vec{\tau} + \delta(t, \vec{x}_1, \vec{x}_2, \vec{\tau}) \right) \right) - \ddot{\vec{x}}$$

where

$$\mathbf{S}_1 = \frac{\partial \mathbf{J}(\vec{x}_1)}{\partial \vec{x}_1} (\vec{x}_2 \otimes \mathbf{I}_n) + \mathbf{J}(\vec{x}_1) \frac{\partial \vec{x}_2}{\partial \vec{x}_1}$$

$$\mathbf{S}_2 = \frac{\partial \mathbf{J}(\vec{x}_1)}{\partial \vec{x}_2} (\vec{x}_2 \otimes \mathbf{I}_n) + \mathbf{J}(\vec{x}_1) \frac{\partial \vec{x}_2}{\partial \vec{x}_2}$$

Obtained a clean and neat linearized system

$$\dot{\vec{e}} = \mathbf{A} \vec{e} + \Delta \mathbf{A} + \mathbf{B} \vec{u} + \Delta \delta$$

$$\mathbf{A} = \begin{bmatrix} 0_{6 \times 6} & \mathbf{I}_6 \\ 0_{6 \times 6} & 0_{6 \times 6} \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0_{6 \times 6} \\ \mathbf{I}_6 \end{bmatrix}$$

$$\Delta \mathbf{A} = \begin{bmatrix} 0_{6 \times 1} \\ \mathbf{S}_2 \Delta f \left(\begin{matrix} \vec{x}_1 \\ \vec{x}_2 \end{matrix} \right) \end{bmatrix}$$

$$\Delta \delta = \begin{bmatrix} 0_{6 \times 1} \\ \mathbf{S}_2 \mathbf{M}^{-1} \delta \left(t, \vec{x}_1, \vec{x}_2, \vec{\tau} \right) \end{bmatrix}$$

Transformed control,
which we will design next

Apply the Input-Output Linearization control law:

$$\vec{\tau} = (\mathbf{S}_2 \mathbf{M}^{-1})^{-1} (-\mathbf{S}_1 \mathbf{J}(\vec{x}_1) \vec{x}_2 + \mathbf{S}_2 \mathbf{M}^{-1} (\mathbf{C}(\vec{x}_2) + \mathbf{D}(\vec{x}_2) \vec{x}_2 + \mathbf{g}(\vec{x}_1)) + \ddot{\vec{x}} + \vec{u})$$

Thrust force (physically)



Methodology: Robust LQR

Now we need to design the \vec{u}

$$\dot{\vec{e}} = \mathbf{A}\vec{e} + \Delta\mathbf{A} + \mathbf{B}\vec{u} + \Delta\delta$$

Proposed control algorithm:

$$\begin{aligned}\vec{u}(t) &= \vec{u}_l(t) + \vec{u}_s(t) \\ \vec{u}_l(t) &= -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}\vec{e}(t) \\ \vec{u}_s(t) &= -(\mathbf{GB})^{-1}\gamma \text{sign}(\vec{s})\end{aligned}$$

- $\vec{u}_l(t)$ is the continuous part which is used to optimize the performance of the nominal error system
- $\vec{u}_s(t)$ is the discontinuous part which guarantees the robustness.

Need to transform back to the actual thrust!



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$$J = \frac{1}{2} \int_0^T [\vec{e}_n(t)^T \mathbf{Q} \vec{e}_n(t) + \vec{u}_l(t)^T \mathbf{R} \vec{u}_l(t)] dt$$

Minimize the
docking time

Minimize the fuel
consumption

According to the optimal control theory:

$$\vec{u}_l(t) = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}\vec{e}_n(t)$$

- The \mathbf{P} represent the covariance matrix which is the solution of the algebraic Riccati equation (ARE).

- Sliding Mode Control is applied to regular the uncertainties and disturbances:

$$\vec{u}_s(t) = -(\mathbf{GB})^{-1}\gamma \text{sign}(\vec{s})$$

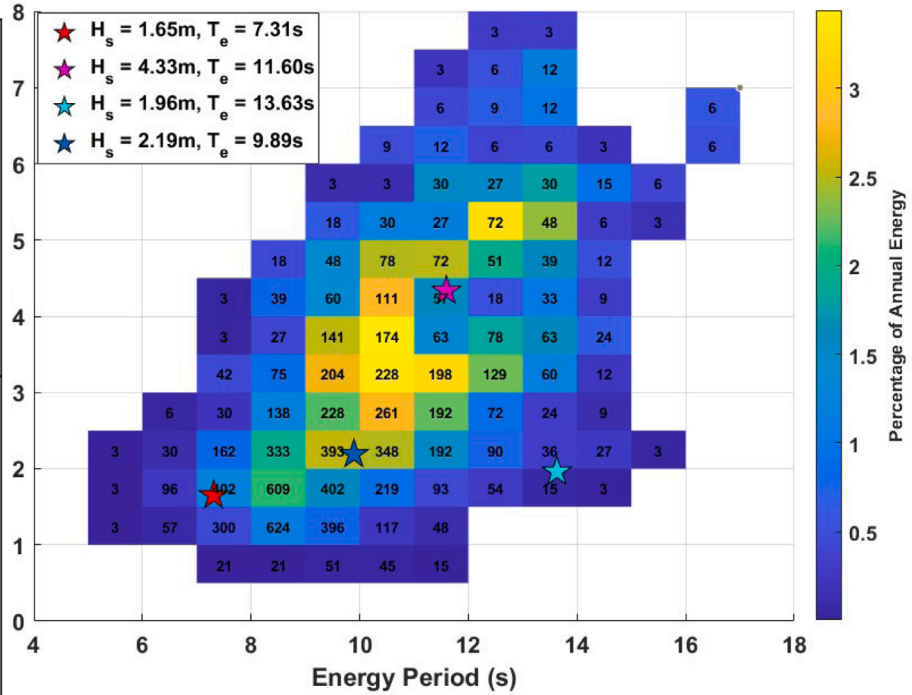
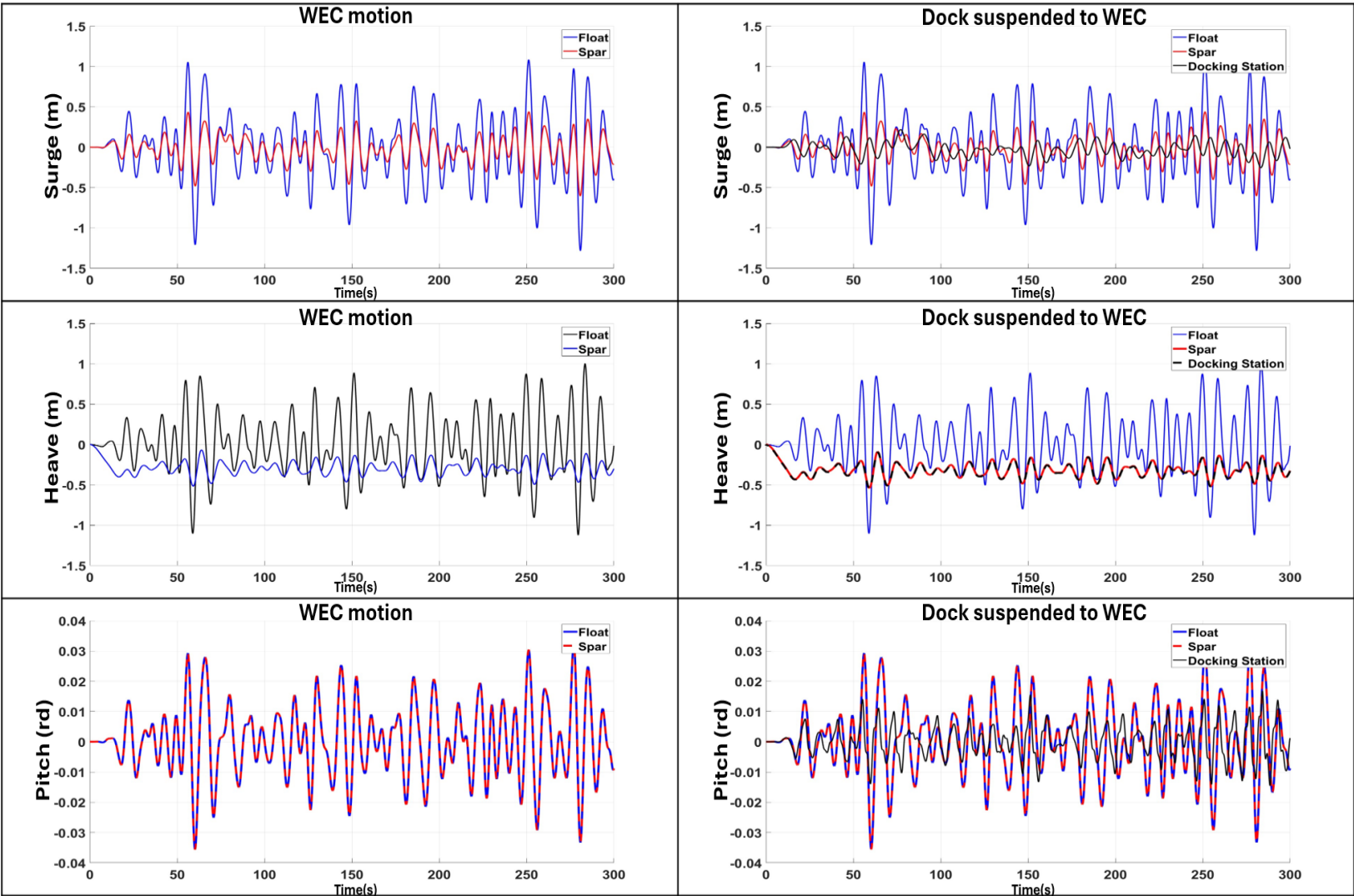
- where the sliding surface is designed as:

$$\begin{aligned}\vec{s}(t, \vec{e}) \\ = \mathbf{G}\vec{e}(t) - \mathbf{G}\vec{e}(0) - \mathbf{G} \int_0^t (\mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P})\vec{e}(\tau) d\tau\end{aligned}$$

where $\mathbf{G} \in \mathbf{R}^{6 \times 12}$ is a constant matrix which is designed such that \mathbf{GB} is nonsingular.

Simulation results: Dock motion analysis

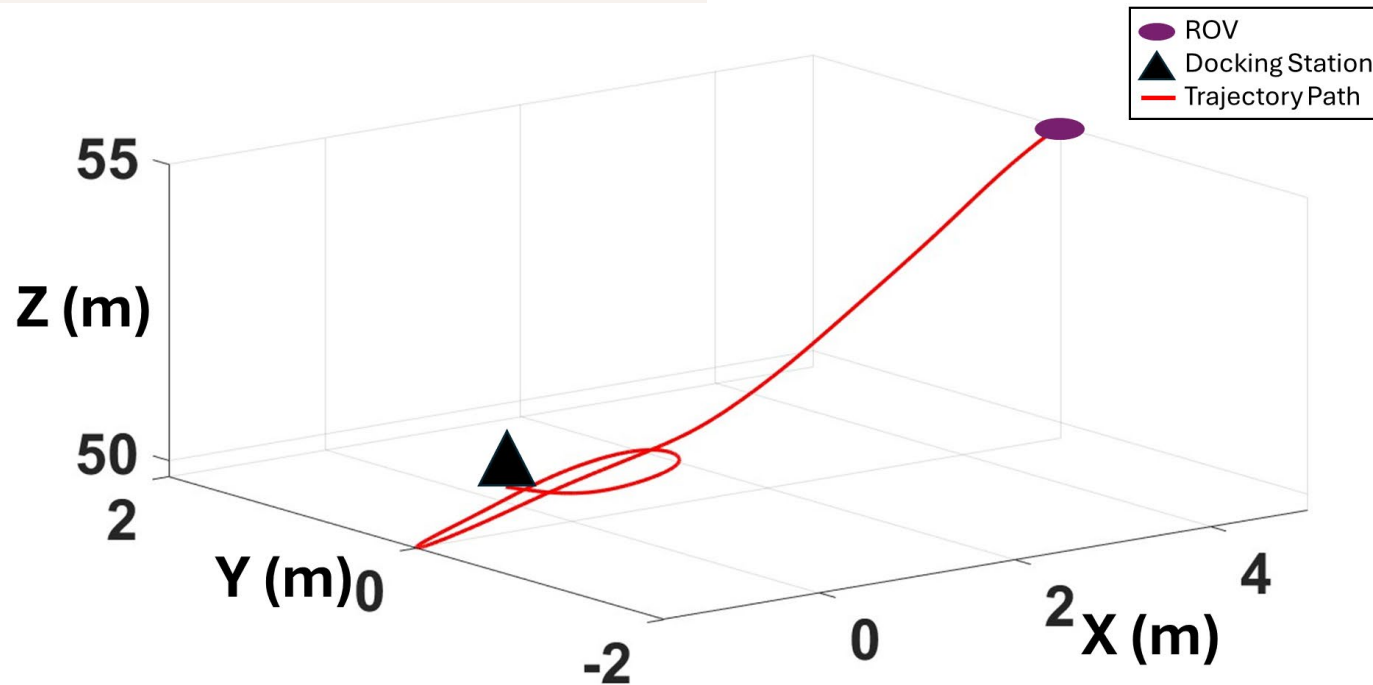
Hs = 1.65m, Tp = 8.91s



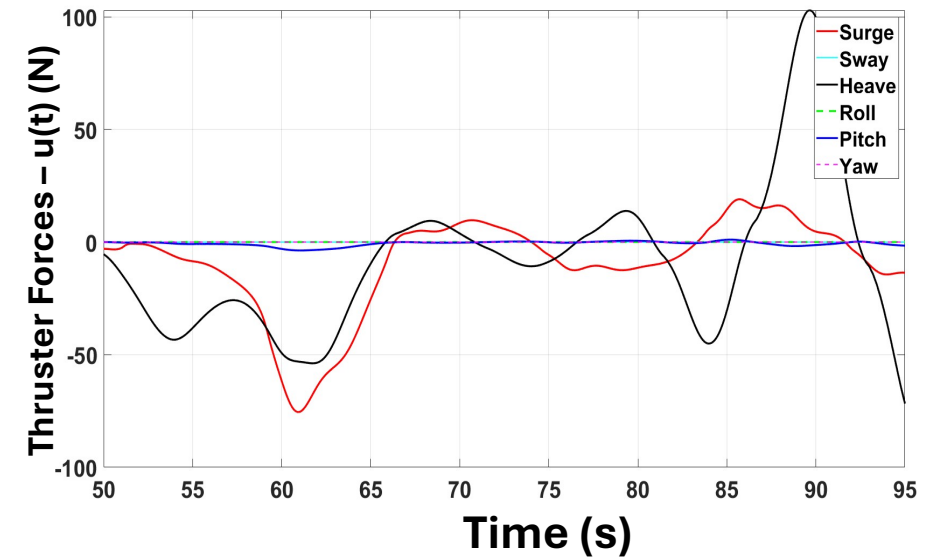
	Hs = 1.65 m, Tp = 8.91s	Hs = 4.33 m, Tp = 13.97s
Surge (m)	[0.26, -0.27]	[1.73, -1.98]
Heave (m)	[0, -0.358]	[1.02, -1.75]
Pitch (rad)	[0.0189, -0.176]	[0.0712, 0.773]

Simulation results: Docking in ideal scenario

Docking trajectory of the UUV



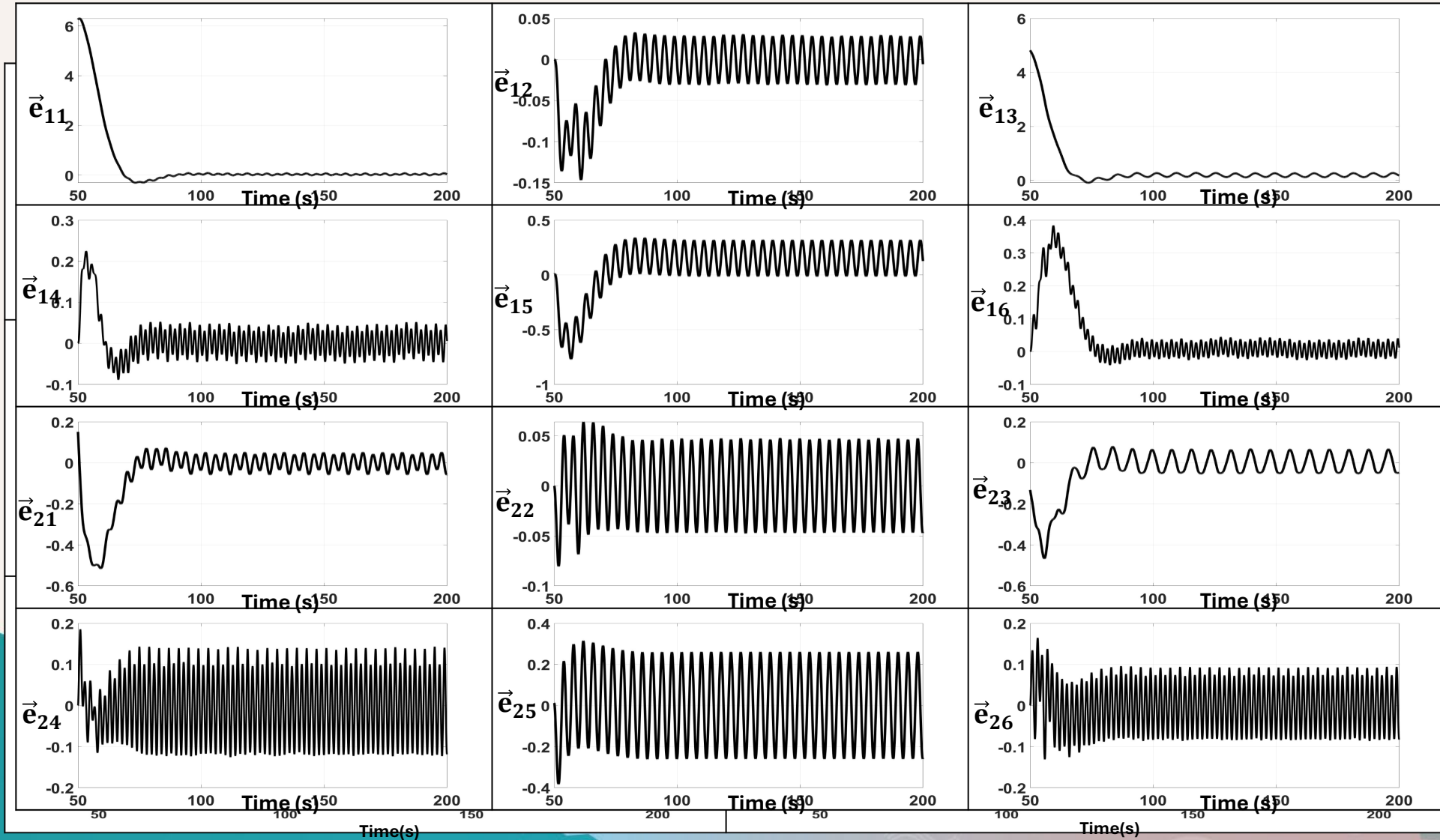
Thruster forces are within the limit



Simulation results: Docking with disturbances (LQR only)

Disturbance signals are added to the dynamics (all 6DoF) which are unaware to the control:

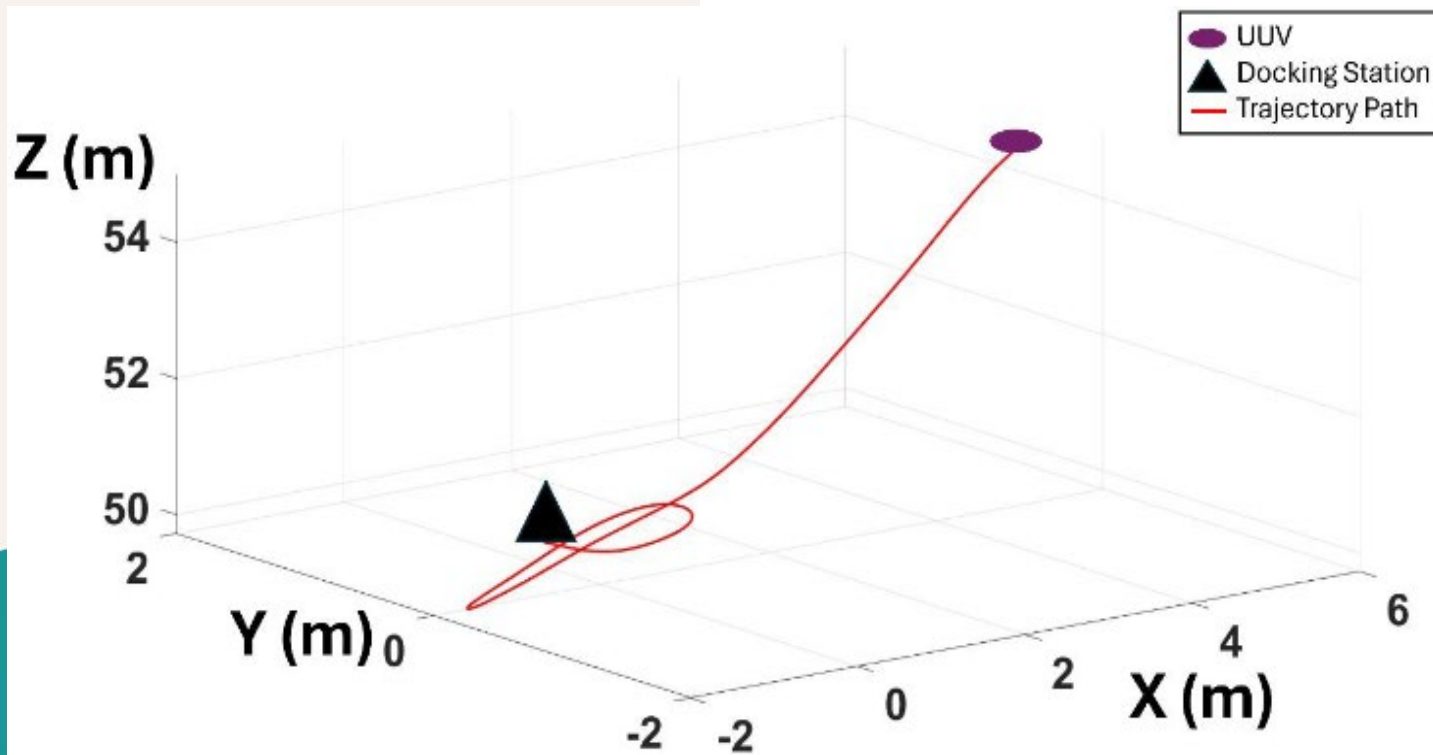
$$d(t) = [1.5 \sin 0.5\pi t; 1.5 \sin 0.5\pi t; 1.5 \sin 0.25\pi t; 0.15 \sin \pi t; 0.15 \sin 0.5\pi t; 0.15 \sin \pi t]$$



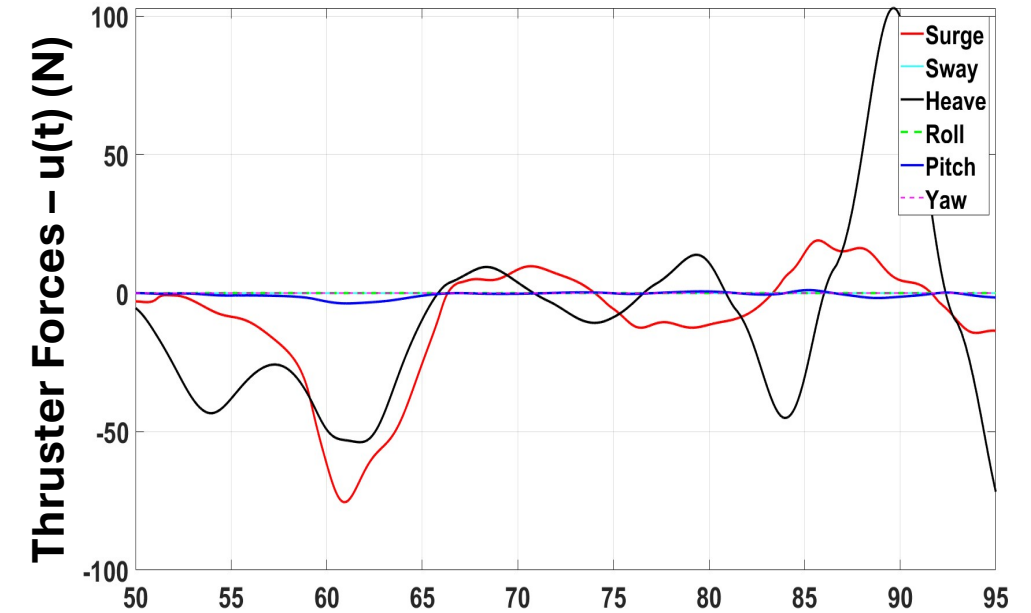
Simulation results: Docking with disturbances (RLQR)

Same disturbances are still applied in this case, but the RLQR control is applied

Docking trajectory of the UUV



Thruster forces are within the limit

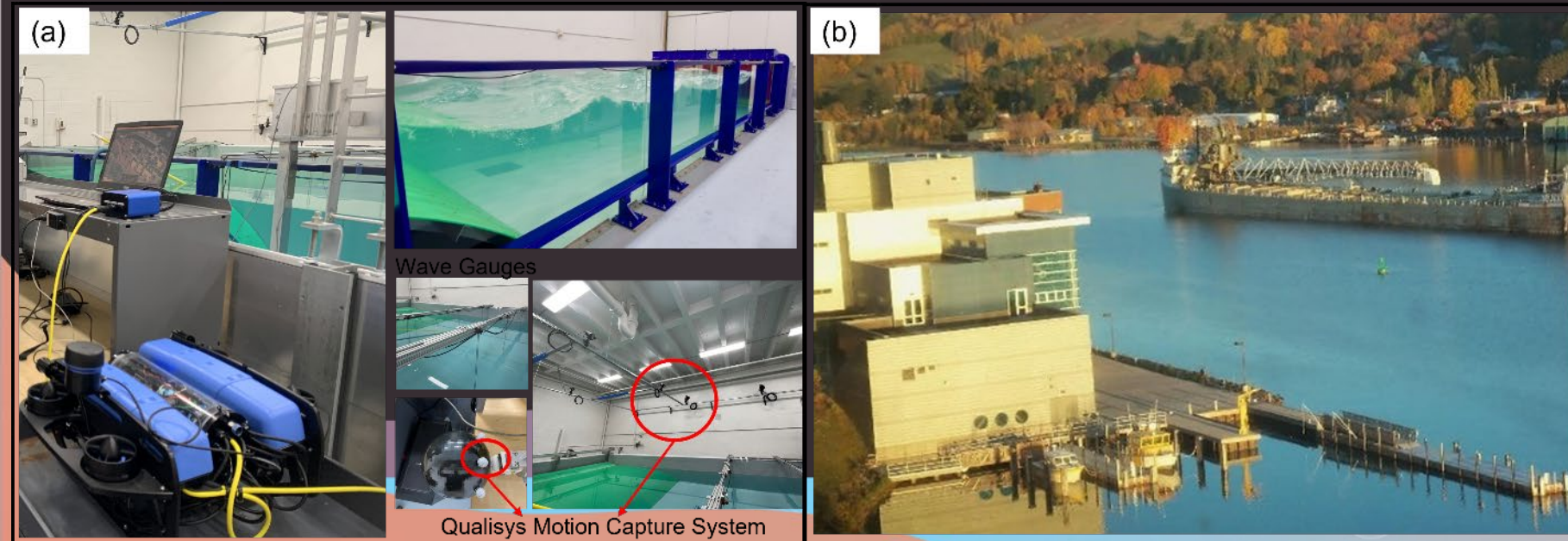
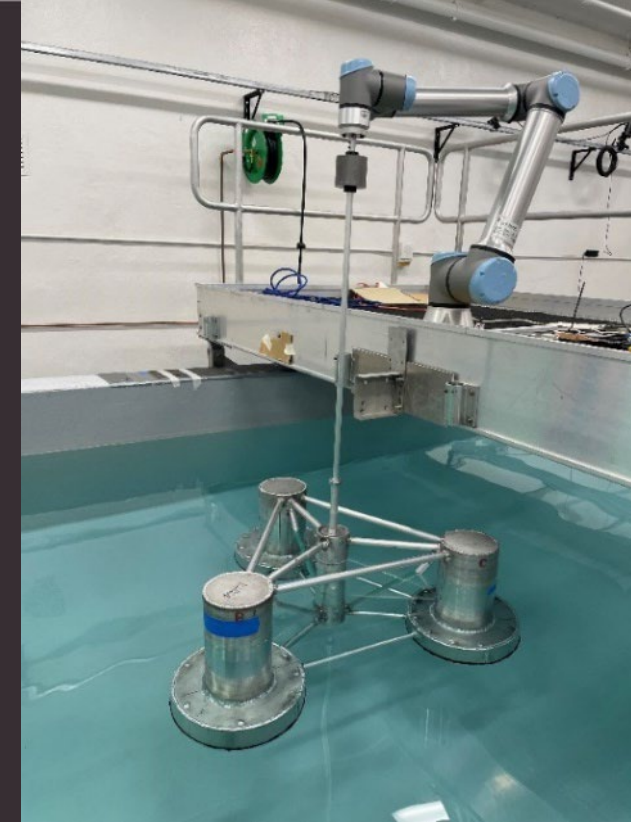


Conclusion

- Docking station motion is significant when it is hanged with the WEC which poses challenges for docking.
- It is critical to consider both the optimality and robustness in designing the docking control.
- ROC effectively controls the UUV to achieve an optimal docking performance in the presence of uncertainties and disturbances.

Future work

- The hydrodynamic model will be improved by incorporating the tether force and ocean current force.
- Experimentally validate the docking control in wave lab at MTU
- Apply UUV for mooring line inspections/fault diagnosis (camera vision-based)



Acknowledgement:

Dr. Shangyan Zou and Abishek Subramanian are supported by NSF under grant CMMI – 2138522.

