



A Robust Optimal Control for Docking and Charging Unmanned Underwater Vehicles Powered by Wave Energy

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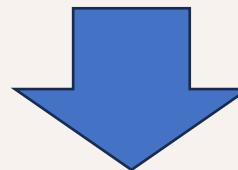
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Introduction

- Unmanned Underwater Vehicles (UUV)

- Increasingly used to **conduct dangerous and remote missions** such as ocean mapping, offshore structure maintenance, detecting and clearing mines, maritime security, recovery, and so forth.
- Encounter significant **operational challenges**, including the need for manual retrieval, recharging, and redeployment.

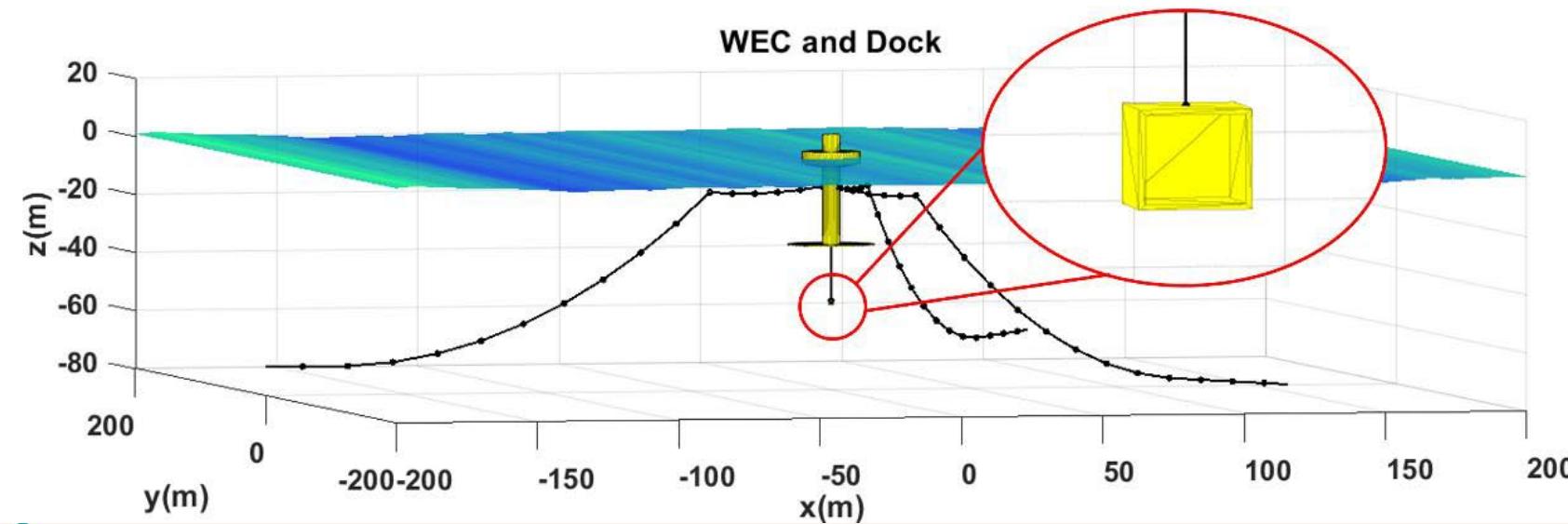


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Introduction

- Wave Energy Converters can be used to charge the UUVs
 - Wave Energy is abundant, consistent, and offers high-power density.
 - WEC can be coupled with a docking/charging station beneath to continuously supply wave power to UUVs.
 - This can enable on-demand, at-sea, and autonomous recharging and surface communication for UUV.



Introduction

- Motivation

- Autonomous control of UUV coupled with WEC system is insufficiently studied.
- Lack of an accurate and efficient integrated simulation framework to describe WEC-UUV behavior.
- Lack of an effective and robust control system that can guide the UUV to dock with the docking station with minimal fuel consumption and time, subject to uncertainties and disturbances.

- Research Focus

- Develop a detailed simulation framework that integrates the WEC, docking station, and UUV, which can simulate the UUV docking performance efficiently and accurately.
- Develop a robust optimal control to optimize the UUV docking performance subject to dynamic ocean environments and uncertainties.



Methodology: WEC modeling

- A generic point absorber WEC, Reference Model 3 (RM3), is used to represent the WEC.

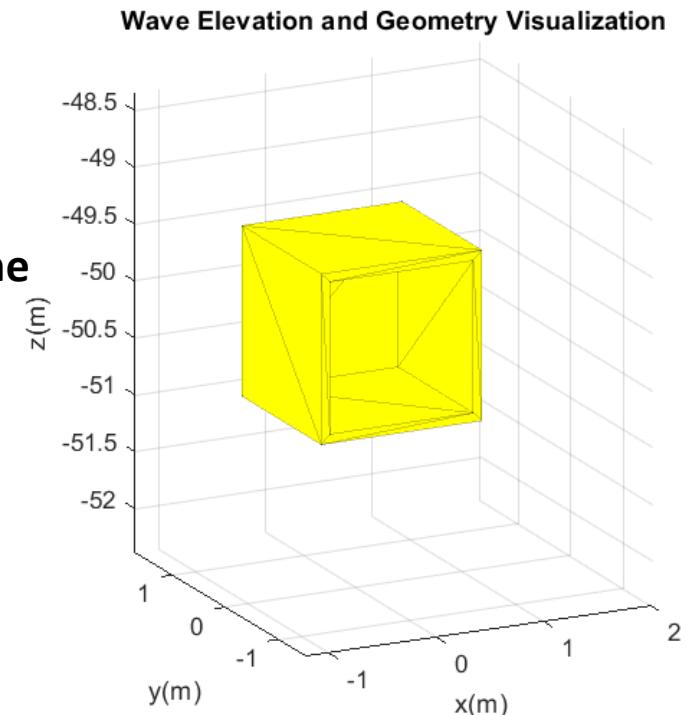
Cummins Equation (Newtons 2nd Law)

$$(M_r + M_\infty) \ddot{\vec{x}} = \vec{F}_e + \vec{F}_{PTO} + \vec{F}_r + \vec{F}_s + \vec{F}_m + \vec{F}_v + \vec{F}_c$$

- $\vec{x} = [x, y, z, \phi, \theta, \psi]$ is the state vector which represents the 6 DoF displacement in the body-fixed frame.
- The mooring force is simulated by using MoorDyn.

- Docking station is modelled as a **drag body** in WecSim
- The hydrostatic force, cable reaction forces, and quadratic drag force are considered.
- The hydrodynamic forces are neglected given that they are deeply submerged in the water

Force between the WEC and the docking station



Docking Station		Suspending cable	
Mass (kg)	1064.621	Stiffness	1000000
Volume (m^3)	0.394	Damping	100
Inertia [I_x, I_y, I_z] ($kg\ m^2$)	[733.352, 733.352, 457.979]	Quadratic Drag Area (m^2)	[10, 10, 10, 0, 0, 0]
Quadratic Drag Area (m^2)	[2.25, 2.25, 2.25, 0, 0, 0]	Quadratic Drag Coefficient (Cd)	[1.4, 1.4, 1.4, 0, 0, 0]
Quadratic Drag Coefficient (Cd)	[1.2, 1.2, 1.2, 0, 0, 0]	Cable top coordinates in global frame (m)	[0, 0, -30]
Center of Gravity (m) in global frame	[0,0, -50]	Cable bottom coordinates in global frame (m)	[0, 0, -49.25]
Center of Buoyancy (m) in body fixed frame	[0, 0, 0]		

Methodology: UUV modeling

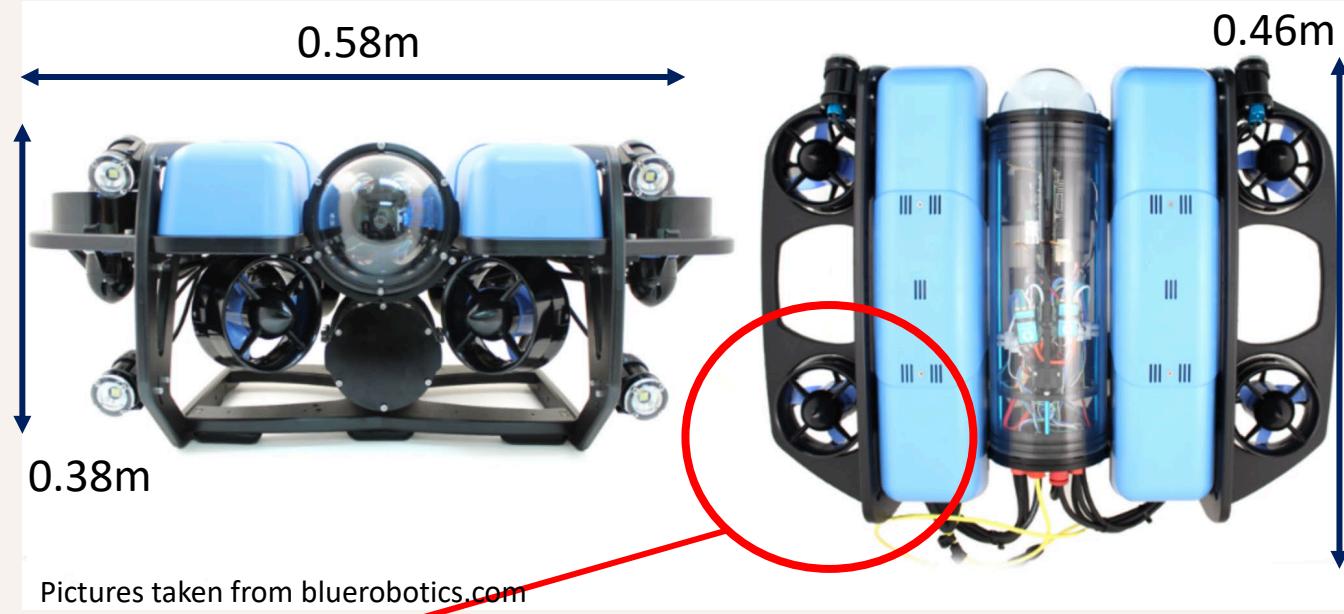
Nonlinear Dynamics of UUV

$$\dot{\vec{\eta}} = J(\vec{\eta})\vec{v}$$
$$\dot{\vec{M}\vec{v}} + \vec{C}(\vec{v}) + \vec{D}(\vec{v})\vec{v} + \vec{g}(\vec{\eta}) = \vec{\tau}$$

- $\vec{\eta} = [x, y, z, \phi, \theta, \psi]$ represents position of the UUV in the global frame
- $\vec{v} = [u, v, w, p, q, r]$ denotes the velocity of the vehicle expressed in the body-fixed frame.
- The model parameters are calibrated with experiments in [1].



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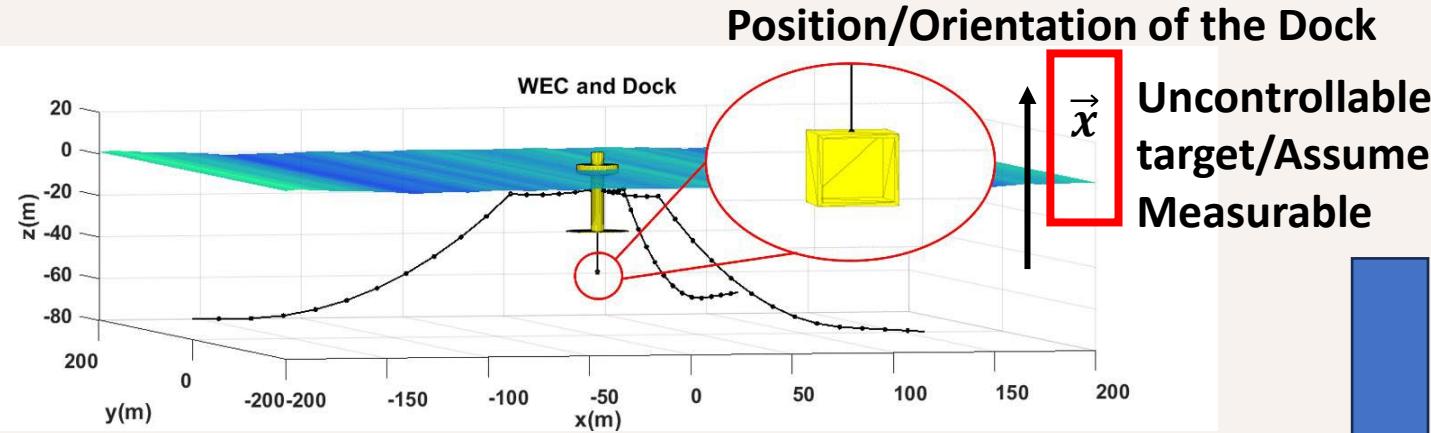


Direction	Max Thrust	
Surge	85 N	
Sway	85 N	
Heave	120 N	
Roll	26 Nm	
Pitch	14 Nm	
Yaw	22 Nm	

Mass (kg)	13.5
Moment of Inertia (kg m ²)	(0.26, 0.23, 0.37)
Damping Coefficients (Nsm ⁻¹)	(137.7, 0, 33, 0, 0.8, 0)
Quadratic Damping Coefficients (Ns ² m ⁻²)	(141, 217, 190, 1.19, 0.47, 1.5)
Added Mass (kg)	(6.36, 7.12, 18, 68)
Added Mass Moment (kg m ²)	(0.189, 0.135, 0.222)

[1] von Benzon, Malte, et al. "An open-source benchmark simulator: Control of a bluerov2 underwater robot." *Journal of Marine Science and Engineering* 10.12 (2022): 1898

Methodology: Docking control



Position/Orientation of the UUV



- The control problem can be formulated as:

$$\text{Min: } \vec{e} = \vec{\eta} - \vec{x}$$

Subject to:

$$\dot{\vec{x}}_1 = \mathbf{J}(\vec{x}_1)\vec{x}_2$$

$$\dot{\vec{x}}_2 = -\mathbf{M}^{-1}(\mathbf{C}(\vec{x}_2) + \mathbf{D}(\vec{x}_2)\vec{x}_2 + \mathbf{g}(\vec{x}_1)) + \Delta f(\vec{x}_1, \vec{x}_2) + \mathbf{M}^{-1}(\vec{\tau} + \delta(t, \vec{x}_1, \vec{x}_2, \vec{\tau}))$$

Control that needs to be designed

Model Uncertainties

Actuation disturbances

- It is difficult to design a linear control subject to this complex dynamics, which also cannot regulate the uncertainties and disturbances. An **input-output linearization technique** is therefore applied.



Methodology: Input-Output Linearization

Nonlinear state transformation

- Define state:

$$\vec{e}_1 = \vec{\eta} - \vec{x}$$

$$\vec{e}_2 = \dot{\vec{\eta}} - \dot{\vec{x}}$$

- According to the UUV dynamics before, we have:

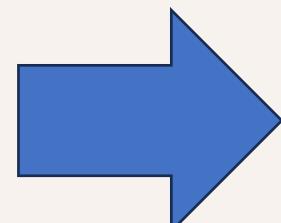
$$\dot{\vec{e}}_1 = \vec{e}_2$$

$$\dot{\vec{e}}_2 = \mathbf{S}_1 \mathbf{J}(\vec{x}_1) \vec{x}_2 + \mathbf{S}_2 \left(-\mathbf{M}^{-1} (\mathbf{C}(\vec{x}_2) + \mathbf{D}(\vec{x}_2) \vec{x}_2 + \mathbf{g}(\vec{x}_1)) + \Delta f(\vec{x}_1, \vec{x}_2) + \mathbf{M}^{-1} (\vec{\tau} + \delta(t, \vec{x}_1, \vec{x}_2, \vec{\tau})) \right) - \ddot{\vec{x}}$$

where

$$\mathbf{S}_1 = \frac{\partial \mathbf{J}(\vec{x}_1)}{\partial \vec{x}_1} (\vec{x}_2 \otimes I_n) + \mathbf{J}(\vec{x}_1) \frac{\partial \vec{x}_2}{\partial \vec{x}_1}$$

$$\mathbf{S}_2 = \frac{\partial \mathbf{J}(\vec{x}_1)}{\partial \vec{x}_2} (\vec{x}_2 \otimes I_n) + \mathbf{J}(\vec{x}_1) \frac{\partial \vec{x}_2}{\partial \vec{x}_2}$$



Obtained a clean and neat linearized system

$$\dot{\vec{e}} = \mathbf{A} \vec{e} + \Delta \mathbf{A} + \mathbf{B} \vec{u} + \Delta \delta$$

$$\mathbf{A} = \begin{bmatrix} 0_{6 \times 6} & I_6 \\ 0_{6 \times 6} & 0_{6 \times 6} \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0_{6 \times 6} \\ I_6 \end{bmatrix}$$

$$\Delta \mathbf{A} = \begin{bmatrix} 0_{6 \times 1} \\ \mathbf{S}_2 \Delta f \left(\vec{x}_1, \vec{x}_2 \right) \end{bmatrix}$$

$$\Delta \delta = \begin{bmatrix} 0_{6 \times 1} \\ \mathbf{S}_2 \mathbf{M}^{-1} \delta \left(t, \vec{x}_1, \vec{x}_2, \vec{\tau} \right) \end{bmatrix}$$

Transformed control, which we will design next

Apply the Input-Output Linearization control law:

$$\vec{\tau} = (\mathbf{S}_2 \mathbf{M}^{-1})^{-1} (-\mathbf{S}_1 \mathbf{J}(\vec{x}_1) \vec{x}_2 + \mathbf{S}_2 \mathbf{M}^{-1} (\mathbf{C}(\vec{x}_2) + \mathbf{D}(\vec{x}_2) \vec{x}_2 + \mathbf{g}(\vec{x}_1)) + \ddot{\vec{x}} + \vec{u})$$

Thrust force (physically)



Methodology: Robust LQR

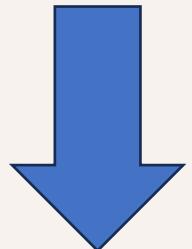
Now we need to design the \vec{u}

$$\dot{\vec{e}} = \vec{A}\vec{e} + \Delta\vec{A} + \vec{B}\vec{u} + \Delta\delta$$

Proposed control algorithm:

$$\begin{aligned}\vec{u}(t) &= \vec{u}_l(t) + \vec{u}_s(t) \\ \vec{u}_l(t) &= -\vec{R}^{-1}\vec{B}^T\vec{P}\vec{e}(t) \\ \vec{u}_s(t) &= -(\vec{G}\vec{B})^{-1}\gamma \text{ sign}(\vec{s})\end{aligned}$$

- $\vec{u}_l(t)$ is the continuous part which is used to optimize the performance of the nominal error system
- $\vec{u}_s(t)$ is the discontinuous part which guarantees the robustness.



Need to transform back to the actual thrust!



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$$J = \frac{1}{2} \int_0^T [\vec{e}_n(t)^T \vec{Q} \vec{e}_n(t) + \vec{u}_l(t)^T \vec{R} \vec{u}_l(t)] dt$$

Minimize the docking time

Minimize the fuel consumption

According to the optimal control theory:

$$\vec{u}_l(t) = -\vec{R}^{-1}\vec{B}^T\vec{P}\vec{e}_n(t)$$

- The \vec{P} represent the covariance matrix which is the solution of the algebraic Riccati equation (ARE).

- Sliding Mode Control is applied to regular the uncertainties and disturbances:

$$\vec{u}_s(t) = -(\vec{G}\vec{B})^{-1}\gamma \text{ sign}(\vec{s})$$

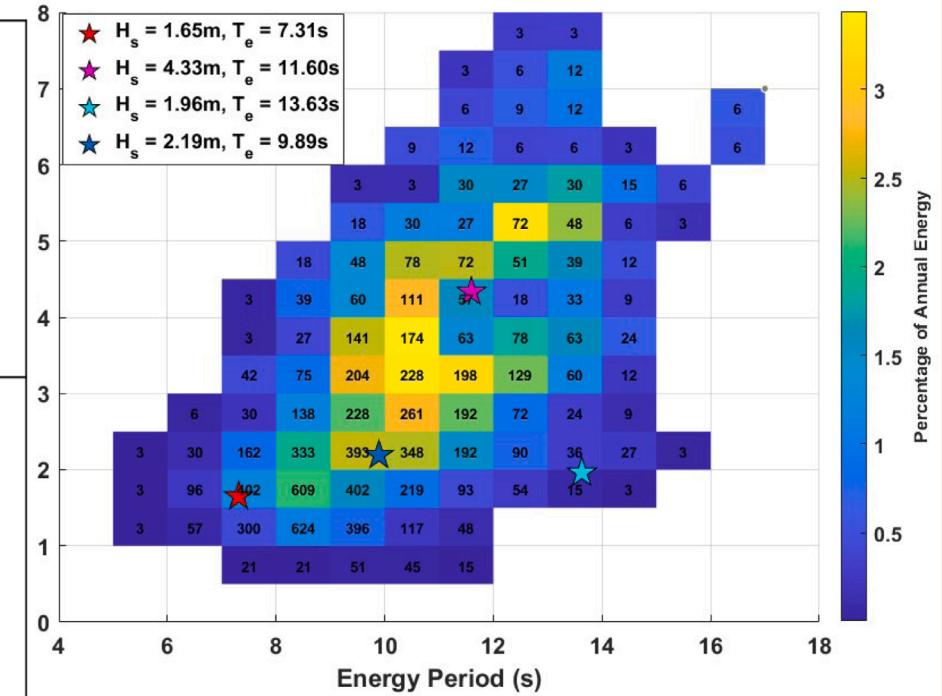
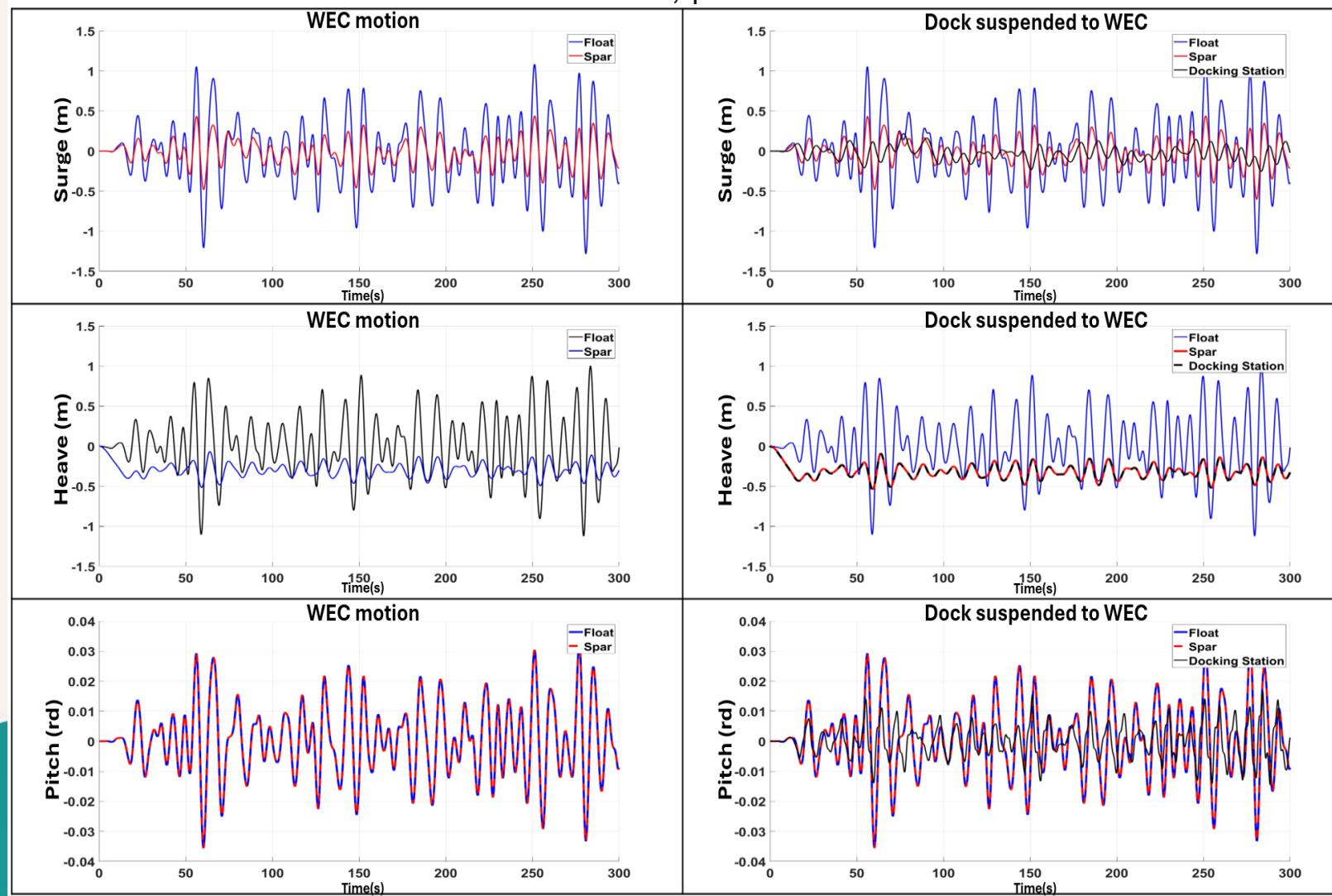
- where the sliding surface is designed as:

$$\begin{aligned}\vec{s}(t, \vec{e}) &= \vec{G}\vec{e}(t) - \vec{G}\vec{e}(0) - \vec{G} \int_0^t (\vec{A} - \vec{B}\vec{R}^{-1}\vec{B}^T\vec{P})\vec{e}(\tau) d\tau\end{aligned}$$

where $\vec{G} \in \mathbb{R}^{6 \times 12}$ is a constant matrix which is designed such that $\vec{G}\vec{B}$ is nonsingular.

Simulation results: Dock motion analysis

$H_s = 1.65\text{m}$, $T_p = 8.91\text{s}$

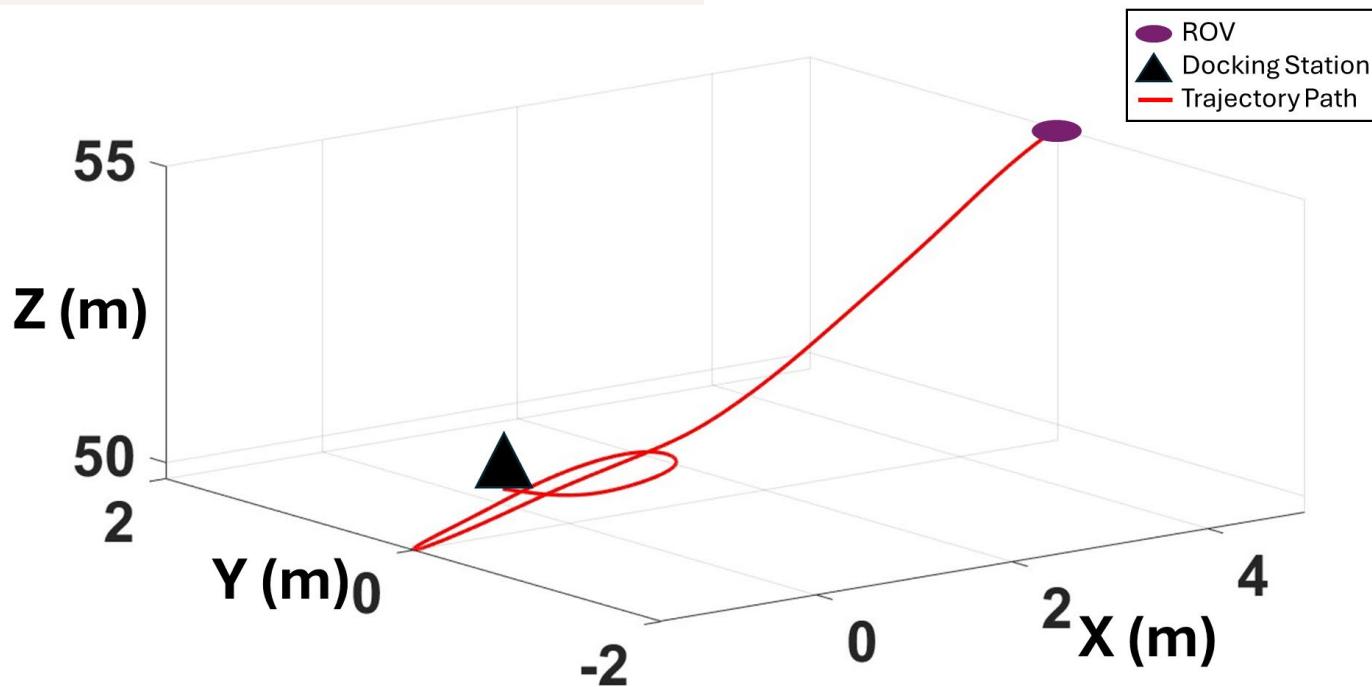


$H_s = 1.65\text{ m}$, $T_p = 8.91\text{s}$ $H_s = 4.33\text{ m}$, $T_p = 13.97\text{s}$

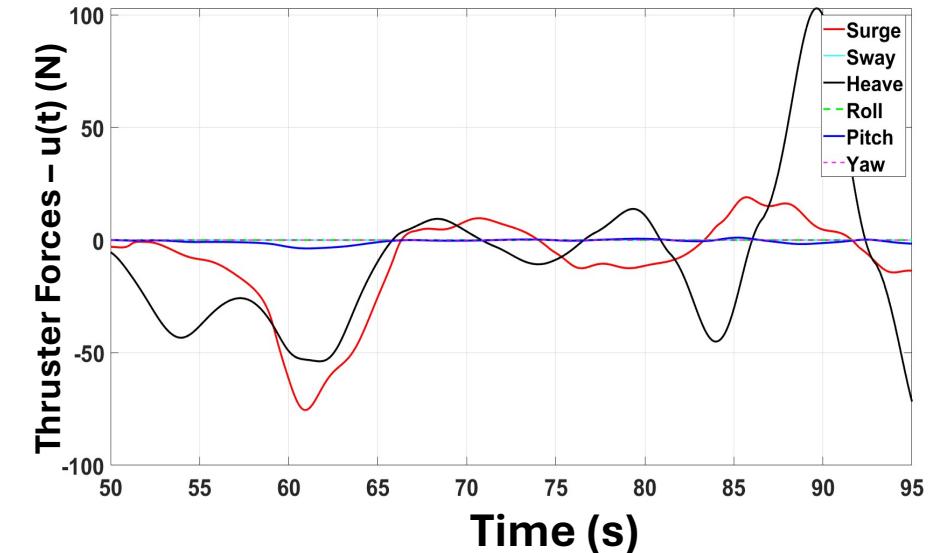
Surge (m)	$[0.26, -0.27]$	$[1.73, -1.98]$
Heave (m)	$[0, -0.358]$	$[1.02, -1.75]$
Pitch (rad)	$[0.0189, -0.176]$	$[0.0712, 0.773]$

Simulation results: Docking in ideal scenario

Docking trajectory of the UUV



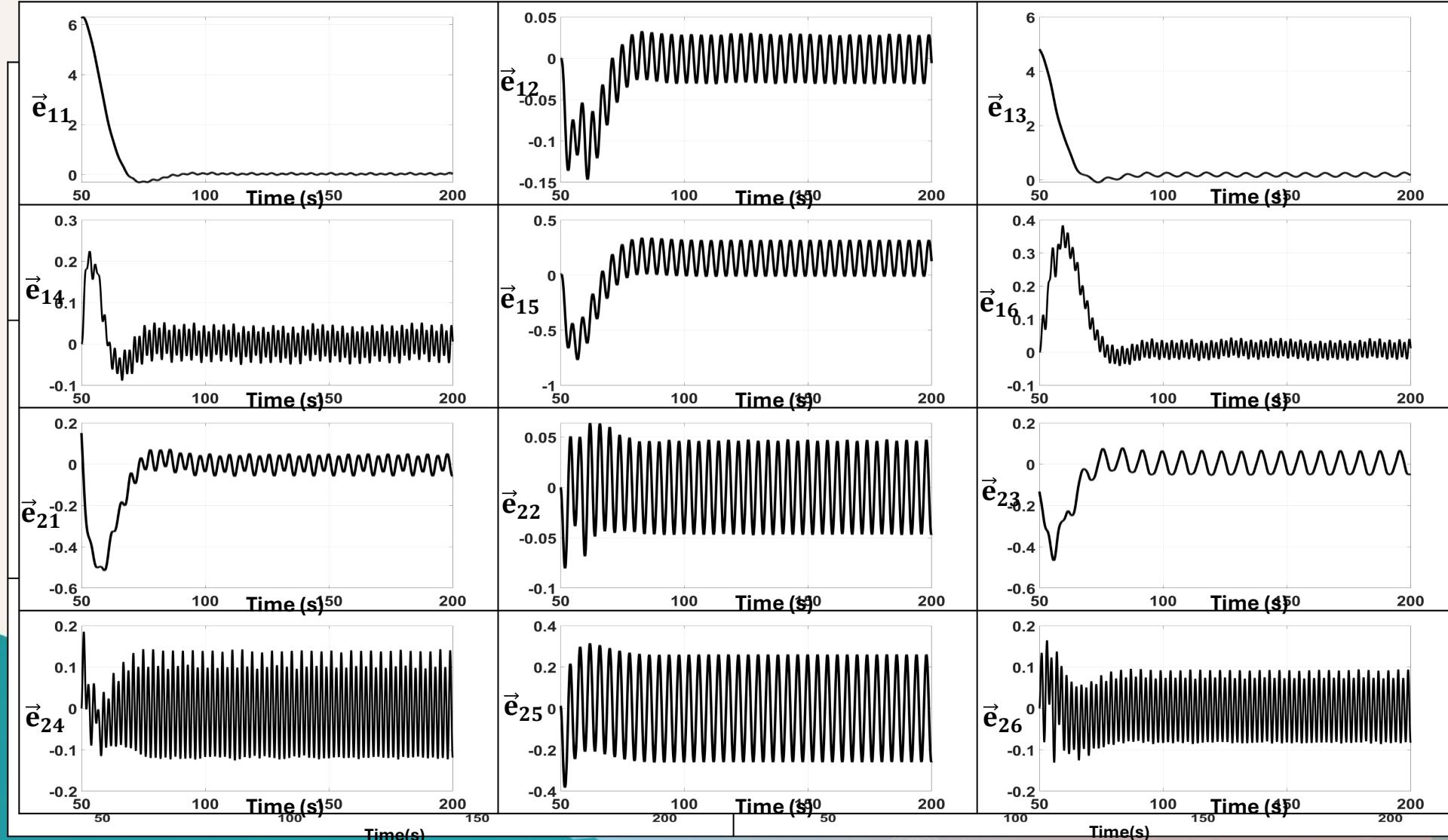
Thruster forces are within the limit



Simulation results: Docking with disturbances (LQR only)

Disturbance signals are added to the dynamics (all 6DoF) which are unaware to the control:

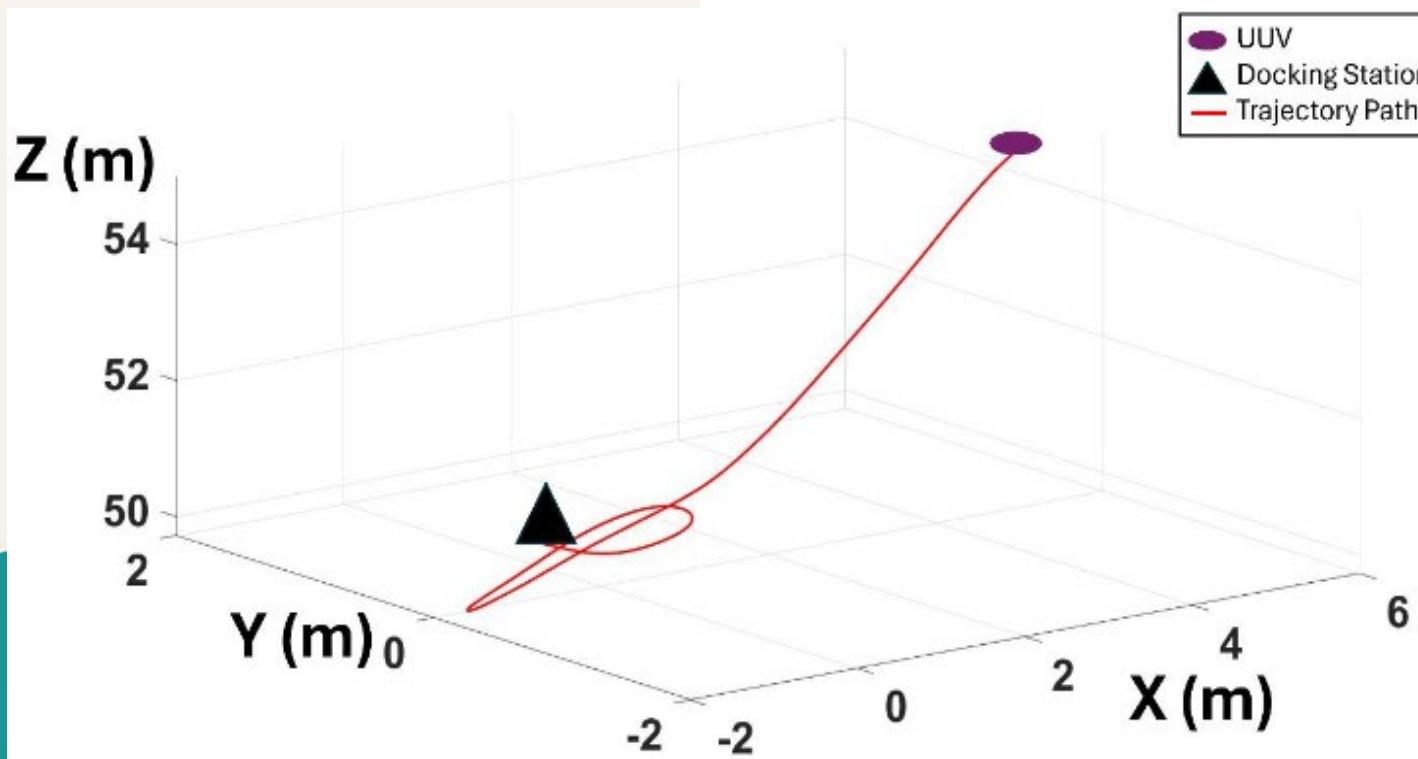
$$d(t) = [1.5 \sin 0.5\pi t ; 1.5 \sin 0.5\pi t ; 1.5 \sin 0.25\pi t ; 0.15 \sin \pi t ; 0.15 \sin 0.5\pi t ; 0.15 \sin \pi t]$$



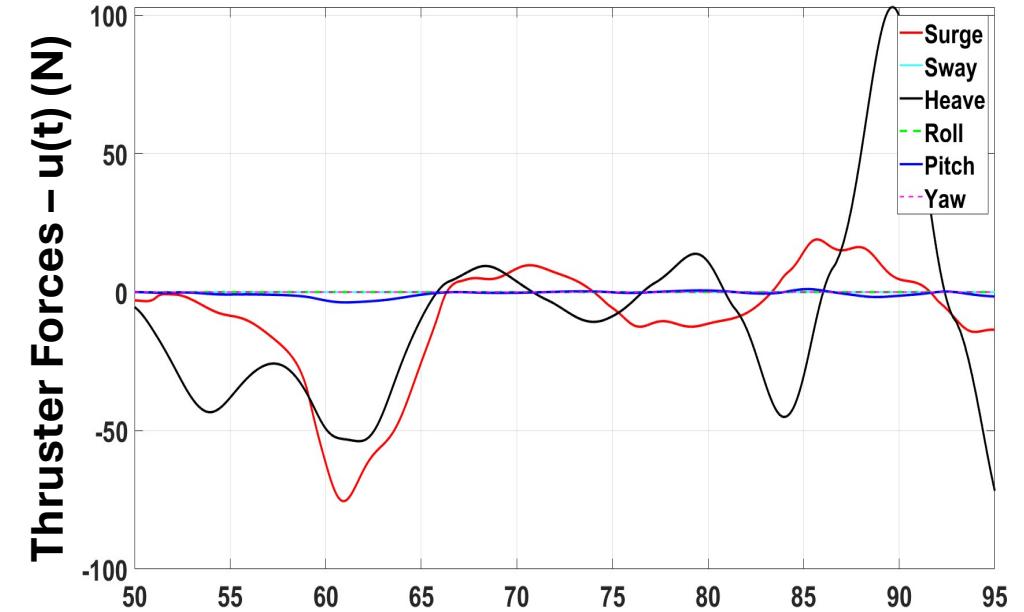
Simulation results: Docking with disturbances (RLQR)

Same disturbances are still applied in this case, but the RLQR control is applied

Docking trajectory of the UUV



Thruster forces are within the limit

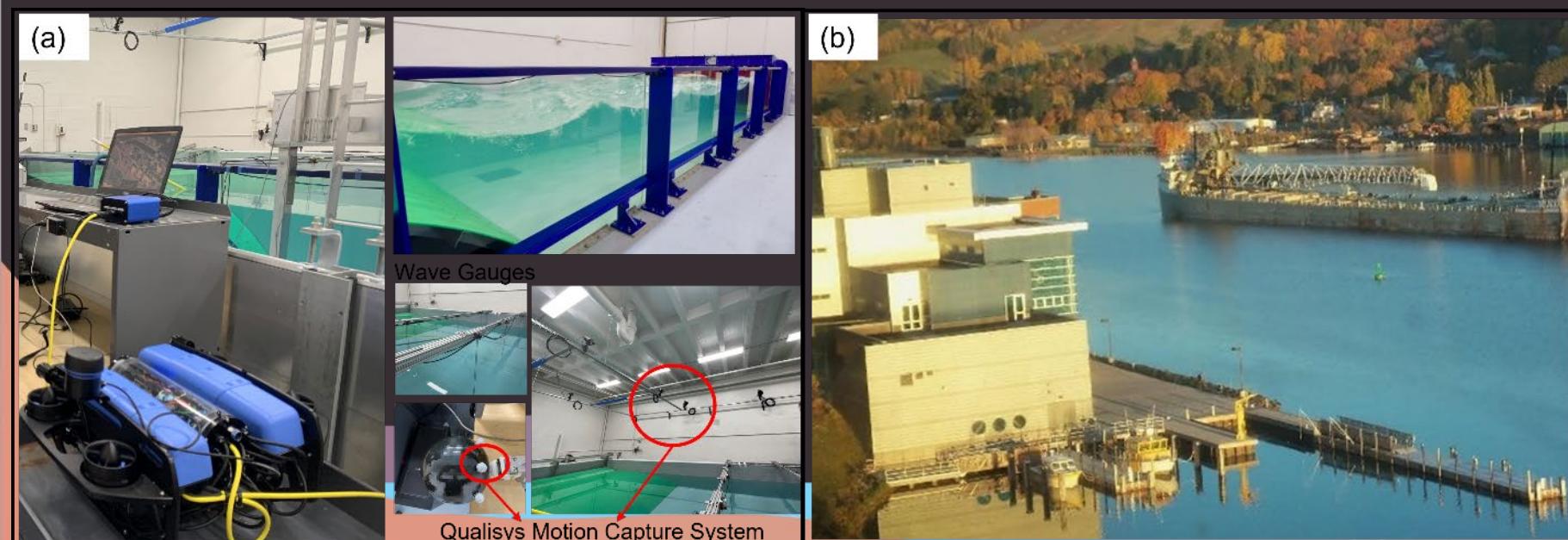


Conclusion

- Docking station motion is significant when it is hanged with the WEC which poses challenges for docking.
- It is critical to consider both the optimality and robustness in designing the docking control.
- ROC effectively controls the UUV to achieve an optimal docking performance in the presence of uncertainties and disturbances.

Future work

- The hydrodynamic model will be improved by incorporating the tether force and ocean current force.
- Experimentally validate the docking control in wave lab at MTU
- Apply UUV for mooring line inspections/fault diagnosis (camera vision-based)



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