Hydrodynamic Characteristics Investigation of Multiple Floating Bodies under Phase Control

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1 Introduction

The motion modes of two rigid barges connected by a hinge joint, and of an infinite array of bodies were extensively studied by Newman [1]. Other researchers went deeper into the issue of arrays of multiple floating bodies using various methodologies [2, 3]. Zhang *et al.* [4] notably demonstrated that the motions and performance of such arrays can be effectively computed in the time domain, even when accounting for interactions between devices. The aspects of particular interest in these studies are the overall stability of the floating array and the effectiveness of energy absorption by the Power Take-Off (PTO) systems between the multiple floating bodies. The relationship between the objective function in phase control and its corresponding control effect is still under exploration.

2 Theory and Methodology

2.1 Dynamic Equations of Multiple Floating Bodies

The theoretical model of N floating bodies without connections is depicted in Fig. 1. The global coordinate o-xyz is a right-handed Cartesian coordinate with x-y axes in the horizontal water plane and z axis oriented in the upward direction. Each body can be simplified as a mass point at their centres of gravity (CoG). The hydrodynamic parameters and motion responses are described in body-fixed coordinates o^j-$x^jy^jz^j$, where j corresponds to the j-th body.

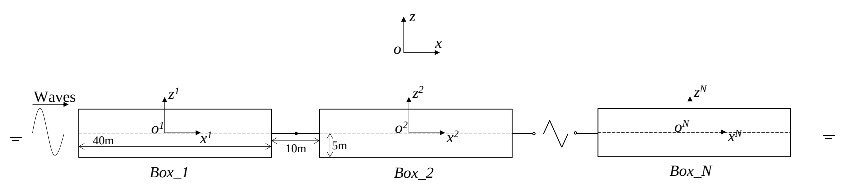


Figure 1: Configuration of N hinged boxes.

The water depth, h, is assumed to be infinite, and the waves are propagating towards the positive x-axis throughout the computation. When considering the nonlinear wave surface memory effect, the time-domain motion equation of multiple floating bodies in wave is,

$$(\boldsymbol{M}+\boldsymbol{m})\ddot{\boldsymbol{\eta}}(t)+\int_0^t \boldsymbol{h}_{\mathrm{r}}(t\text{-}\tau)\dot{\boldsymbol{\eta}}(\tau)d\tau+\boldsymbol{K}\boldsymbol{\eta}(t)=\boldsymbol{f}_{\mathrm{e}}(t) \tag{1}$$

$\boldsymbol{\eta}$, $\dot{\boldsymbol{\eta}}$ and $\ddot{\boldsymbol{\eta}}$ are the displacement, velocity and acceleration vectors respectively, where the motion of the j-th body is defined as $\boldsymbol{\eta}^j=\left[\eta_1^j;\ \eta_2^j;\ \eta_3^j;\ \eta_4^j;\ \eta_5^j;\ \eta_6^j\right]=\left[x^j;\ y^j;\ z^j;\ \varphi^j;\ \theta^j;\ \psi^j\right]$, representing the surge motion, sway motion, heave motion, roll angle, pitch angle, and yaw angle respectively.

According to the constraint matrix method, the motion equation of the mechanically connected multi-body system can be derived with the coefficient matrix of constraints, S. $\boldsymbol{\eta}$, $\dot{\boldsymbol{\eta}}$, $\ddot{\boldsymbol{\eta}}$ can be replaced by $\boldsymbol{\eta}'$, $\dot{\boldsymbol{\eta}}'$, $\ddot{\boldsymbol{\eta}}'$:

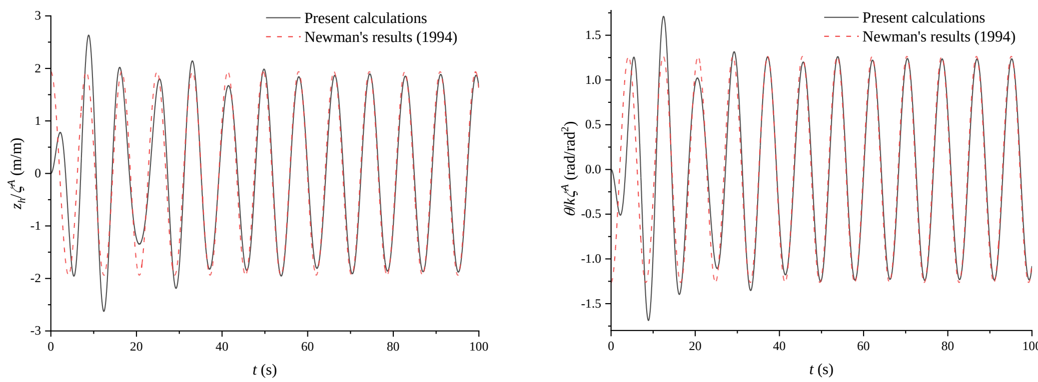


Figure 2: The time series for normalised heave motion (left) and angular deflection (right) at hinge point.

$$(\boldsymbol{M}+\boldsymbol{m})\boldsymbol{S}\ddot{\boldsymbol{\eta}}'(t)+\int_0^t \boldsymbol{h}_{\mathrm{r}}(t-\tau)\boldsymbol{S}\dot{\boldsymbol{\eta}}(\tau)d\tau+\boldsymbol{K}\boldsymbol{S}\boldsymbol{\eta}'(t)=\boldsymbol{f}_{\mathrm{e}}(t)+\boldsymbol{f}_{\mathrm{PTO}}(t)+\boldsymbol{f}_{\mathrm{h}}(t) \tag{2}$$

In the coordinate of multi-body system, the connection forces $\boldsymbol{f}_{\mathrm{h}}$ are internal forces. In order to eliminate the internal hinge forces $\boldsymbol{f}_{\mathrm{h}}$, multiplies the matrix $\boldsymbol{S}^{\mathrm{T}}$ at both sides of Eq. (2):

$$\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{M}+\boldsymbol{m})\boldsymbol{S}\ddot{\boldsymbol{\eta}}'(t)+\boldsymbol{S}^{\mathrm{T}}\int_0^t \boldsymbol{h}_{\mathrm{r}}(t-\tau)\boldsymbol{S}\dot{\boldsymbol{\eta}}(\tau)d\tau+\boldsymbol{S}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{S}\boldsymbol{\eta}'(t)=\boldsymbol{S}^{\mathrm{T}}\boldsymbol{f}_{\mathrm{e}}(t)+\boldsymbol{S}^{\mathrm{T}}\boldsymbol{f}_{\mathrm{PTO}}(t) \tag{3}$$

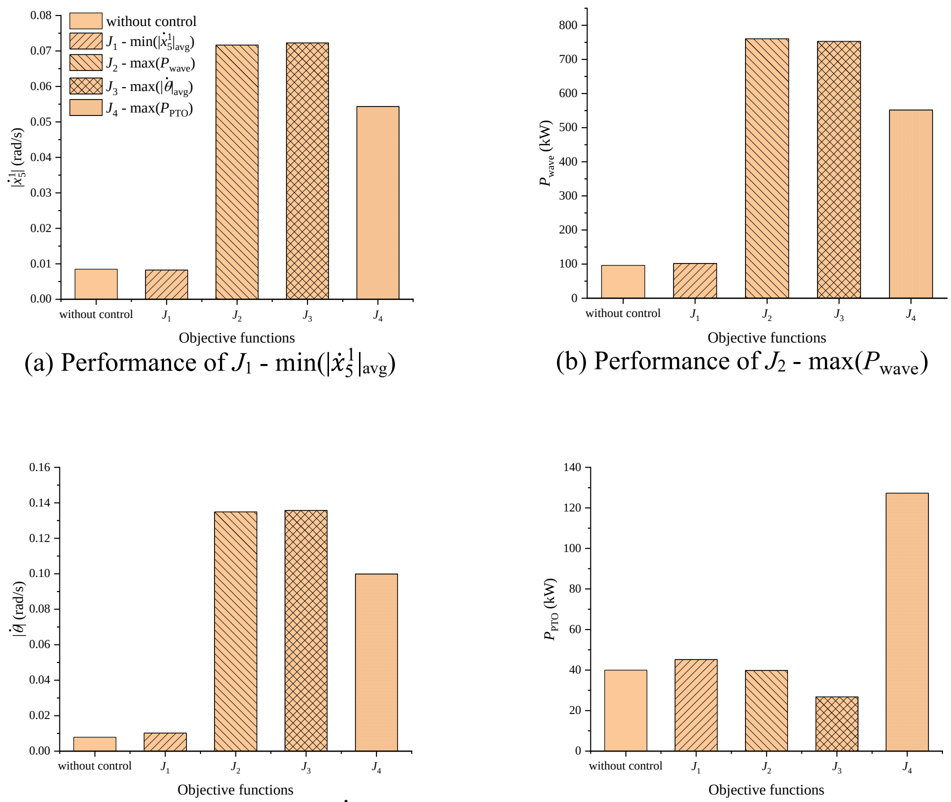
2.2 Optimal Phase Control Theory

In discrete phase control, the control command $\beta(t)$ is binary, which means the command is either 0 or 1. The control force $\boldsymbol{f}_{\mathrm{PTO}}(t)=\beta(t)B_{\mathrm{PTO}}\dot{\theta}(t)$ is also discrete. In order to minimise or maximise the target objective function, we need to minimise or maximise the Hamiltonian, H, calculated by the state of the system:

$$\begin{aligned} &H=L+\boldsymbol{\lambda}\left(\gamma\cdot\boldsymbol{\eta}'+\zeta\right) \\ &\dot{\boldsymbol{\lambda}}=-\frac{\partial H}{\partial \boldsymbol{\eta}'}=-\frac{\partial L}{\partial \boldsymbol{\eta}'}-\boldsymbol{\lambda}\gamma \end{aligned} \tag{4}$$

where $\boldsymbol{\lambda}$ is the Lagrange multiplier and L is the Lagrangian function, i.e., the performance index. By solving the value of $\boldsymbol{\lambda}$, it is possible to derive the Hamiltonian H that includes β. Maximising H needs the term that contains β to be positive or 0, while minimising H needs the term containing β to be negative or 0. After β is determined, the updated responses $\boldsymbol{\eta}'$ with control can be computed. The responses with control are introduced to the iteration as the initial state until the results converges and reaches its numerical optimum. The objective function $J=\frac{1}{T}\int_0^T L dt$ is defined as a physical value that is optimised during the numerical optimisation process in the time interval [0, T], representing the performance of the system in a period T. J reaches its maximum or minimum value corresponding to the maximisation or minimisation of H. To serve different objectives in varying optimisation scenarios, several alternative objective functions in the case of two hinged boxes (N=2) are investigated, which are expressed by:

- $J_1=\left|\dot{x}_5^1\right|_{\mathrm{avg}}=\frac{1}{T}\int_0^T\left|\dot{x}_5^1(t)\right|dt$
- $J_2=P_{\mathrm{wave}}=\frac{1}{T}\int_0^T\{f_{\mathrm{e},5}^1(t)\dot{x}_5^1(t)+f_{\mathrm{e},5}^2(t)\dot{x}_5^2(t)\}dt$
- $J_3=\left|\dot{\theta}\right|_{\mathrm{avg}}=\frac{1}{T}\int_0^T\left|\dot{\theta}(t)\right|dt$
- $J_4=P_{\mathrm{PTO}}=\frac{1}{T}\int_0^T\beta(t)B_{\mathrm{PTO}}(t)\dot{\theta}(t)^2dt$



(a) Performance of J_1 - min($|\dot{x}_5^1|_{avg}$)

(b) Performance of J_2 - max(P_{wave})

(c) Performance of J_3 - max($|\dot{\theta}|_{avg}$)

(d) Performance of J_4 - max(P_{PTO})

Figure 3: Performance of J_1 to J_4 when applying different objectives.

When J_1 is applied, the physical meaning is that the average pitch speed of *Box_1* is minimised. This corresponds to a scenario of stabilising floating platforms or substructures. When J_2 is applied, the total power of wave force, $f_{e,5}\dot{x}_5$, is maximised. When J_3 is applied, the average relative angular speed of rotation between the two boxes is maximised. When J_4 is applied, the energy absorption of PTO is maximised. It corresponds to a scenario of increasing the power output of wave energy converters which harness relative motion between sections. The selection of the objective function depends on the optimisation goal.

3 Results and Discussions

This section validates the established hinged multiple floating bodies model with the results of two hinged boxes in Neman's research [1]. In regular waves, $\boldsymbol{f}_e^j=[\zeta^A f_W^j \sin(\omega_W t+\varepsilon_i^j)]^T$, i=1, 2, ⋯6, are the components of the j-th wave excitation force vector, where f_W is the wave force transfer function; ζ^A is the incoming wave amplitude; ω_W is the angular wave frequency; ε_i^j are the phases of harmonic components of a periodic wave. The responses for heave and hinge deflection when the wave frequency is specified as 0.76 rad/s are computed in the time domain. The result in Newman's research at a wave frequency of 0.76 rad/s is transformed into the time domain using Fourier transformation. Figure 2 shows a comparison of the results of the present study and Newman's research, showing good agreement after convergence.

The following computations are also based on the model of two hinged boxes. When wave frequency is 1 rad/s, the time-averaged results under optimal phase control are shown in Fig. 3. Fig. 3 compares the performance of different objective functions when applying four corresponding objectives. The results show that each objective function can effectively minimise or maximise its corresponding objective. It is also found that each performance index can only be optimised when applying its corresponding objective function J. The results under other objective functions are suboptimal.

Figure 4 illustrates the numerical optimisation effects of different performance indices in different wave frequencies. The results are optimised under their respective objective functions. In general, Fig. 4 shows when the optimal phase control is applied to the system, all the objectives can be achieved. However, it should be noted that the effect of the control depends on the wave conditions. It indicates that in certain wave conditions, the control strategy proposed in this study can be effectively applied to control the multiple floating bodies with different application scenarios.

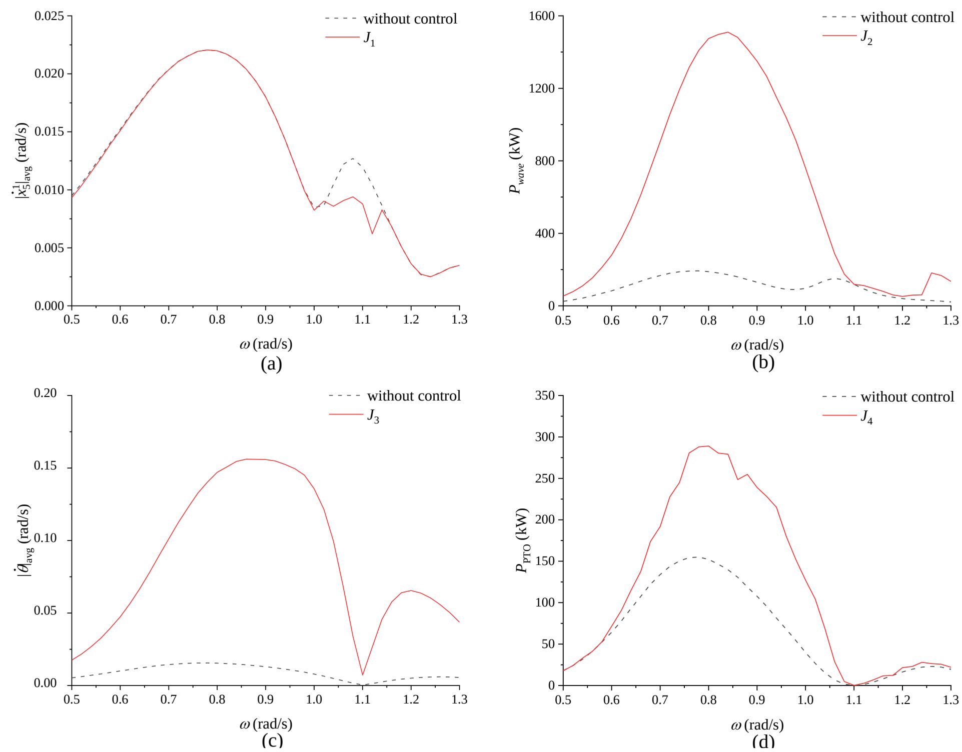


Figure 4 Performances of different objective functions in different wave frequencies.

4 Conclusions

The effect of different phase control strategies on multiple floating bodies is presented. When applied to two rigid boxes with a hinge connection, the phase control can either enhance or reduce the wave energy absorption depending on its objective functions effectively. Different control strategies, especially objective functions, can lead to variations in energy conversion efficiency within a floating multi-body system. The effectiveness of optimal phase control is highly dependent on the system's hydrodynamic parameters and wave conditions.

References

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