Hydrodynamic performance of a U-shaped oscillating water column consisting of a flexible bottom-standing front wall

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1 Introduction

A novel U-shaped oscillating water column device (UOWC) is proposed in this paper, where the front bottom-standing wall is considered to be flexible rather than rigid. To study the performance of the flexible UOWC, a theoretical model is developed using linear potential flow theory, dry mode expansion, and eigenfunction expansion methods. A Galerkin approximation approach is adopted to deal with the strong singularities at the sharp edges of the device's front bottom-standing and front surface-piercing wall. Our results show that three peaks of the frequency response of the maximum wave power absorption efficiency can be obtained, two of which are determined by the natural frequencies of the effective oscillating water column and the 1st natural mode of the flexible wall, respectively, and the remaining one is due to wave near-trapping.

2 Mathematical model

Fig. 1 shows a sketch of the device studied in this paper. A flexible bottom-standing wall with a height of l is placed in front of an OWC chamber in water depth h. The OWC chamber is of width a with the draft of the front wall denoted as d_1. The spacing distance between the front wall and the flexible wall is b. A power take-off (PTO) system consisting of a Wells turbine is installed at the top of the chamber to capture wave power, and the PTO damping is denoted as c_{PTO}.

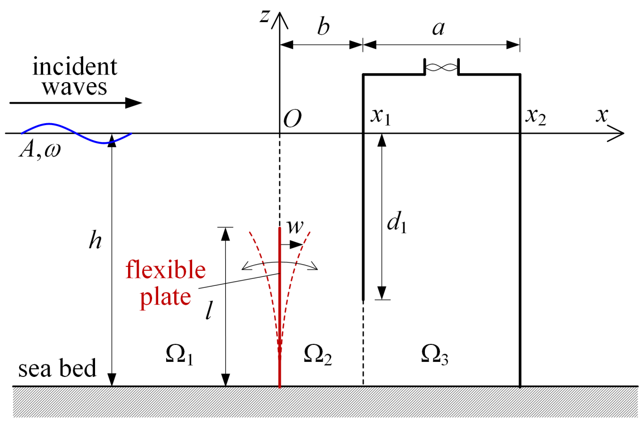

Figure 1: Sketch of the U-shaped OWC consisting of a flexible bottom-standing wall.

The fluid is assumed to be incompressible and inviscid, the flow motion is irrotational and described by a velocity potential $\Phi(x, z, t) = \text{Re}[\phi(x, z)exp(-\text{i}\omega t)]$, where ω denotes the angular frequency of the incident waves and t the time. Similarly, the dynamic air pressure inside the chamber $P(t)$ and the deflection of the flexible wall $W(z,t)$ for $z \in [-h, -d_0]$, where $d_0 = h - l$, can be expressed as $P(t) = \mathrm{Re}[pexp(-\mathrm{i}\omega t)]$ and $W(z,t) = \mathrm{Re}[w(z)exp(-\mathrm{i}\omega t)]$, respectively.

The velocity potential should satisfy the dynamic equation of the flexible wall

$$EI\partial^4 w/\partial z^4 - \omega^2 \rho_p h_p w = \mathrm{i}\omega\rho(\phi|_{x=0^-} - \phi|_{x=0^+}), \tag{1}$$

where EI represents the flexural rigidity, in which E is the Young's modulus of the wall material, $I = h_p^3/12$ is the area moment of inertia of the wall; h_p the thickness of the flexible wall; ρ_p density of the wall.

By using the dry mode expansion method, $w(z)$ may be further expanded as $w(z) = \sum_{n=1}^{\infty} W_n w_n(z)$, where W_n is the complex amplitude of the nth natural mode denoted by $w_n(z)$. $w_n(z)$ satisfies $\partial^4 w_n/\partial z^4 - \kappa_n^4 w_n = 0$, where $\kappa_n^4 = \omega_n^2 \rho_p h_p/(EI)$ and ω_n is the relative eigenfrequency, and the boundary conditions at the two ends of the flexible wall

$$w_n|_{z=-h} = \partial w_n/\partial z|_{z=-h} = 0, \quad \text{and} \quad \partial^2 w_n/\partial z^2|_{z=-d_0} = \partial^3 w_n/\partial z^3|_{z=-d_0} = 0. \tag{2}$$

The solution to the above gives

$$w_n(z) = \cos[\kappa_n(z+h)] - \cosh[\kappa_n(z+h)] - \xi_n\{\sin[\kappa_n(z+h)] - \sinh[\kappa_n(z+h)]\}, \tag{3}$$

where $\xi_n = [\cos(\kappa_n l) + \cosh(\kappa_n l)]/[\sin(\kappa_n l) + \sinh(\kappa_n l)]$; $\kappa_n \in \mathbb{R}^+$ are the solutions of $\cosh(\kappa_n l)\cos(\kappa_n l) + 1 = 0$ and are numbered in ascending order of their magnitude.

The velocity potential is decomposed into an incident wave potential, ϕ_I, a diffracted wave potential, ϕ_D, and a series of radiated wave potentials due to the air pressure change inside the OWC chamber and the deflection of the flexible wall as

$$\phi = \phi_{-1} + p\phi_0 + \sum_{n=1}^{\infty} \dot{W}_n \phi_n, \tag{4}$$

where $\phi_{-1} = \phi_I + \phi_D$, in which $\phi_I = -\mathrm{i}gA\mathrm{e}^{\mathrm{i}kx}\cosh[k(z+h)]/[\omega\cosh(kh)]$, A is the incident wave amplitude, k the wave number, and g the gravitational acceleration.

ϕ_0 describes the fluid motion due to unit-amplitude of dynamic pressure change inside the chamber in the absence of incident waves and deflection of the flexible wall; $\dot{W}_n = -\mathrm{i}\omega W_n$, and ϕ_n for $n > 0$ denotes the radiated wave velocity potential due to the unit-amplitude of the velocity change of the flexible wall in the nth mode.

ϕ_{-1} shall satisfy the Laplace equation in the fluid domain with $\partial\phi_{-1}/\partial z|_{z=-h} = 0$ and $\partial\phi_{-1}/\partial z|_{z=0} = K\phi_{-1}|_{z=0}$, where $K = \omega^2/g$, and $\partial\phi_{-1}/\partial x = 0$ on the walls of the UOWC. Additionally, the far-field radiation boundary condition $\partial\phi_D/\partial x = -\mathrm{i}k\phi_D$ should be satisfied at $x \to -\infty$.

The above-mentioned conditions should also be satisfied by ϕ_0 except on the free surface inside the chamber, $z = 0$, $x \in [x_1, x_2]$ with $x_1 = b$, $x_2 = a + b$, with internal boundary conditions

$$\partial\phi_0/\partial z = K\phi_0 + \mathrm{i}\omega/(\rho g). \tag{5}$$

Note, ϕ_n for $n > 0$ satisfy the above-mentioned conditions with an exception at the front bottom-standing wall, i.e., $x = 0$, $z \in [-h, -d_0]$, where $\partial\phi_n/\partial x = w_n(z)$.

The fluid domain is divided into three regions, Ω_1 ($x \in (-\infty, 0^-], z \in [-h, 0]$), Ω_2 ($x \in [0^+, x_1^-], z \in [-h, 0]$), and Ω_3 ($x \in [x_1^+, x_2], z \in [-h, 0]$). The expressions of the velocity potential ϕ_n at these regions are therefore

$$\phi_n = \begin{cases} \sum_{j=0}^{\infty} A_{n,j} \mathrm{e}^{\lambda_j x} Z_j(z) + \delta_{-1,n}\phi_I, & \in \Omega_1 \\ \sum_{j=0}^{\infty} \left(B_{n,j}\mathrm{e}^{\lambda_j x} + C_{n,j}\mathrm{e}^{-\lambda_j x}\right) Z_j(z), & \in \Omega_2 \\ \sum_{j=0}^{\infty} D_{n,j}\left(\mathrm{e}^{\lambda_j (x-x_2)} + \mathrm{e}^{-\lambda_j (x-x_2)}\right) Z_j(z) - \dfrac{\mathrm{i}\delta_{n,0}}{\rho\omega}, & \in \Omega_3 \end{cases} \tag{6}$$

where $A_{n,j}$, $B_{n,j}$, $C_{n,j}$, and $D_{n,j}$ are unknown coefficients to be determined,

$$Z_j(z) = \cos[\lambda_j(h+z)]/\sqrt{N_j},\ N_j = 0.5\left[1 + \sin(2\lambda_j h)/(2\lambda_j h)\right], \tag{7}$$

in which λ_j are solutions of $K + \lambda_j g \tan(\lambda_j h) = 0$, with $\lambda_0 = -\mathrm{i}k$ corresponds to the progressive waves, and $\lambda_j \in \mathbb{R}^+$ for $j > 0$ related to the evanescent waves numbered in ascending order of their magnitude. The eigenfunctions $Z_j(z)$ form a complete set of orthogonal functions over $[-h, 0]$.

The fluid velocity and the hydrodynamic pressure are square-root singular at the edge of a thin wall. We adopted the Galerkin approximation approach proposed by Porter & Evans (1995) to incorporate the known null velocity potential jump and the square-root behavior of velocity at the edge of a thin wall. The continuity conditions of the velocity and pressure at the interfaces of the three regions should be satisfied and can be turned into a system of algebraic equations. The unknown coefficients can be obtained after solving the algebraic system numerically.

The upward flux at the water surface inside the OWC chamber due to the contributions of ϕ_{-1}, i.e., so-called the excitation volume flow, and the upward flux at the water surface inside the OWC chamber due to the contributions of the radiated velocity potential ϕ_n $(n \geq 0)$ can be written as

$$\begin{aligned} F_{e,0} &= \int_{x_1}^{x_2} \partial\phi_{-1}/\partial z|_{z=0} \mathrm{d}x = K\int_{x_1}^{x_2} \phi_{-1}|_{z=0}\mathrm{d}x, \\ F_{0,n}^{R} &= \int_{x_1}^{x_2} \partial\phi_n/\partial z\big|_{z=0}\mathrm{d}x = \int_{x_1}^{x_2} \left[K\phi_n|_{z=0} + \delta_{n,0}\mathrm{i}\omega/(\rho g)\right]\mathrm{d}x = -(c_{0,n} - \mathrm{i}\omega m_{0,n}), \end{aligned} \tag{8}$$

where $c_{0,n}$ and $m_{0,n}$ are the hydrodynamic coefficients related to the upward flux inside the OWC chamber due to the radiated waves induced by the dynamic change of the air pressure inside the chamber $(n = 0)$ and the flexible wall's motion in nth mode $(n > 0)$.

The generalized wave excitation force acting on the nth mode of the flexible wall $(n > 0)$ and the wave radiation force acting on the nth mode of the flexible wall $(n > 0)$ due to radiated velocity potential ϕ_j $(j \geq 0)$ may be expressed as

$$\begin{aligned} F_{e,n} &= \mathrm{i}\omega\rho\int_{-h}^{-d_0} \left(\phi_{-1}|_{x=0^-} - \phi_{-1}|_{x=0^+}\right) w_n(z)\mathrm{d}z, \\ F_{n,j}^{R} &= \mathrm{i}\omega\rho\int_{-h}^{-d_0} \left(\phi_j|_{x=0^-} - \phi_j|_{x=0^+}\right) w_n(z)\mathrm{d}z = -(c_{n,j} - \mathrm{i}\omega m_{n,j}), \end{aligned} \tag{9}$$

where $c_{n,j}$ and $m_{n,j}$ are the hydrodynamic coefficients related to the wave radiation force acting on the nth mode of the flexible wall due to the radiated waves induced by the dynamic change of the air pressure inside the OWC chamber $(j = 0)$ and the oscillation of the flexible wall in the jth mode $(j > 0)$.

After multiplying w_ζ ($\zeta > 0$) by the dynamic equation of the flexible wall, integrating over $z \in [-h, -d_0]$ and utilizing the orthogonality of the natural modes, together with the pneumatic power take-off (PTO) dynamic equation, we have

$$
\begin{aligned}
(c_{PTO} + c_{0,0} - \mathrm{i}\omega m_{0,0})p + \sum_{n=1}^{\infty}(c_{0,n} - \mathrm{i}\omega m_{0,n})\dot{W}_n &= F_{e,0}, \\
W_\zeta(EI\kappa_\zeta^4 - \omega^2\rho_p h_p)l + (c_{\zeta,0} - \mathrm{i}\omega m_{\zeta,0})p + \sum_{n=1}^{\infty}(c_{\zeta,n} - \mathrm{i}\omega m_{\zeta,n})\dot{W}_n &= F_{e,\zeta}.
\end{aligned} \tag{10}
$$

The time-averaged absorbed wave power $P_e = 0.5c_{PTO}|p|^2$ can be nondimensionalized as wave power capture efficiency, η. An optimized PTO damping $c_{PTO} = c_{opt}$ can be predicted, resulting in the maximum efficiency, η_{max}.

3 Results

Fig. 2 illustrates a comparison between the performance of a flexible UOWC, a rigid UOWC, in which the bottom-standing wall now is rigid $EI/(\rho g h^4) = 10^6$, and an OWC with a rather small bottom-standing wall, $l/h = 0.01$, using $\rho_p/\rho = 0.4526$, $h_p/h = 0.0148$, $a/h = 0.4237$, $b/h = 0.2119$, $d_1/h = 0.7615$. There is merely one peak of the η_{max} curve in the computed range of wave conditions for the OWC case. A similar result is also observed for the rigid UOWC case but with its position moving towards low frequencies due to the increase of the length of the effective oscillating water column. As the front bottom-standing wall becomes flexible, interestingly, three peaks of the η_{max} curve are observed, in which the first peak around $kh = 0.65$ is dominated by the natural frequency of the effective oscillating water column, the third around $kh = 1.96$ is determined by the natural frequency related to the 1st natural mode of the flexible wall, and the second one at $kh = 1.37$ is due to wave near-trapping.

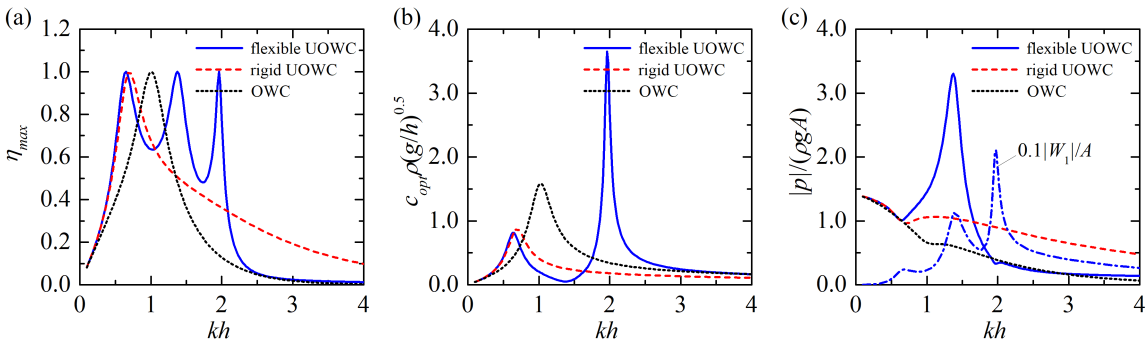

Figure 2: Frequency responses of (a) η_{max}; (b) $c_{opt}\rho(g/h)^{0.5}$; and (c) $|p|/(\rho g A)$, in which $|W_1|/A$ for the flexible UOWC is also included. Flexible UOWC: $EI/(\rho g h^4) = 0.0203$, $l/h = 0.7096$; rigid UOWC: $EI/(\rho g h^4) = 10^6$; OWC: $l/h = 0.01$.

Acknowlegement: The authors acknowledge support from the University of Plymouth Centre for Decarbonisation and Offshore Renewable Energy (CDORE) Seedcorn Funding.

References

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